# THE REPEATED PRISONER'S DILEMMA



# **REAL LIFE GAMES:** HOW GAME THEORY SHAPES HUMAN

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# How did the Prisoner's Dilemma come about?



MELVIN DRESHER



Amidst concern about nuclear war and political instability, we devised a little game to show that the Nash equilibrium sometimes makes strange predictions.



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	Game	AA	JW	AA's comments	JW's comments
	1	D	Ċ	JW will play (D)— sure win. Hence if I play [C)—I lose.	Hope he's bright.
	2	D	с	What is he doing?!!	He isn't but maybe he'll wise up.
	3	D	D	Trying mixed?	Okay, dope.
	4	D	D	Has he settled on [D]?	Okay, dope.
_	5	с	D	Perverse!	It isn't the best of possible worlds.
	6	D	с	I'm sticking to [D] since he will mix for at least 4 more times.	Oh ho! Guess I'll have to give him another chance.
	7	D	С		Cagey, ain't he? Well
	8	D	D		In time he could learn, but not in te moves so:

Poundstone, W. (1993). Prisoner's Dilemma: John Von Neumann, Game Theory and the Puzzle of the Bomb. Anchor Books.





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**MELVIN DRESHER** For all the confusion, mutual cooperation occurred 60 out of the 100 trials.







John Nash You know guys...

Playing the Prisoner's Dilemma once is not the same as playing it 100 times in succession.

Playing the game over and over again is like playing a different, multi-round game.

And repeating the game might open the door for cooperation...

# Cooperation involves providing a *benefit* to someone else, at a personal *cost*.

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also called altruism

## **SELF-SACRIFICING ANTS**

At night, colonies of the Brazilian ant *Forelius pusillus* retreat into their nest underground.

- For protection, they seal off the entrance to the nest with sand.
- Every night, a few ants remain outside to finish the job.
- Unable to survive outside the nest, these ants die.
- So as not to alert predators to the location of the nest, they scatter around at a distance.

Tofilski, A., Couvillon, M. J., Evison, S. E. F., Helanterä, H., Robinson, E. J. H., & Ratnieks, F. L. W. (2008). Preemptive defensive self-sacrifice by ant workers. *The American Naturalist*, 172(5), 239-243.

Raihani, N. (2022). The Social Instinct: What Nature Can Teach Us About Working Together. Vintage.





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## NICOLA RAIHANI From the cells that make up organs inside our bodies, to social insects, to the hyper-social species that is *Homo* sapiens.

Raihani, N. (2022). The Social Instinct: What Nature Can Teach Us About Working Together. Vintage.



# Humans are also capable of selfless behavior.

## **SELF-SACRIFICING HUMANS**

Postman's Park, in London, features moving testimonies of acts of self-sacrifice.

Postman's Park, King Edward St, London EC1A 7BT, United Kingdom

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In the simplest scenario we model the dilemma of cooperation with the Donation Game.





A cooperator pays a cost c for the other player to receive a benefit b, with b > c > 0.

A defector does not pay any cost, and provides no benefit.

Nowak, M.A. (2006). Evolutionary Dynamics. Belknap Press







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Coope

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		payoffs
	Cooperate	Defect
erate	b - c, b - c	- <i>c</i> , <i>b</i>
efect	<i>b</i> , - <i>c</i>	0, 0

# Clearly, the Donation Game is a type of Prisoner's Dilemma.

Clearly, the Donation Game is a type of Prisoner's Dilemma. And we've seen what happens there.



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Defect	<i>b</i> , - <i>c</i>	0,  0

pure Nash equilibria
(Cooperate, Cooperate)
(Cooperate, Defect)
(Defect, Cooperate)
(Defect, Defect)

Does cooperation ever make sense in Prisoner's Dilemma-type situations?

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Does cooperation ever make sense in Prisoner's Dilemma-type situations? It might make sense to pay a cost today, if that means I get something back. Tomorrow...



A cooperator pays a cost c for the other player to receive a benefit b, with b > c > 0.

