LMU LUDWIG-MAXIMILIANS-UNIVERSITÄT MÜNCHEN

JUNE 16, 2025

NETWORKED MINDS: OPINION DYNAMICS AND COLLECTIVE INTELLIGENCE IN SOCIAL NETWORKS

NODE CENTRALITY

Adrian Haret
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The centrality of a node measures the importance of the node, as a function of its connections.

DEFINITION (DEGREE CENTRALITY)

The *degree centrality** $C_d(i)$ of node i is the number of nodes connected to i:

$$C_d(i) = d_i.$$

*Note that this definition applies to undirected networks.

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To make it easier to compare centralities across different networks, we can normalize it relative to the total number n of nodes:

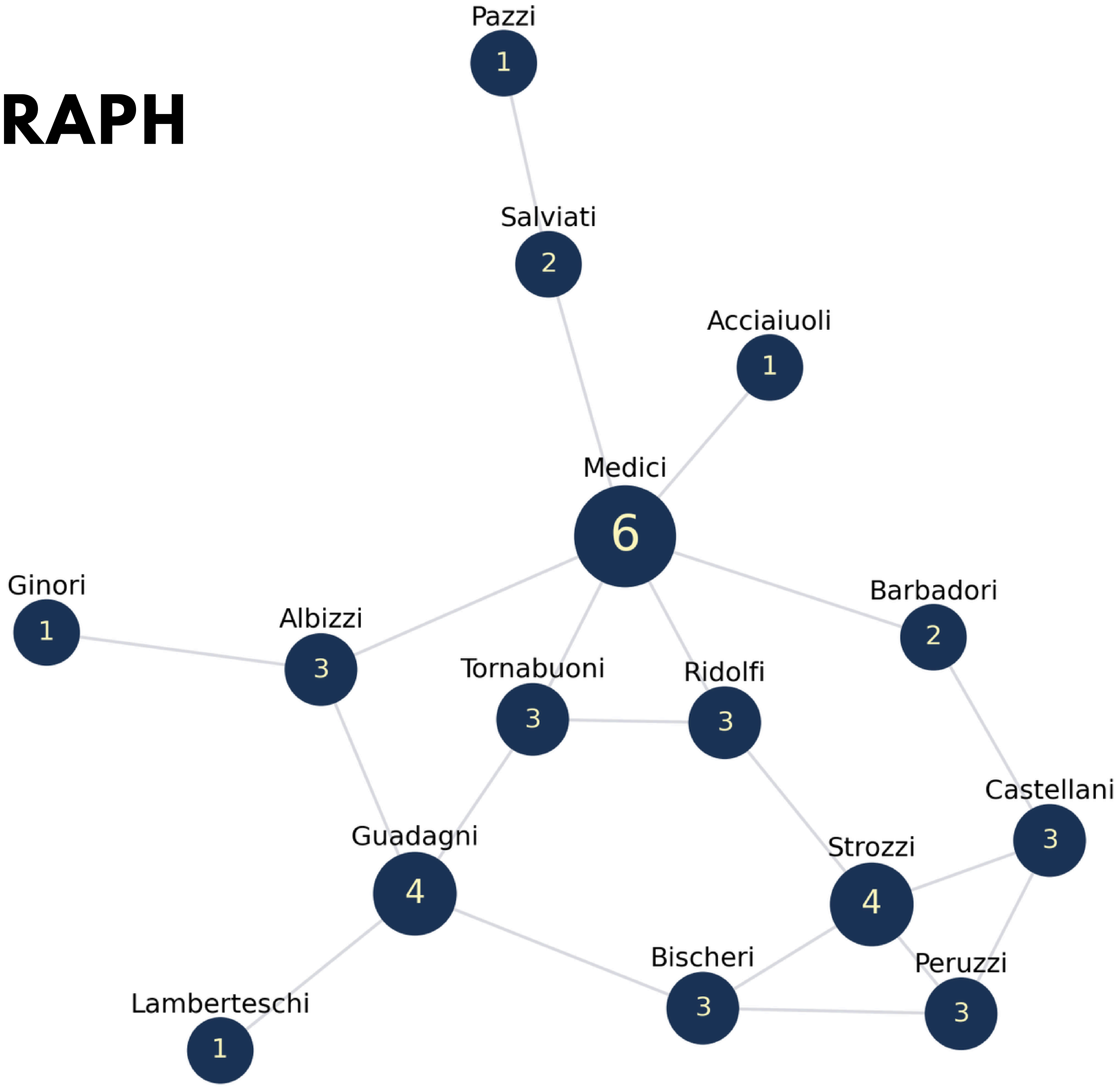
$$C_d(i) = \frac{d_i}{n-1}.$$

*Note that this definition applies to undirected networks.

FLORENTINE FAMILIES GRAPH

Degrees

The Medici have the highest degree.



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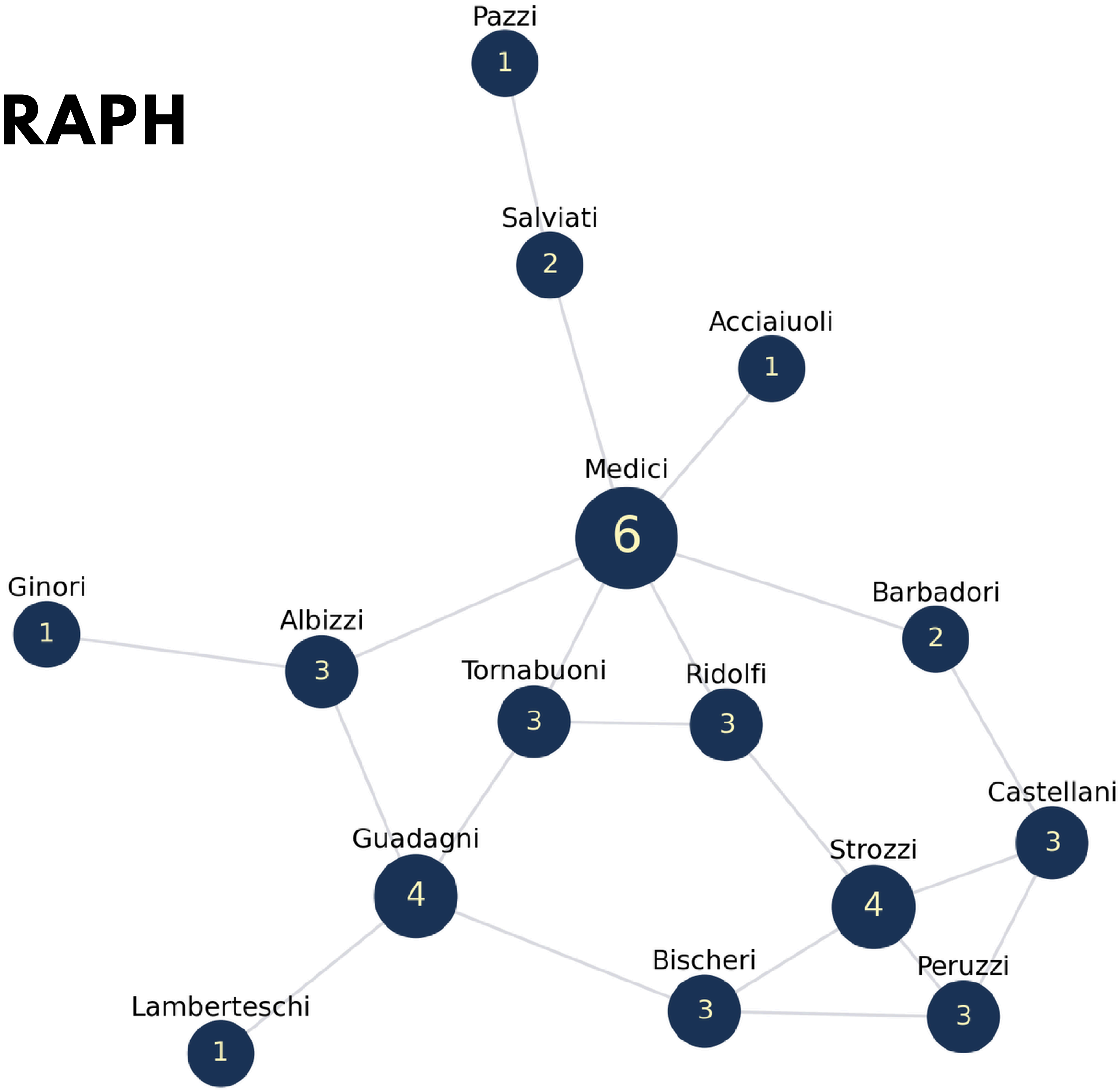
The Medici have the highest degree.

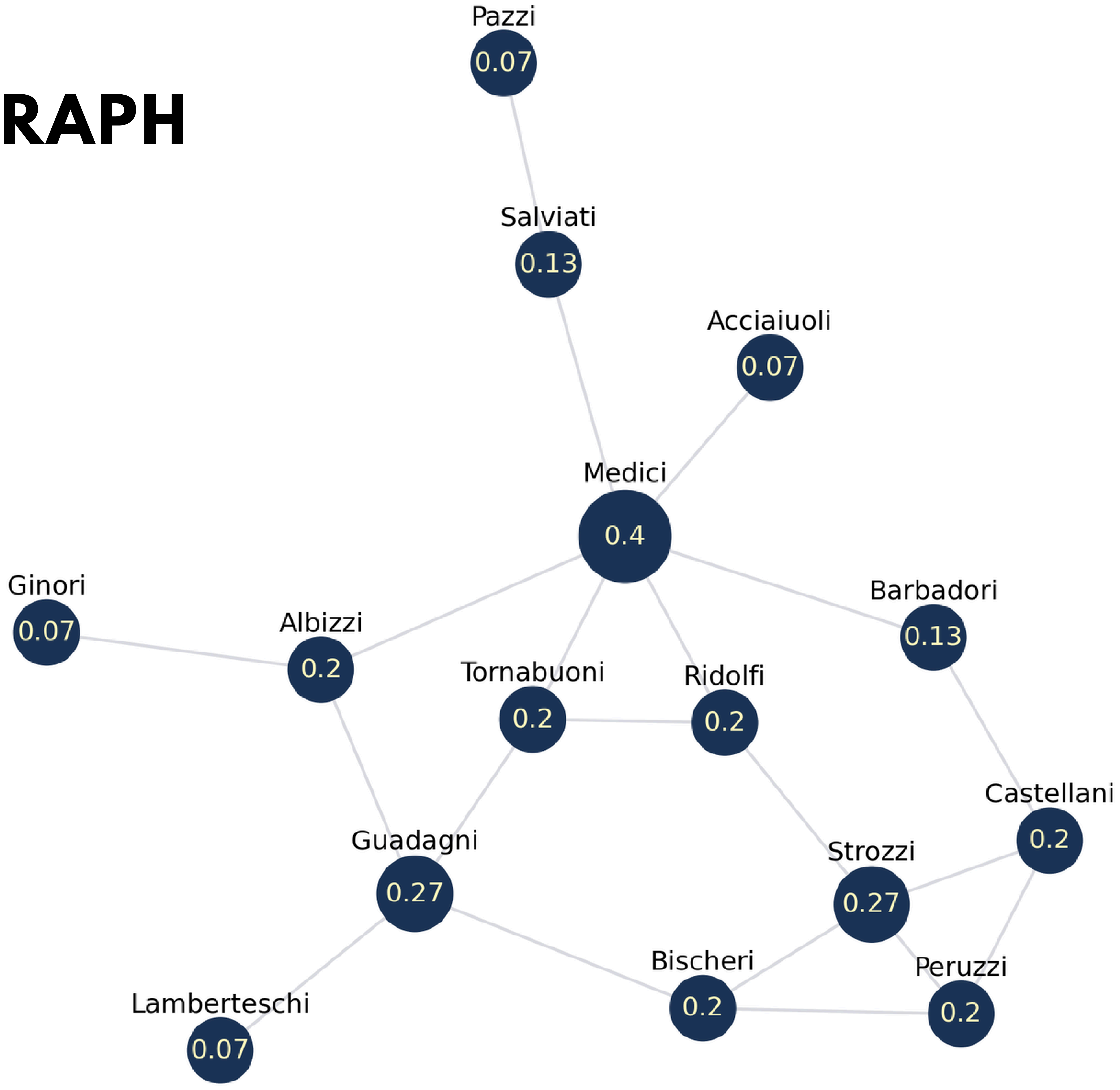
About twice as high as the second most connected families.



