#### LUDWIG-MAXIMILIANS-UNIVERSITÄT MÜNCHEN MAY 12, 2025

#### REAL LIFE GAMES: HOW GAME THEORY SHAPES HUMAN DECISIONS

## GAME THEORY THE ULTIMATUM GAME & PARETO OPTIMALITY

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## Shall we play another game?

#### The Ultimatum Game

Player 1 has an endowment.

They propose a split with Player 2.

If Player 2 accepts, they divide the money according to the proposed split.

If Player 2 rejects, they both get nothing.



## How do we think through this?

Player 2, as a self-interested agent, should accept any split where they get more than 0.



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Knowing this, Player 1 will offer the smallest possible amount to Player 2, and keep the difference.



## What about Nash equilibria?

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#### The Ultimatum Game 🚰

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If Player 2 accepts, they divide the money according to the proposed split.

If Player 2 rejects, they both get nothing.

Let's say the endowment is 4, and consider two possible splits.



When we switch to the table view, we get more states.

Subtle, but important: Player 2's actions are the moves it would make at *every* choice node of the tree. Even if the game doesn't reach that node.

				payoffs
	Yes, Yes	Yes, No	No, Yes	No, No
3, 1)	3,1	3,1	0, 0	0, 0
2, 2)	2,2	0, 0	2,2	0, 0

#### pure Nash equilibria

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Note that the Nash equilibria reflect this.

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	Yes, Yes	Yes, No	No, Yes	No, No	
(3, 1)	3,1	3,1	0,  0	0, 0	
(2, 2)	2,  2	0,  0	2,2	0, 0	
pure Nash equilibria					
	(3, 1), (Yes, Yes)				

(2, 2), (No, Yes)

2/2

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#### PURE NASH EQUILIBRIA FOR ULTIMATUM GAME





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The profile ((2, 2), (No, Yes)) is an equilibrium *because* Player 2 commits to refusing a (3, 1) split, *if* it occurs.

This is like a threat that Player 2 makes.

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	Yes, Yes	Yes, No	No, Yes	No, No	
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pure Nash equilibria					
	(3, 1), (Yes, Yes) (3, 1), (Yes, No)				

(2, 2), (No, Yes)

2/2

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Though we might wonder whether such a threat is credible.

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pure Nash equilibria				
(3, 1), (Yes, Yes) (3, 1), (Yes, No)				

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2/2

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The strongest rationality assumption\* is that Player 2's threat (or commitment) is not credible. In other words, that they will never leave money on the table. This narrows down the set of equilibria.

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#### THE ULTIMATUM GAME IN EXPERIMENTS

Interestingly, in experiments people do not behave according to this prediction.

Early experiments found that Proposers offered, on average, around 40% - 50% of the total amount.

Güth, W., Schmittberger, R., & Schwarze, B. (1982). An experimental analysis of ultimatum bargaining. *Journal of Economic Behavior & Organization*, 3(4), 367–388.

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Early experiments found that Proposers offered, on average, around 40% - 50% of the total amount.

Interestingly, Responders were willing to pay a cost to punish unfair splits: offers below 20-30% of the total sum were frequently rejected.

Güth, W., Schmittberger, R., & Schwarze, B. (1982). An experimental analysis of ultimatum bargaining. *Journal of Economic Behavior & Organization*, 3(4), 367–388.

Joe Henrich and colleagues tested out people in 15 communities across the world.



Henrich, J., Boyd, R., Bowles, S., Camerer, C., Fehr, E., Gintis, H., & McElreath, R. (2001). In Search of Homo Economicus: Behavioral Experiments in 15 Small-Scale Societies. *The American Economic Review*, 91(2), 73–78.

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In some groups (e.g. the Machiguenga), low offers were common and often accepted.

Lamalera Aché Pittsburgh Shona Orma Achuar Sangu Gnau Tsimane Kazakh Torguud Mapuche Hadza Machiguenga Quichua



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In others (e.g., the Lamalera, Indonesian whale hunters), proposers gave more than 50%, sometimes offering hyper-fair splits. Responders rejected low offers harshly.