A defector does not pay any cost, and provides no benefit.

The game is repeated for a known, finite number *k* of rounds.

Payoffs are the sum of payoffs from each round.



Round 1

Round k

•••

payoffs

payoffs1CooperateDefect1Cooperateb - c, b - c-c, bDefectb, -c0, 0



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$$u_i(s_1, s_2) = \sum_t u_i^t(s_1, s_2)$$
# But what is a strategy in this repeated setting?

But what is a strategy in this repeated setting? It is a specification of the player's action at each round.

Game is repeated for a grand total of two rounds.



Round

Round

			payoffs
		Cooperate	Defect
1	Cooperate	b - c, b - c	- <i>c</i> , <i>b</i>
•	Defect	<i>b</i> , - <i>c</i>	0, 0
		Coorento	Defect

		Cooperate	Defect
2	Cooperate	b - c, b - c	-c, b
_	Defect	<i>b</i> , - <i>c</i>	0,  0

#### pure Nash equilibria

Game is repeated for a grand total of two rounds.

Take the benefit to be 3, and the cost 1.



			payoffs
		Cooperate	Defect
Round 1	Cooperate	2, 2	-1, 3
	Defect	3, -1	0, 0
	L	:	
	_	Cooperate	Defect
Round 2	Cooperate	2, 2	-1, 3
	Defect	3, -1	0, 0
	pı	ure Nash e	equilibria

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Suppose both players' strategies are to always cooperate (ALLC):



Round

Round

.....

			payoffs
		Cooperate	Defect
1	Cooperate	2,  2	-1, 3
	Defect	3, -1	0, 0
	L		
		Cooperate	Defect
2	Cooperate	2, 2	-1, 3
	Defect	3, -1	0,  0
	рι	ure Nash	equilibria
		(A)	LLC, ALLC)
			2/2

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		(AL	LC, ALLC)
			2/2

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Suppose now one of the players defects at the last round:

ALLC: 
$$\begin{array}{c} 2 & -1 \\ c & c \end{array} > 2 - 1 = 1 \\ \begin{array}{c} 2 \\ c \\ c \end{array} > 2 - 1 = 1 \end{array}$$
  
C, D:  $\begin{array}{c} 2 \\ c \\ c \\ c \end{array} > 2 + 3 = 5 > 4 \end{array}$ 



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		Cooperate	Defect
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	Defect	3, -1	0, 0
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			2/2

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Suppose now one of the players defects at the last round:



Obviously, this is a profitable deviation.







Anticipating this, the other player will want to cut their losses by also defecting.



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So at the last round both players get 0.



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But now the same reasoning applies at the penultimate round.



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Assuming players are rational, they end up always defecting!

Reasoning from the end of the game leads both players to always defect (ALLD):







#### **ROBERT AUMANN**

Assuming players do what's best for them at every point in the game, including the very end, leads to subgame perfect equilibria.

And no trust, hence no cooperation, in the Prisoner's Dilemma.

# No progress so far.



#### ROBERT AUMANN But what if we assume players don't actually know when the game ends?

#### **DEFINITION (DISCOUNTING)**

Assume there is a probability  $\delta$ , called the *discount factor*, that the game is played again at round t + 1, given that it was played at round  $t \ge 1$ .

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Assume there is a probability  $\delta$ , called the *discount factor*, that the game is played again at round t + 1, given that it was played at round  $t \ge 1$ .

Payoffs are calculated as expected values, depending on  $\delta$ .



INDEFINITELY REPEATED VERSION

There are two players, each with two actions: Cooperate or Defect.

A cooperator pays a cost c for the other player to receive a benefit b, with b > c > 0.

A defector does not pay any cost, and provides no benefit.

The game is repeated for an indefinite number of rounds, with a new round taking place with probability  $\delta$ .

Payoffs are the infinite sums depending on  $\delta$ .