FLORENTINE FAMILIES GRAPH

Normalized degree centralities

The degree gets divided by 14, the number of nodes in the graph, minus one.



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VACCINATE THE SUPERSPREADERS

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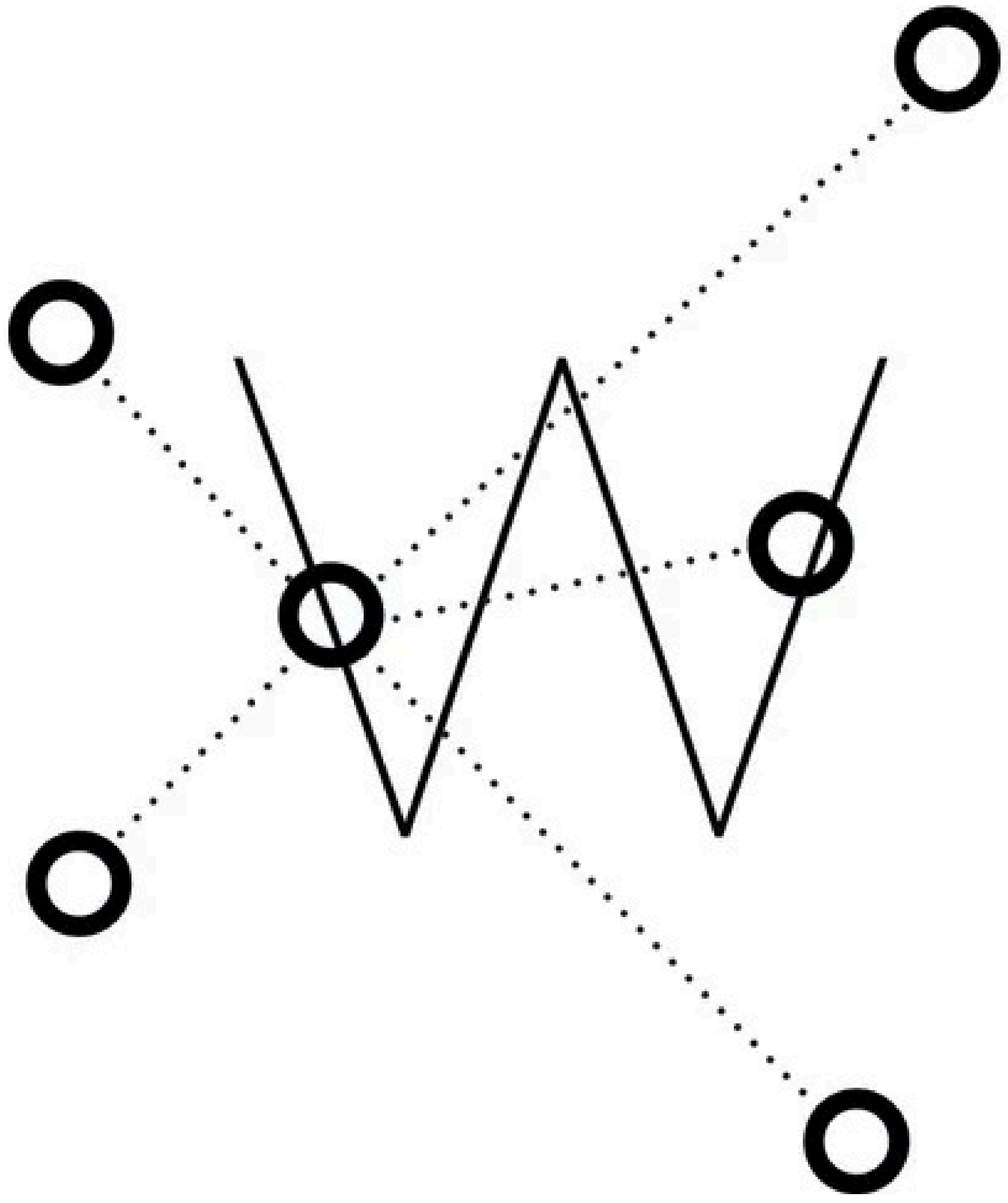

Cox, C. (2020, November 10). The Vulnerable Can Wait. Vaccinate the Super-Spreaders First. *WIRED*.

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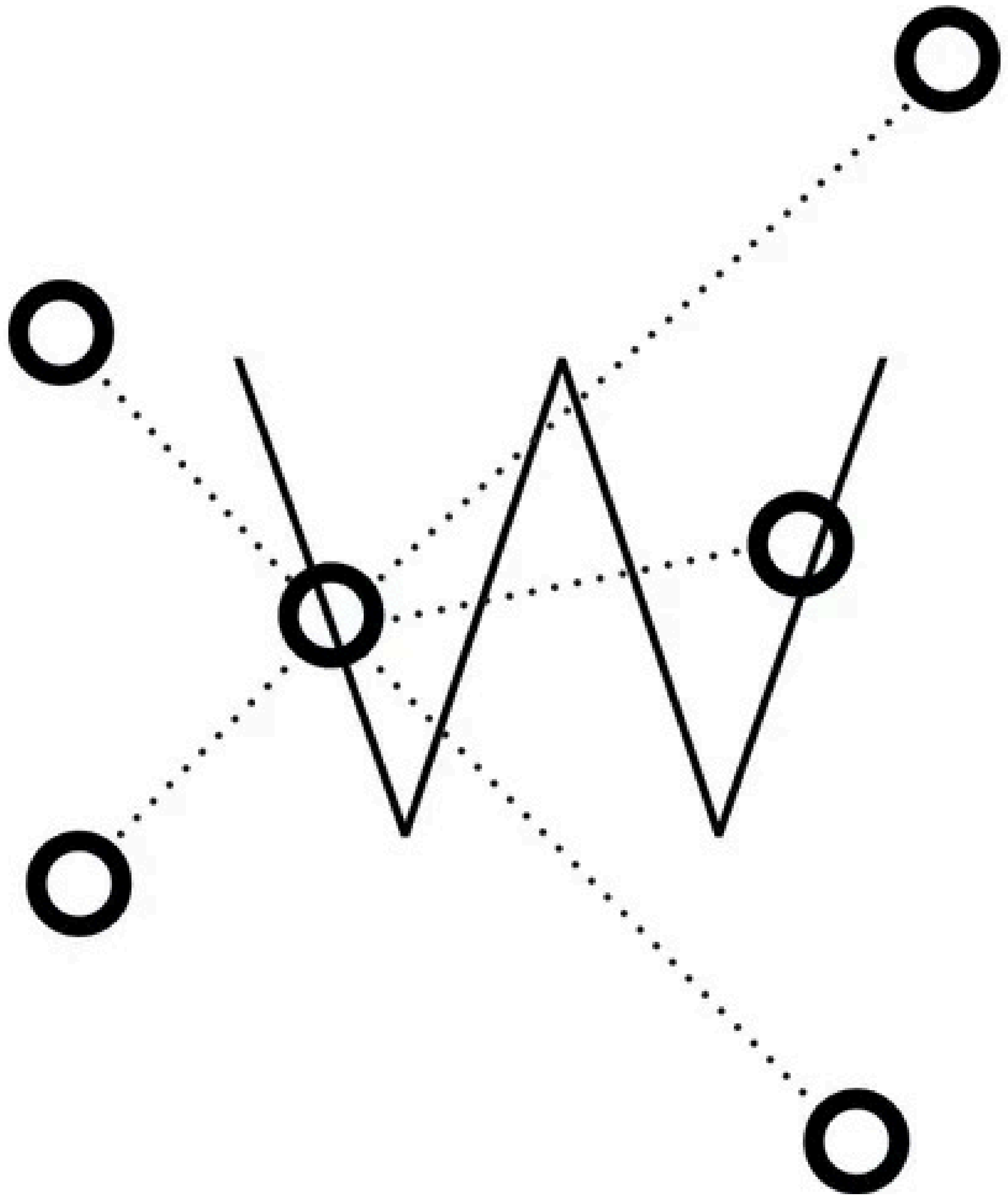
People with many social connections (i.e., high degree) acted as *superspreaders* during COVID.



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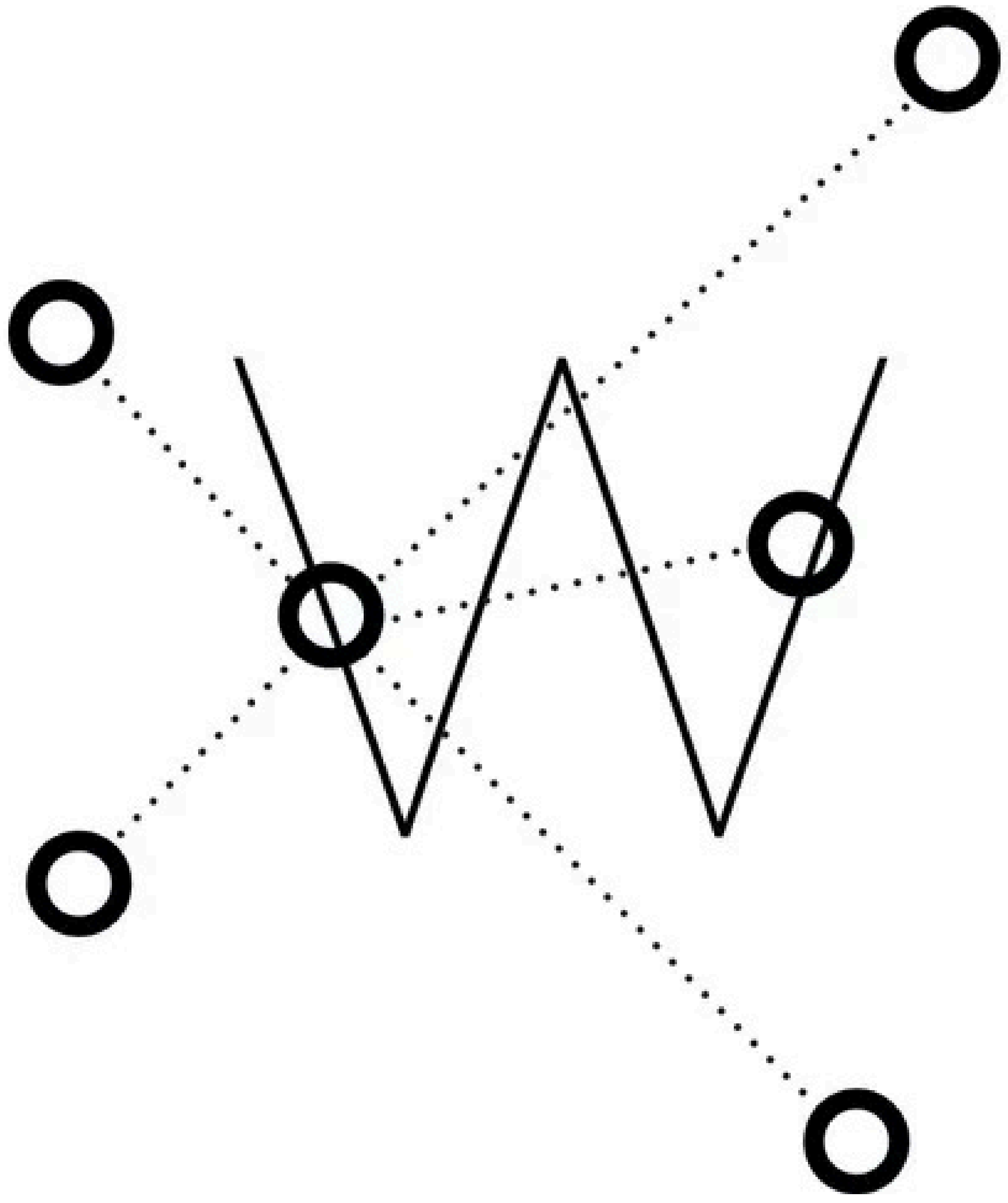