Lamalera Aché Pittsburgh

Shona

Sangu

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Societies with more cooperative labor (e.g., group fishing or hunting) tend to offer and expect fairer splits. Market integration also matters.

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Sangu

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## In social dilemmas there's something weird about the equilibria: everyone hates them, and would prefer a different outcome.

#### Vilfredo Pareto 1848 - 1923

Mathematician and many other things.





#### VILFREDO PARETO How about we look at outcomes where people are (jointly) as well-off as they can be.



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In a Pareto optimal outcome no one can be made better off without making someone else worse off.

#### **PARETO DOMINATION & OPTIMALITY**

**DEFINITION (PARETO DOMINATION)** A strategy profile *s* Pareto dominates strategy profile s' if:

(i)  $u_i(s) \ge u_i(s')$ , for every agent *i*, and

(ii) there exists an agent j such that  $u_j(s) > u_j(s')$ .



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**DEFINITION (PARETO OPTIMALITY)** 

A strategy profile s is *Pareto optimal* if there is no (other) strategy profile s' that Pareto dominates s.



#### PARETO OPTIMALITY INTUITION

Pareto optimality doesn't necessarily mean an outcome is *fair*. Just that the pie is distributed equally.

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If a sum of 4 is distributed among two players, then (2, 1) is dominated by (2, 2) and by (3, 1).

But (2, 2) and (3, 1) are both Pareto optimal.

What dominates what in the Prisoner's Dilemma?

		payoff table
	Cooperate	Defect
Cooperate	-20, -20	-100, 0
Defect	0, -100	-50, -50
	pure Na (Coopera (Coo (Defe (Pareto optin	ash equilibria ate, Cooperate) perate, Defect) ect, Cooperate) Defect, Defect) nal outcomes

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Everything *but* the Nash equilibrium is Pareto optimal!

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#### PARETO OPTIMALITY IN THE TRUST GAME

What dominates what in the Trust Game?



	p	ayoff table
	Кеер	Share
Кеер	1,1	1, 1
Invest	0, 4	2, 2
	pure Nash	equilibria
	(К	еер, Кеер)
Pare	to optimal	outcomes
		2/2

#### PARETO OPTIMALITY IN THE TRUST GAME

What dominates what in the Trust Game?

(Keep, Keep) and (Keep, Share) are dominated by (Invest, Share).

(Invest, Keep) and (Invest, Share) are not dominated by anything.





All these games are examples of *social dilemmas*.

#### **SOCIAL DILEMMAS**

#### DEFINITION

A social dilemma is a situation in which individual incentives are at odds with group incentives. Individual rationality leads members of a group to an outcome that is suboptimal.

> Carpenter, J., & Robbett, A. (2022). *Game Theory and Behavior*. MIT Press. Dawes, R. M. (1980). Social Dilemmas. Annual Review of Psychology, 31 (80), 169–193.

#### SOCIAL DILEMMAS

#### DEFINITION

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More formally, a social dilemma is a game in which the equilibria are Pareto dominated by some other outcome.

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## Social dilemmas show up a lot.

## Social dilemmas show up a lot. They're the reason we can't have nice things.



#### LANCE ARMSTRONG Sports people face a social dilemma when deciding whether to take performance enhancing drugs.

Schneier, B. (2006, August 10). <u>Drugs: Sports' Prisoner's Dilemma</u>. *Wired*.



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#### Or countries deciding whether to cut down carbon

THE UN own carbon emissions.





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JON HAIDT Social media use is similar: it makes teens miserable, but everyone is locked in for fear of missing out.

> Haidt, J. (2024). The Anxious Generation: How the Great Rewiring of Childhood Is Causing an Epidemic of Mental Illness. Penguin Books.

THE UN emissions



# Can't we just expect that players will gravitate towards a Pareto-optimal outcome?

#### PARETO IS FRAGILE

Supposing players end up in a situation where both cooperate, they each have a strong incentive to defect.

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#### **PARETO IS FRAGILE**

Supposing players end up in a situation where both cooperate, they each have a strong incentive to defect.

Pareto-optimal outcomes may not survive, in the long run.



## Nash equilibria are not necessarily good.

## Nash equilibria are not necessarily *good*. They're just hard to escape, if end up in them.