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			payoffs
		Cooperate	Defect
nd 1	Cooperate	2,2	-1, 3
o. 1)	Defect	3, -1	0, 0
	L	Cooperate	Defect
d 2	Cooperate	2, 2	-1, 3
ο. δ)	Defect	3, -1	0, 0
	L		
		Cooperate	Defect
d k	Cooperate	2,2	-1, 3
$\delta^{k-1}$ )	Defect	3, -1	0, 0
	L		·
offs	$u_i$	$s(s_1, s_2) = \sum_t s_t$	$u_i^t(s_1, s_2) \cdot \delta^{t-1}$

# When computing payoffs, the following sum is useful:

 $1 + \delta + \delta^2 + \dots = \frac{1}{1 - \delta}.$ 

## **INDEFINITE ROUNDS EXAMPLE**



Suppose both players' strategies are to always cooperate (ALLC):



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			payoffs
		Cooperate	Defect
nd 1	Cooperate	2,2	-1, 3
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#### ROBERT AUMANN Assume players can condition their strategies on the other player's past actions.



**ROBERT AUMANN** Assume players can condition their strategies on the other player's past actions.

And players commit to punish defection with eternal defection.

## **GRIM TRIGGER**

Suppose both players' play Grim Trigger (GRIM) strategies: start by cooperating; if the other player defects, defect forever starting with the next round.



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Suppose a player deviates by defecting at some round k:



# Is such a deviation profitable?

## **GRIM TRIGGER EQUILIBRIUM CONDITION**

The deviation is not worth it just in case:

 $2 + 2\delta + 2\delta^2 + \dots + 2\delta^{k-1} + 2\delta^k + \dots > 2 + 2\delta + 2\delta^2 + \dots + 2\delta^{k-1} + 3\delta^k$ 

#### iff

## **GRIM TRIGGER EQUILIBRIUM CONDITION**

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$$2 + 2\delta + 2\delta^2 + \dots + 2\delta^{k-1} + 2\delta^k + \dots \ge 2 + 2\delta + 2\delta^k + 2\delta^k + 2\delta^{k+1} + \dots \ge 3\delta^k$$
$$2\delta + 2\delta^2 + \dots \ge 1$$
$$2\delta(1 + \delta + \dots) \ge 1$$

# $2\delta^2 + \dots + 2\delta^{k-1} + 3\delta^k$ iff iff iff iff iff iff

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$$2\delta + 2\delta^{2} + \dots \ge 1$$
$$2\delta(1 + \delta + \dots) \ge 1$$
$$2\delta \cdot \frac{1}{1 - \delta} \ge 1$$
$$\delta \ge \frac{1}{3}.$$

### $2\delta^2 + \dots + 2\delta^{k-1} + 3\delta^k$ iff iff iff iff iff



We've just shown that as long as the chance of the game continuing is high enough, cooperation is an equilibrium.



# **ROBERT AUMANN** And all we need is players being willing to punish each other mercilessly.

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**ROBERT AXELROD** sensible strategies.

- But we can get similar effects with more
- Like Tit for Tat (TFT)! Start by cooperating, then repeat the other player's last action.



**ROBERT AUMANN** And all we need is players being willing to punish each other mercilessly.



- But we can get similar effects with more
- Like Tit for Tat (TFT)! Start by cooperating, then repeat the other player's last action.
- That is, use reciprocity: reward kindness with kindness, and punish defection with

**ROBERT AXELROD** sensible strategies.

defection.



# For general payoffs, we get cooperation at equilibrium as long as:

 $\delta \ge \frac{c}{b}.$
# For general payoffs, we get cooperation at equilibrium as long as: $\delta \ge \frac{c}{h}$ . But the story doesn't end here...



Cooperation is everywhere among living things.



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**???** But how, when everyone is in it only for themselves?



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Game theory is the perfect tool to study the puzzle of cooperation.



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The challenge is to find plausible mechanisms that can facilitate the emergence of cooperation.



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or selection based on kinship and group membership.



trust and reputation...



punishments and rewards...



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