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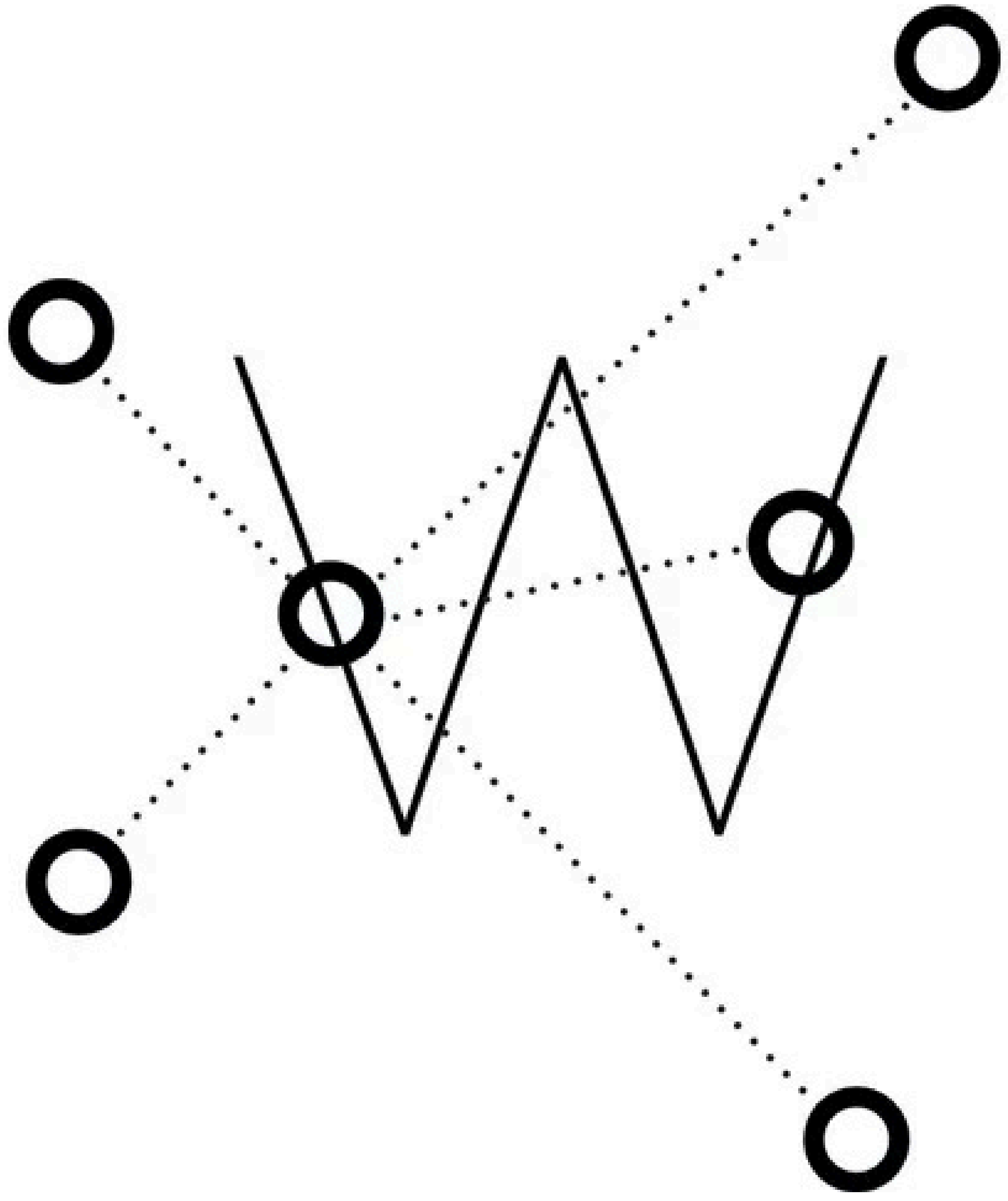
One, perhaps counterintuitive, idea was to start by vaccinating these people.

Rather than, say, the more vulnerable people.



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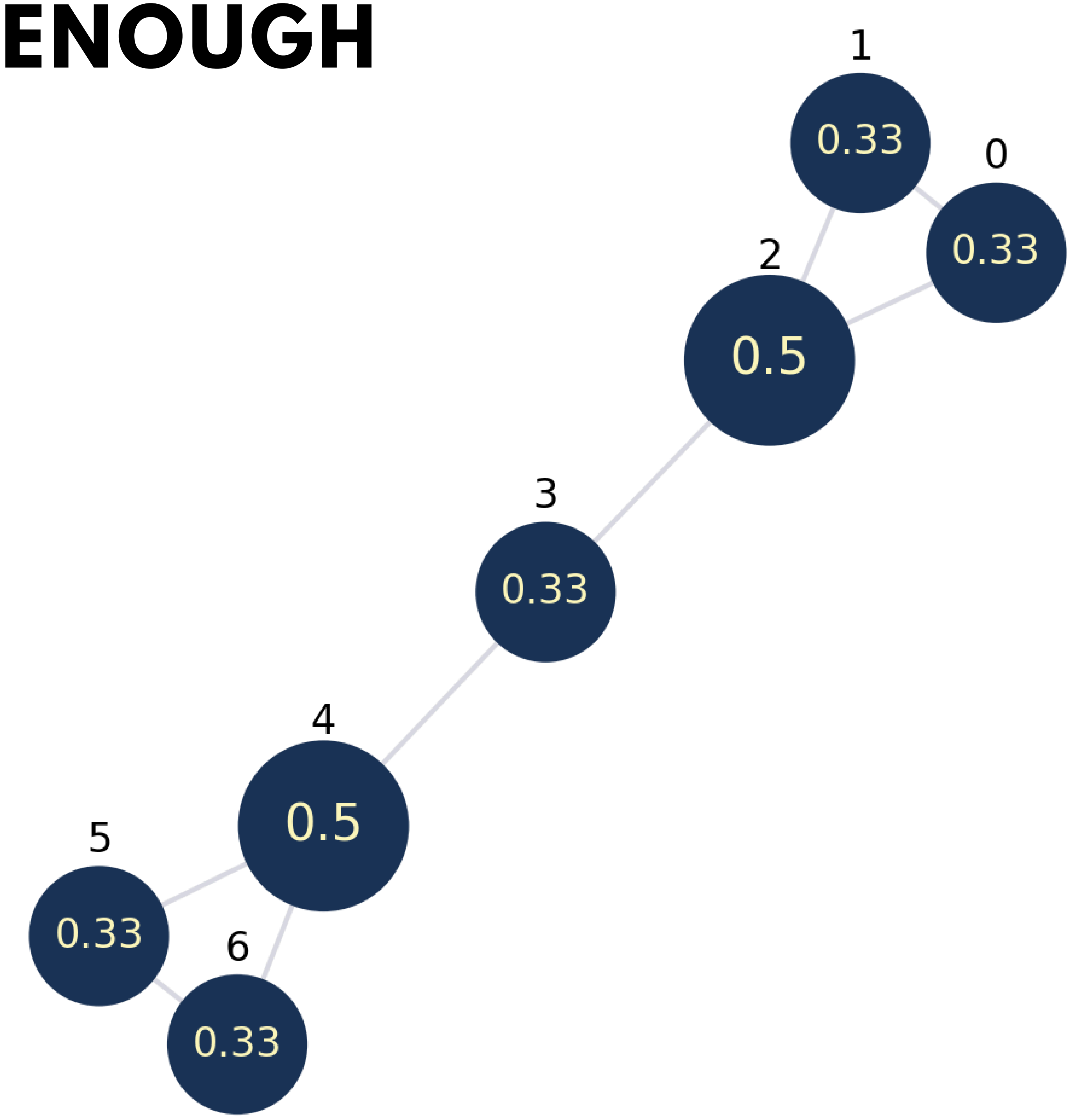
But simply counting degrees is not always what we want.

DEGREE CENTRALITIES NOT ENOUGH

Barbell graph

Node 3 has lower degree (centrality) than nodes 2 and 4.

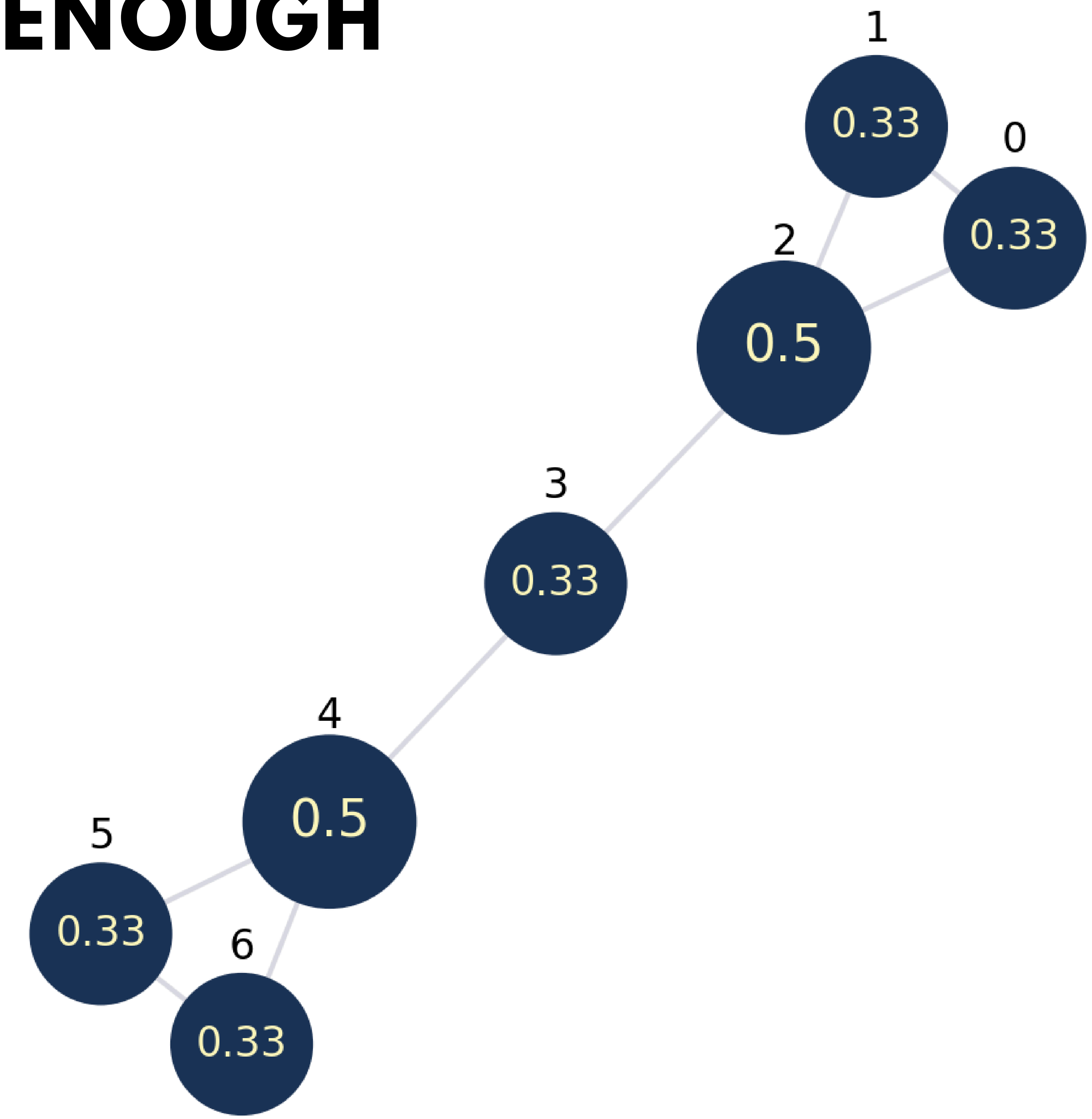


DEGREE CENTRALITIES NOT ENOUGH

Barbell graph

Node 3 has lower degree (centrality) than nodes 2 and 4.

But, intuitively, node 3 is important: all information from one side of the graph to the other has to pass through it.



DEFINITION (BETWEENESS CENTRALITY)

Take $\sigma_{j,k}$ to be *the number of shortest paths between j and k*, and $\sigma_{j,k}(i)$ to be the number of shortest paths between j and k *that pass through i*.

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The *betweeness centrality* of node i is defined as:

$$C_b(i) = \frac{1}{(n-1)(n-2)/2} \cdot \sum_{j \neq k, j \neq i, k \neq i} \frac{\sigma_{j,k}(i)}{\sigma_{j,k}},$$

i.e., the average fraction of shortest paths that pass through i.

THE BARBELL GRAPH

Betwenness centralities

There are $1/2 \cdot (7-1) \cdot (7-2) = 15$ pairs of nodes that do not include node 3:

$$
\begin{aligned}
&(0,1),(0,2),(0,4),(0,5),(0,6),\\
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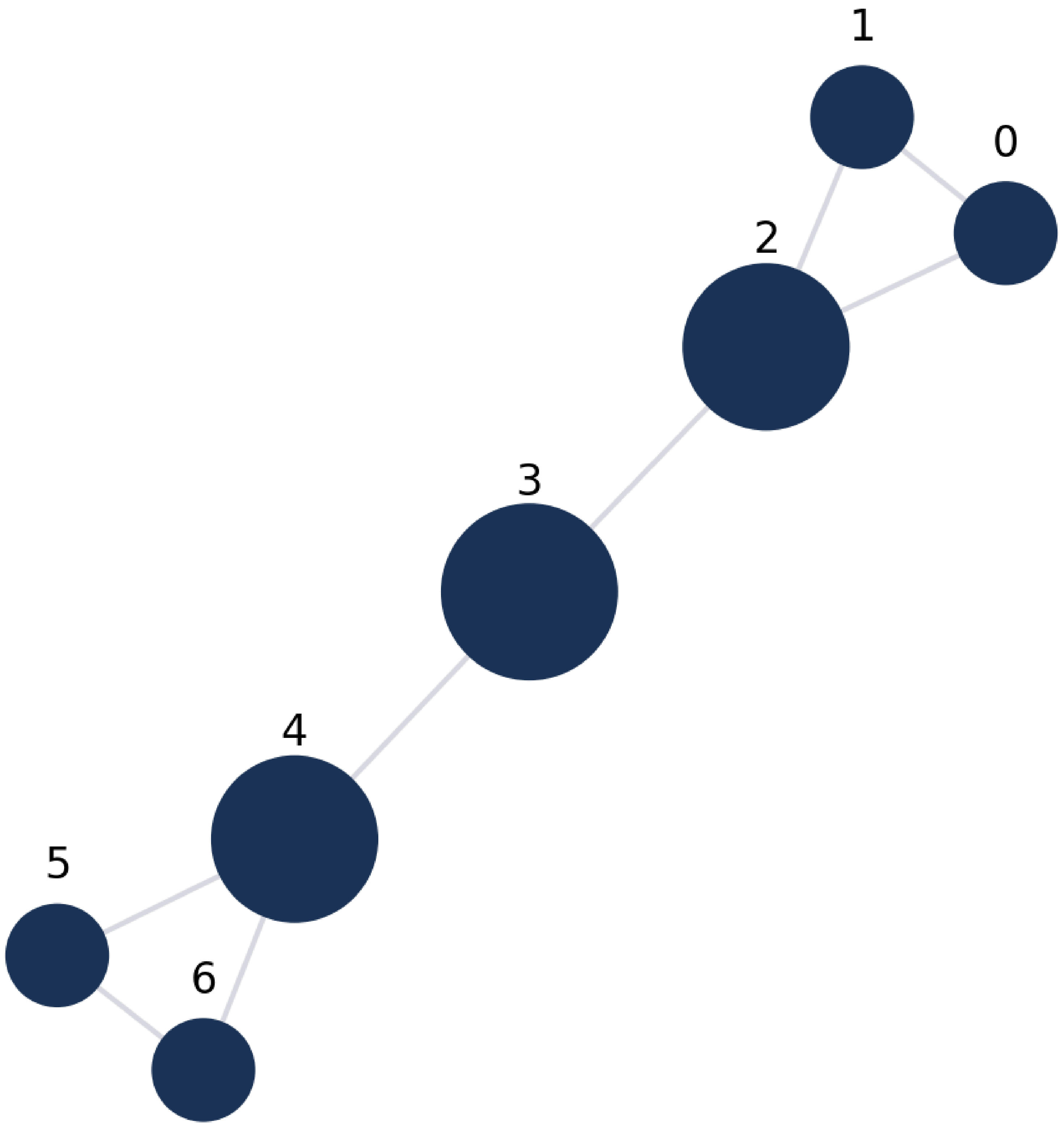

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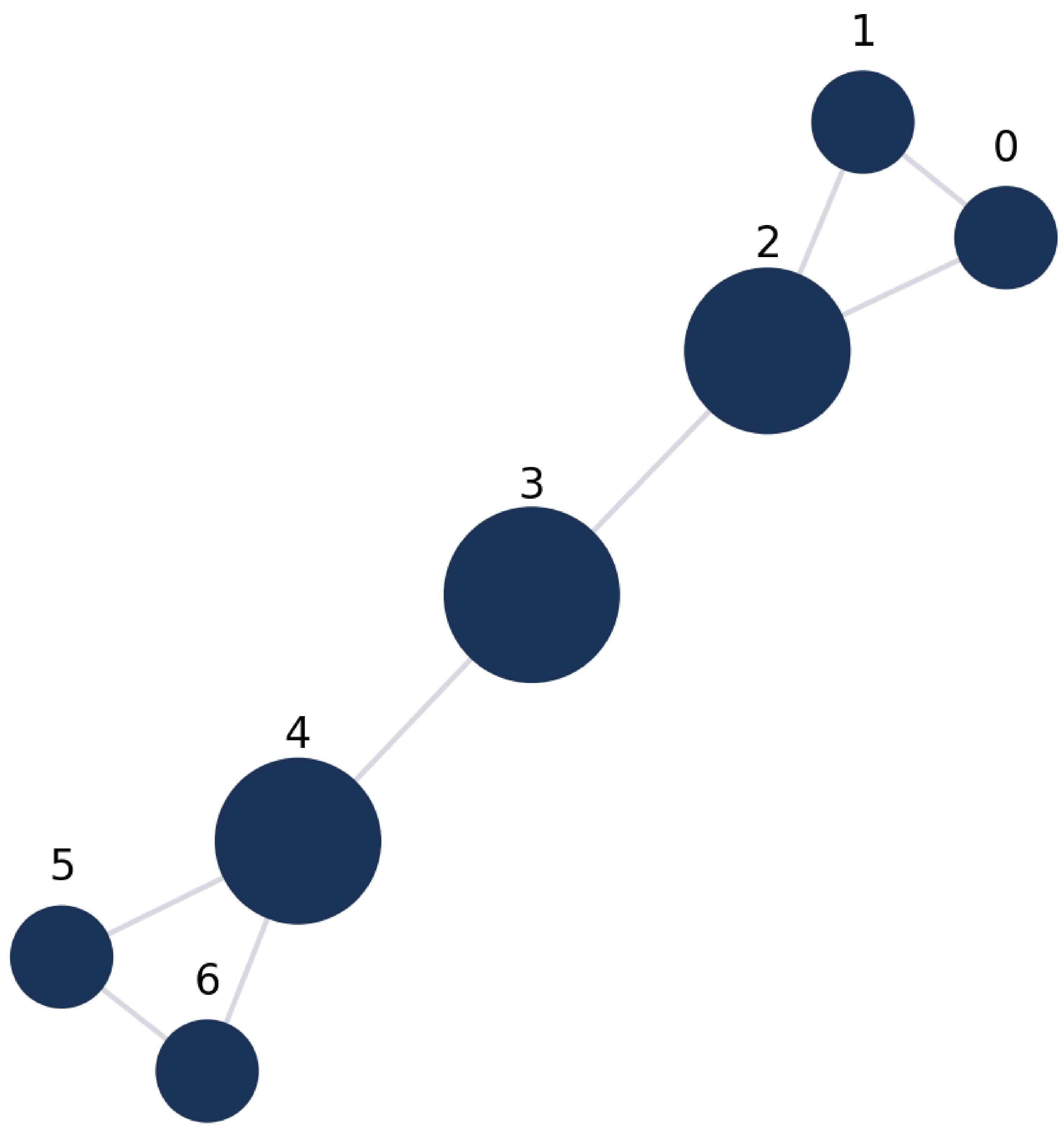
The shortest path between 0 and 1 is $(0,1)$. The shortest path between 0 and 4 is $(0,2,3,4)$…



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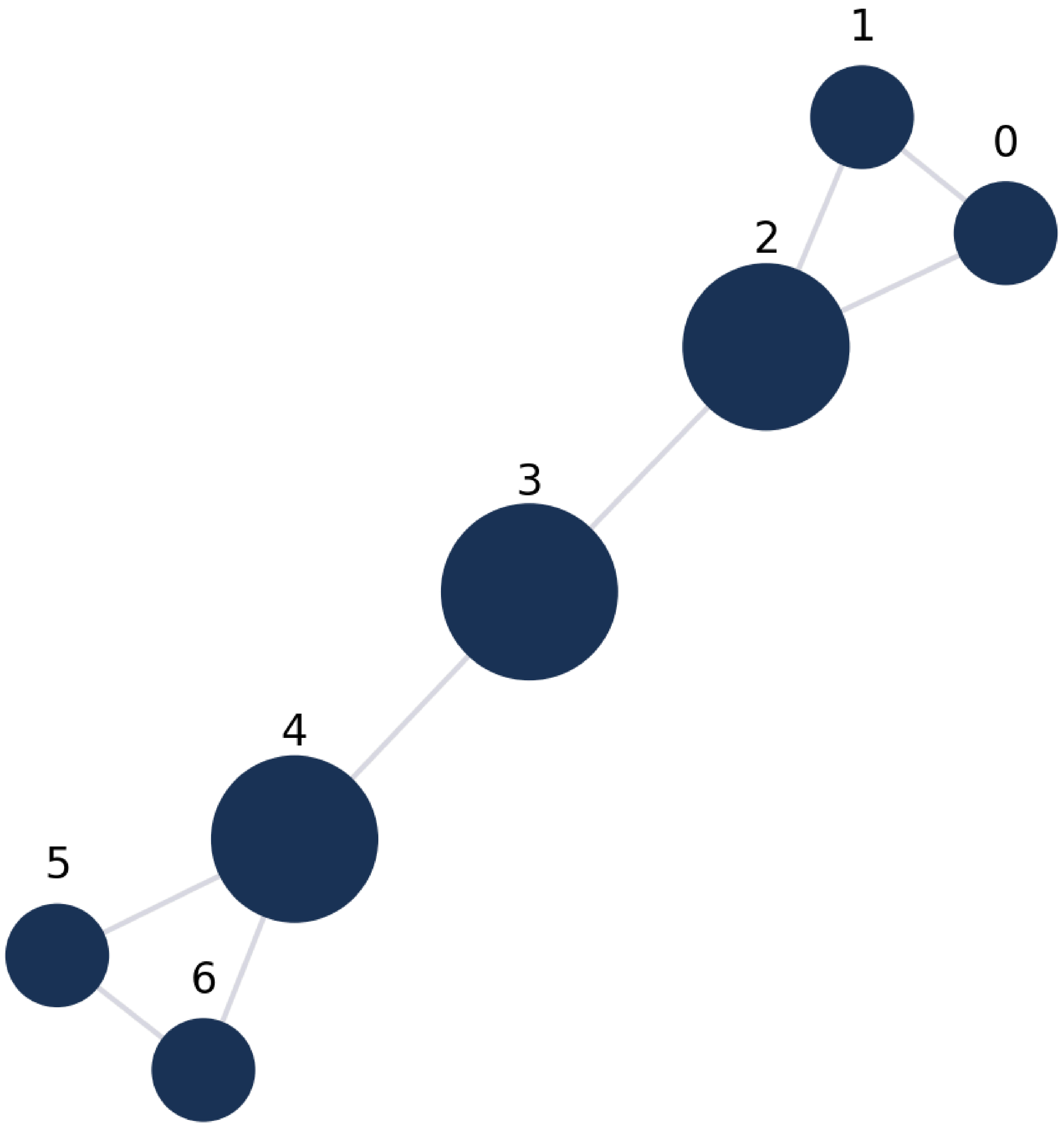
Every pair of nodes has a unique shortest path, i.e., $\sigma_{j,k} = 1$ for al $j \neq 3$ and $k \neq 3$.



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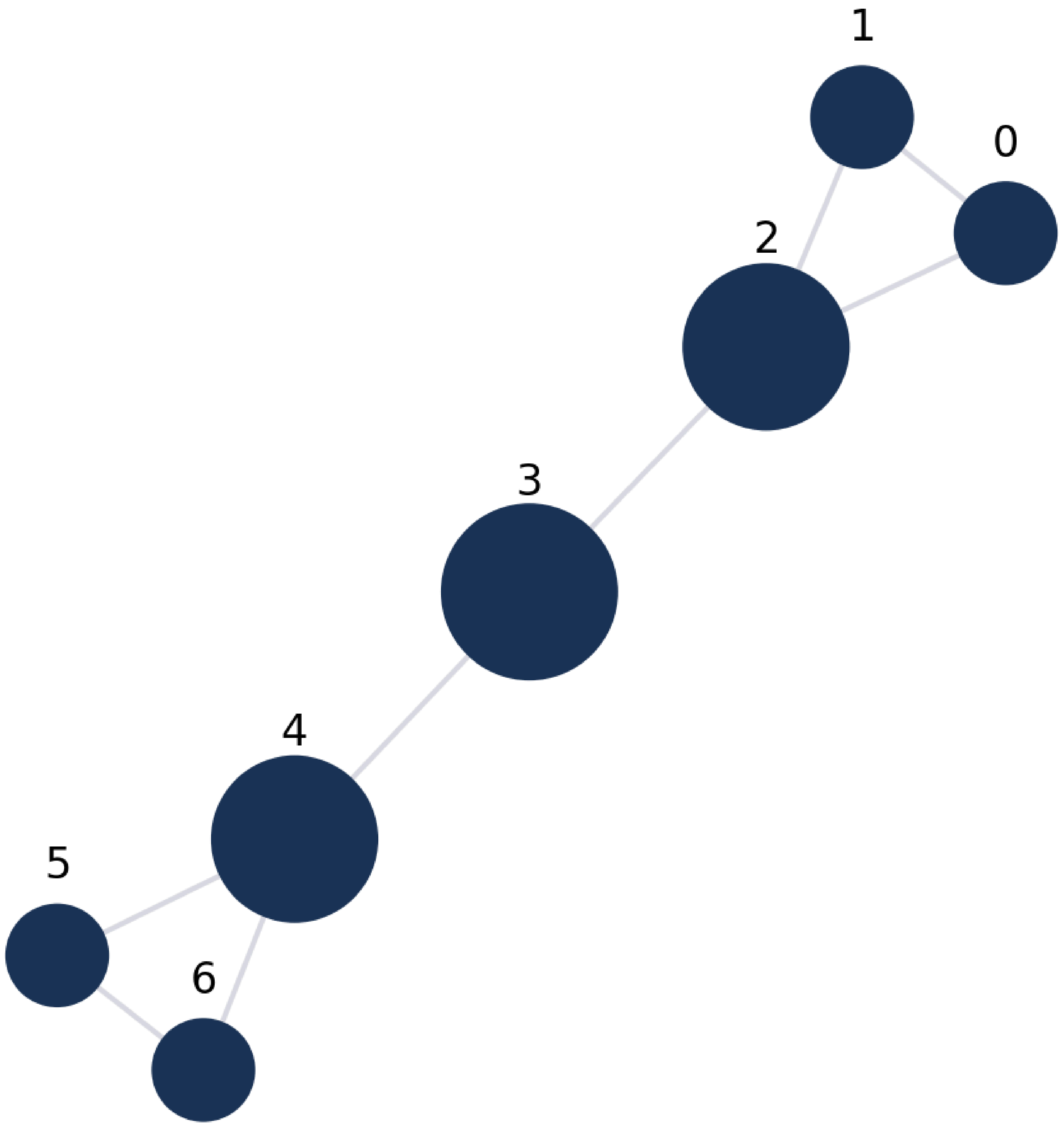

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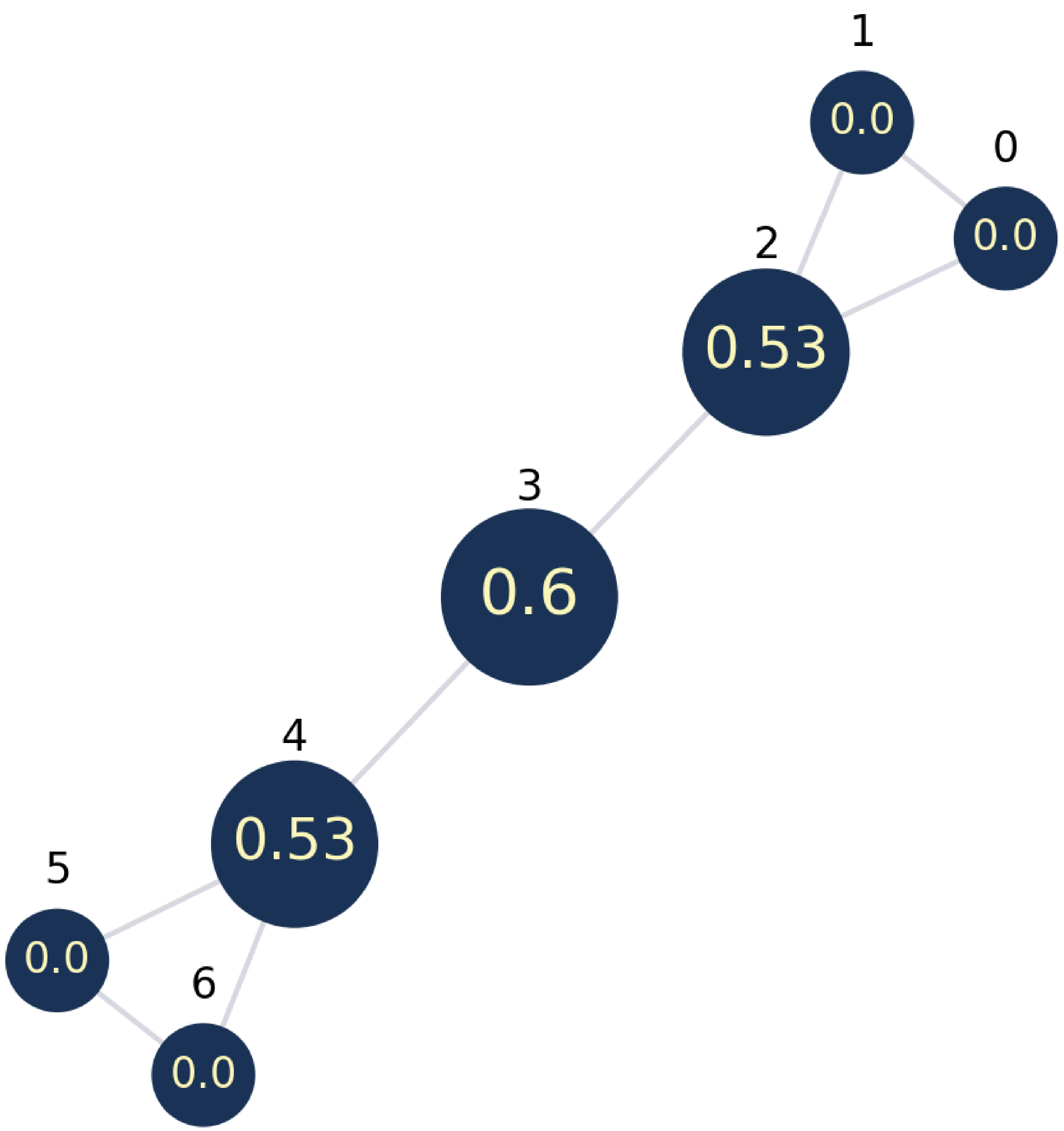
The *betweeness centrality* of node 3 is thus:

$$\begin{aligned}
C_b(3) &= \frac{1}{(7-1)(7-2)/2} \cdot 9\\
&= 0.6.
\end{aligned}$$



Betweenness centrality identifies nodes that have strategic power by controlling information flows.

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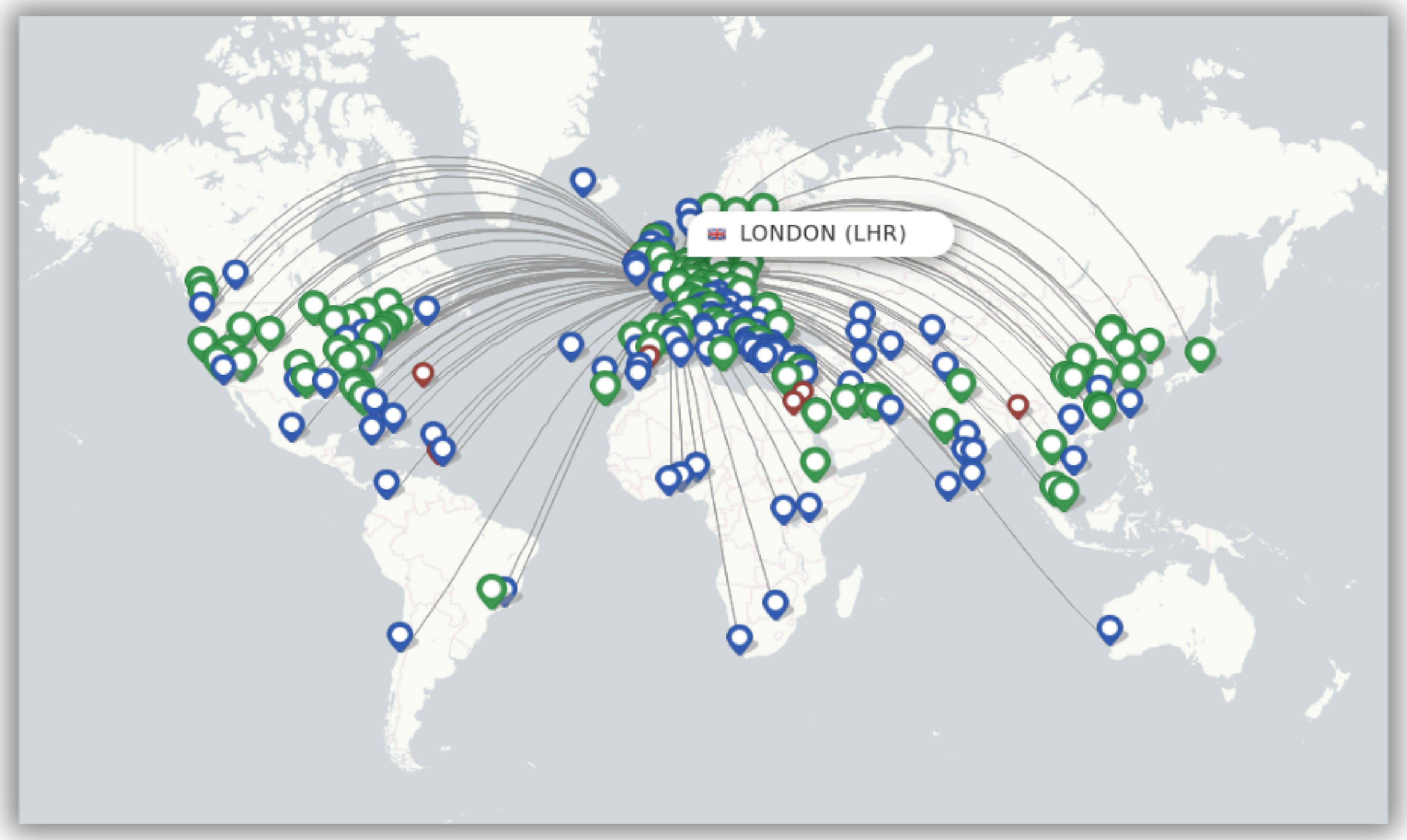
DIRECT FLIGHT NETWORK

Two airports are connected if there is a direct flight between them.



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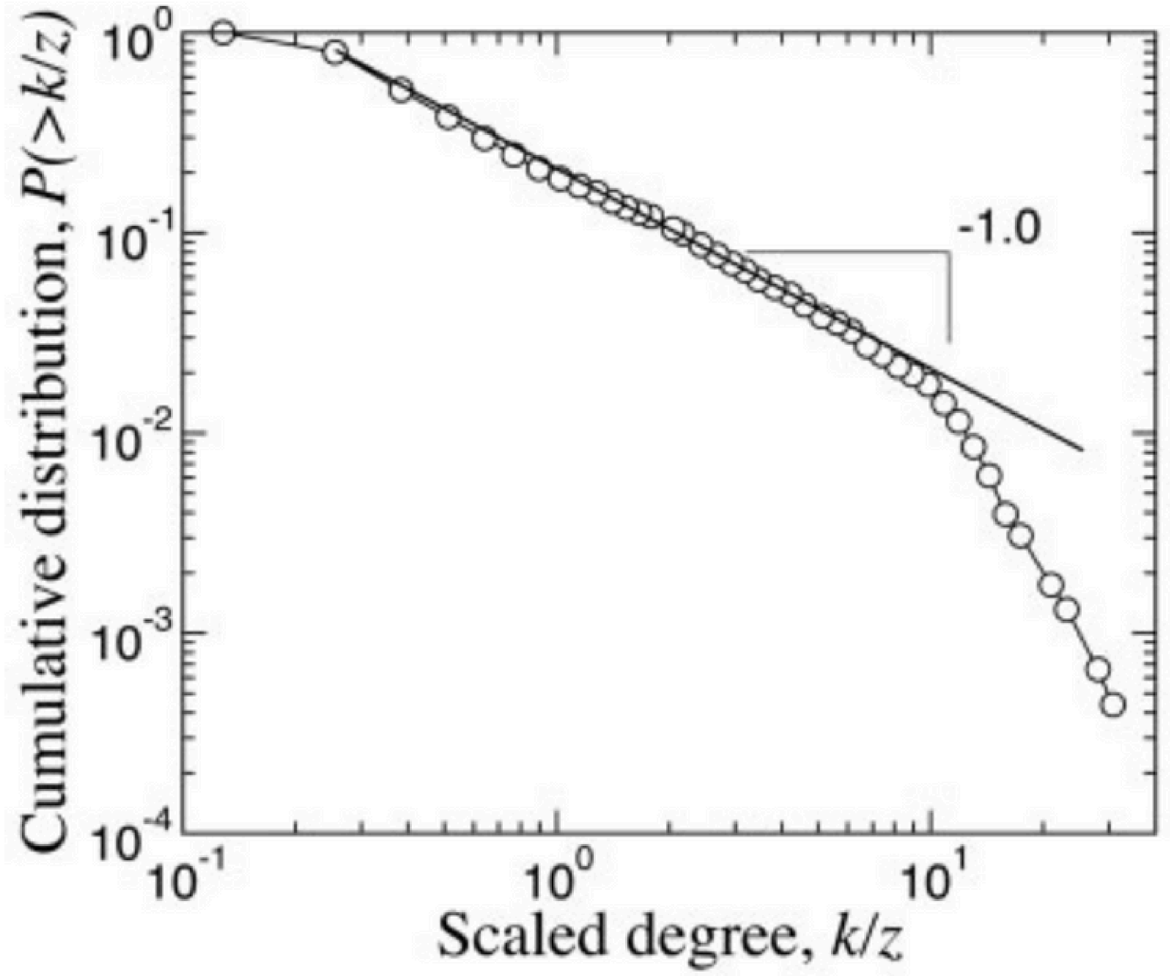

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betweenness centrality

Rank	City	b	b/b_{ran}	Degree
1	Paris	58.8	1.2	250
2	Anchorage*	55.2	16.7	39
3	London	54.7	1.2	242
4	Singapore*	47.5	4.3	92
5	New York	47.2	1.6	179
6	Los Angeles	44.8	2.3	133
7	Port Moresby*	43.4	13.6	38
8	Frankfurt	41.5	0.9	237
9	Tokyo	39.1	2.7	111
10	Moscow	34.5	1.1	186
11	Seattle*	34.3	3.3	89
12	Hong Kong*	30.8	2.6	98
13	Chicago	28.8	1.0	184
14	Toronto	27.1	1.8	116
15	Buenos Aires*	26.9	3.2	76
16	São Paulo*	26.5	2.8	82
17	Amsterdam	25.9	0.8	192
18	Melbourne*	25.5	4.5	58
19	Johannesburg*	25.4	2.6	84
20	Manila*	24.4	3.5	67
21	Seoul*	24.3	2.1	95
22	Sydney*	23.1	3.2	70
23	Bangkok*	22.9	1.8	102
24	Honolulu*	21.1	4.4	51
25	Miami*	20.1	1.4	110

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But also surprising names, like Anchorage and Port Moresby.

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Thus, Anchorage connects different communities.

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TORE OPSAHL

However, if we *weight* the connections (e.g., by the number of flights between airports)...

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However, if we *weight* the connections (e.g., by the number of flights between airports)...

... and adapt the centrality notion accordingly...

Anchorage does not appear as central anymore.

Rank	Betweenness			
	Binary Analysis		Weighted Analysis	
	Airport	Score	Airport	Score
1	FRA (Frankfurt, Germany)	587531	LHR (London, United Kingdom)	1858349
2	CDG (Paris, France)	520707	LAX (Los Angeles, United States)	1310287
3	ANC (Anchorage, United States)	481044	JFK (New York, United States)	1084392
4	DXB (Dubai, United Arab Emirates)	443314	BKK (Bangkok, Thailand)	797785
5	GRU (Sao Paulo, Brazil)	402882	SIN (Singapore)	739981
6	YYZ (Toronto, Canada)	398869	SEA (Seattle, United States)	723145
7	LHR (London, United Kingdom)	389846	MAD (Madrid, Spain)	707354
8	LAX (Los Angeles, United States)	356600	GRU (Sao Paulo, Brazil)	684057
9	DME (Moscow, Russia)	353902	NRT (Tokyo, Japan)	639074
10	BKK (Bangkok, Thailand)	352682	DXB (Dubai, United Arab Emirates)	610765
...	...	...	...	...
14	...	...	ANC (Anchorage, United States)	469203
18	...	...	FRA (Frankfurt, Germany)	392418

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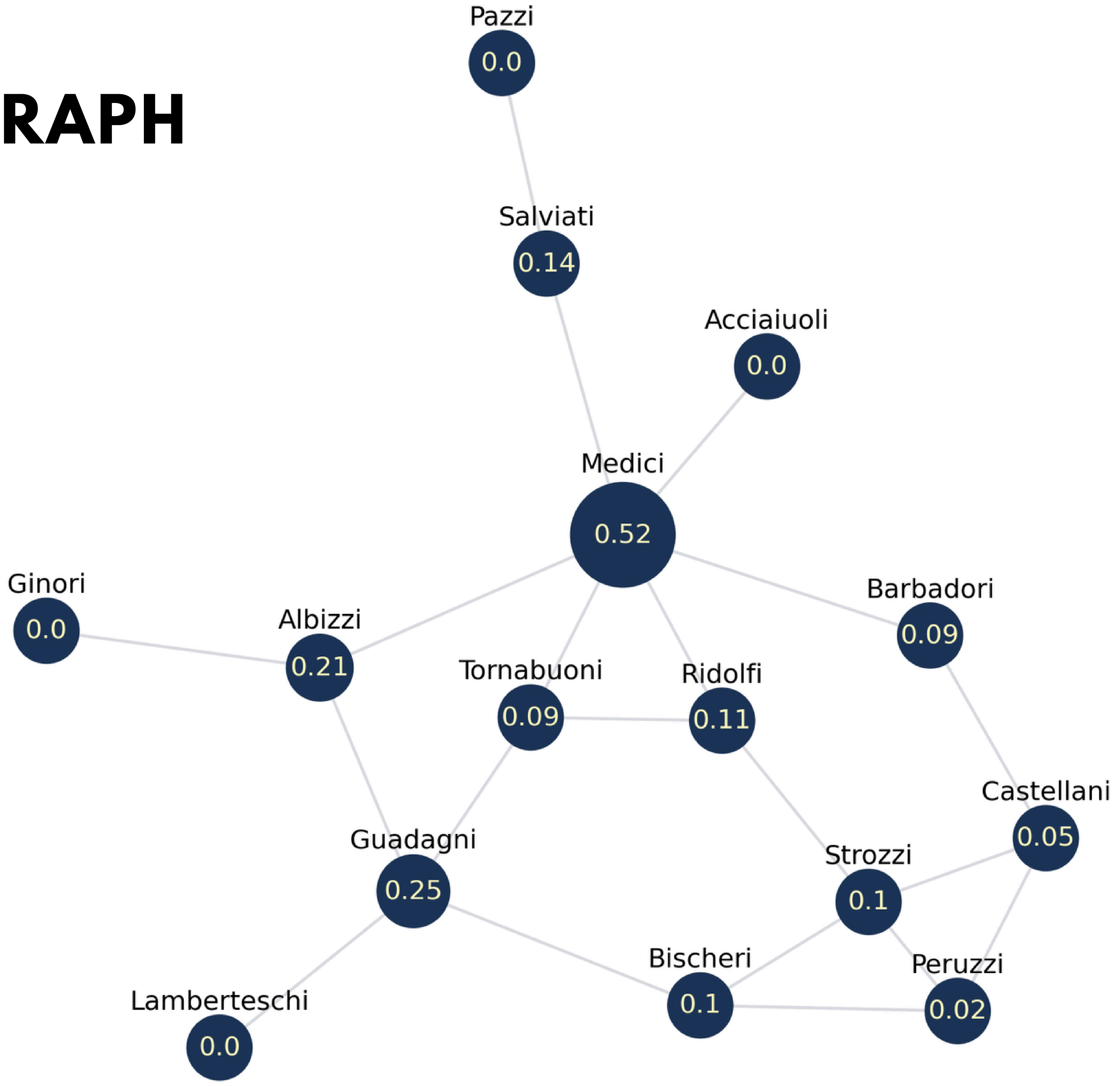
Betweenness centralities

The Medici have more than double the betweenness centrality of the Albizzi and Guadagni.



FLORENTINE FAMILIES GRAPH
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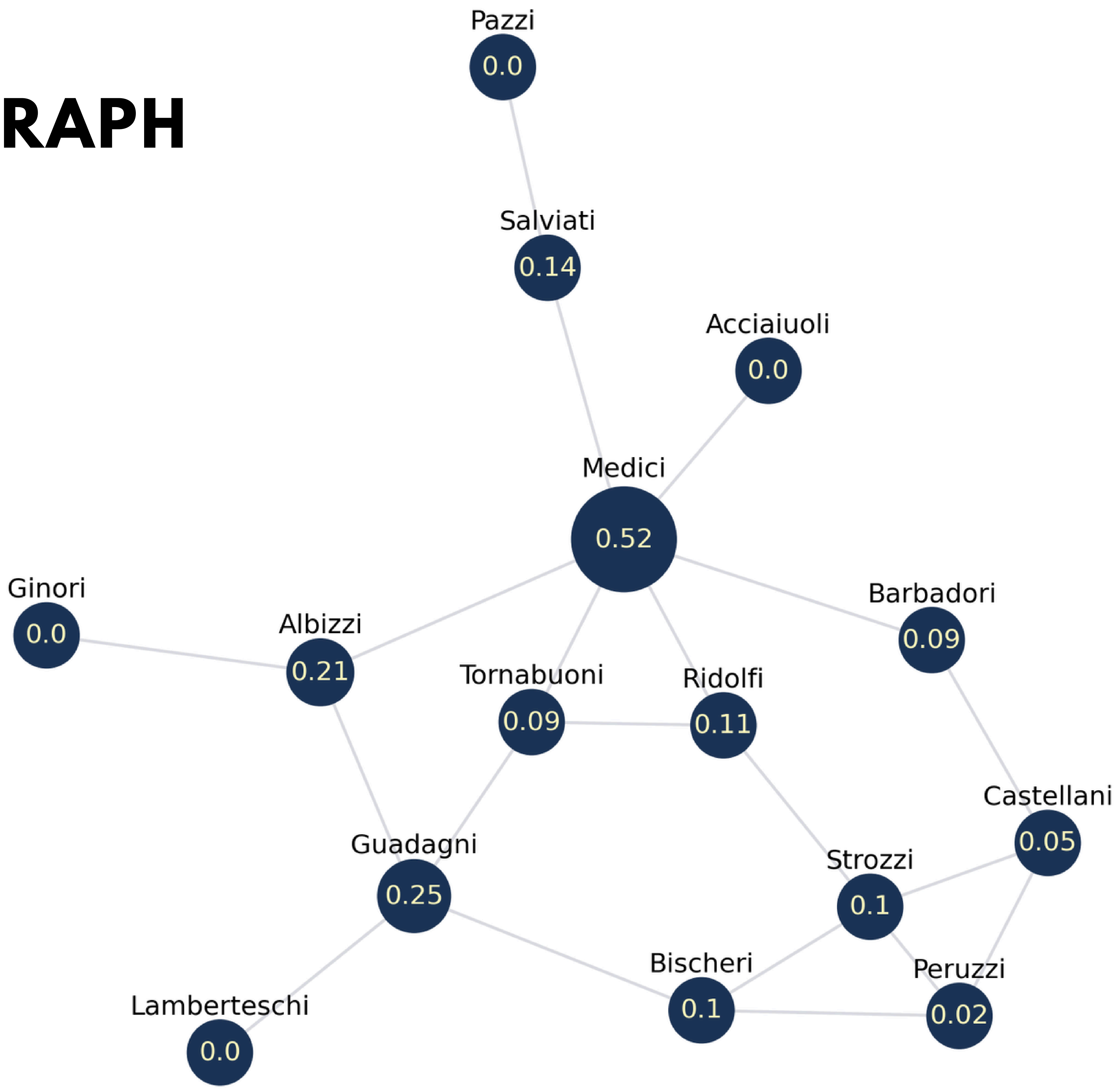
The Medici have more than double the betweenness centrality of the Albizzi and Guadagni.

And about five times the centrality of the Strozzi, Ridolfi and Tornabuoni.



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For the next notion of centrality, we need a bit of linear algebra.

DEFINITION (EIGENVECTORS)

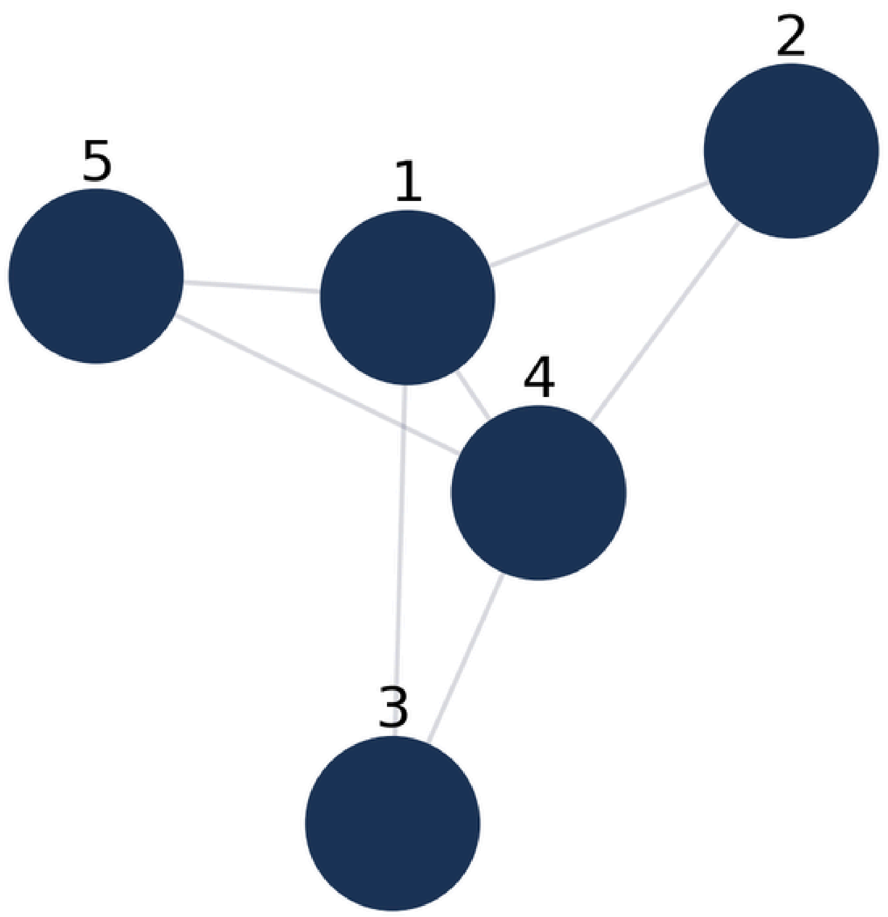
For an $n \times n$ matrix $\boldsymbol{M} = \begin{bmatrix} m_{11} & \dots & m_{1n} \\ \vdots & \ddots & \vdots \\ m_{n1} & \dots & m_{nn} \end{bmatrix}$, an *eigenvector* $\boldsymbol{v} = \begin{bmatrix} v_1 \\ \vdots \\ v_n \end{bmatrix}$ is a non-zero $n \times 1$ vector such that:

$$\boldsymbol{M}\boldsymbol{v} = \lambda \boldsymbol{v},$$

where λ is a scalar called the *eigenvalue* of $\boldsymbol{M}$ corresponding to the eigenvector $\boldsymbol{v}$.

What does this have to do with networks?

ADJACENCY MATRIX



The *adjacency matrix* $\boldsymbol{A}$ of a network with n nodes is an $n \times n$ matrix where entry a_{ij} is 1 if there is an edge between nodes i and j, and 0 otherwise.

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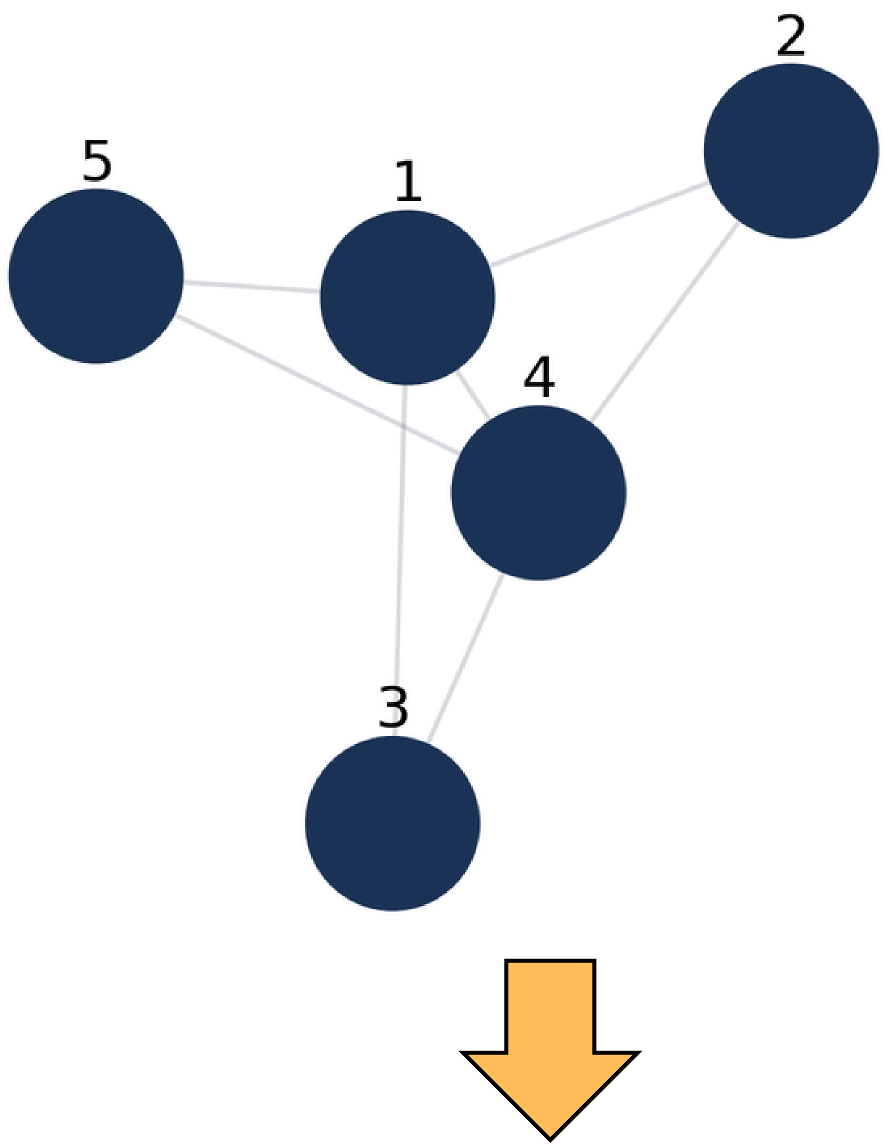

$$\boldsymbol{A} = \begin{bmatrix} 0 & 1 & 1 & 1 & 1 \\ 1 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 & 0 \\ 1 & 1 & 1 & 0 & 1 \\ 1 & 0 & 0 & 1 & 0 \end{bmatrix}$$

Eigenvector centrality measures a node's based on the quality or influence of its connections.

Eigenvector centrality measures a node's based on the quality or influence of its connections. Formally, a node is influential if it is connected to other influential nodes. This is circular, but doable…

DEFINITION (EIGENVECTOR CENTRALITY)

The *eigenvector centrality* $C_e(i)$ of node i is defined as:

$$C_e(i) = \frac{1}{\lambda} \sum_{j \in N(i)} C_e(j),$$

where λ is the largest eigenvalue of the adjacency matrix A of the network, and $N(i)$ is the neighborhood of node i.

FINDING EIGENVECTOR CENTRALITIES

To find the eigenvector centralities, we just solve the system of equations:

$$\boldsymbol{A}\boldsymbol{v} = \lambda \boldsymbol{v}.$$

$$\begin{bmatrix} 0 & 1 & 1 & 1 & 1 \\ 1 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 & 0 \\ 1 & 1 & 1 & 0 & 1 \\ 1 & 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} = \lambda \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix}$$

LARRY PAGE

The idea of eigenvector centrality is behind the PageRank algorithm.

Which we used to organize the web.

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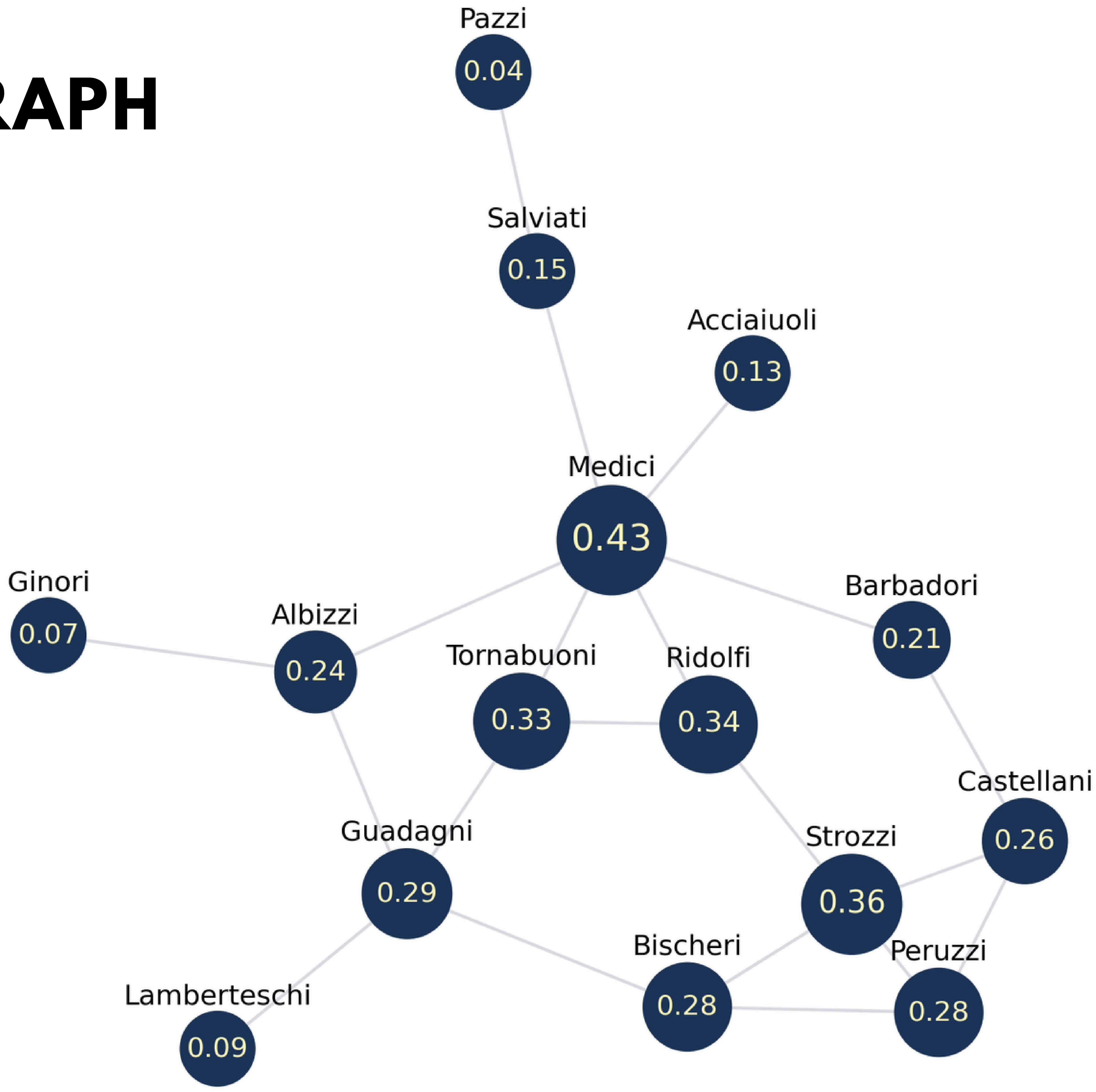
SERGEY BRIN

And make a lot of money!

FLORENTINE FAMILIES GRAPH

Eigenvector centralities

Again, the Medici have the highest eigenvector centrality.

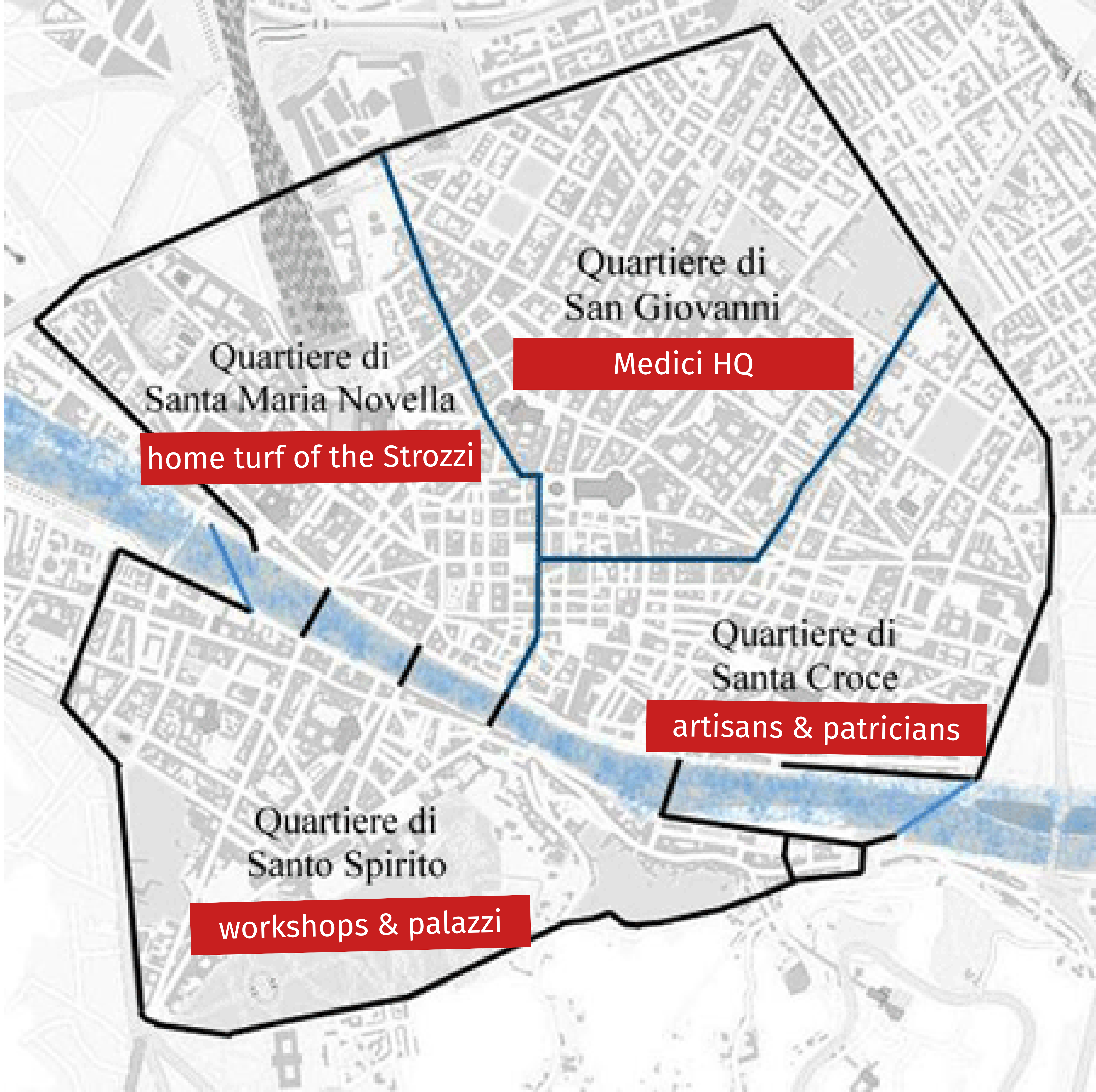


And with this we can start to see how the Medici got so successful.

FLORENCE QUARTERS

Everything north of the Arno is uptown, everything south is Oltrarno (i.e., beyond the Arno).

Each quarter divided into four gonfaloni.



Patrician houses built tower-blocks on their corners and married close.

Patrician houses built tower-blocks on their corners and married close. Cosimo's hack was to act as a broker between different types (of rich people).

JOHN F. PADGETT

Cosimo lent money inside San Giovanni, to ‘new-men clients’.

Padgett, J. F., & Ansell, C. K. (1993). Robust action and the rise of the Medici, 1400-1434. *American Journal of Sociology*, 98(6), 1259–1319.

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Cosimo lent money inside San Giovanni, to 'new-men clients'.

But married across the river and eastward, in families of old patricians.

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CHRISTOPHER K. ANSELL

The two sets almost never bumped into each other in daily street life, so he became the only safe bridge.

Padgett, J. F., & Ansell, C. K. (1993). Robust action and the rise of the Medici, 1400-1434. *American Journal of Sociology*, 98(6), 1259–1319.

MEDICI MARRIAGE AND ECONOMIC TIES

By keeping patrician kin and merchant debtors in separate silos, Cosimo could speak in different registers to each, stay enigmatically above factions, and let both sides depend on his brokerage.

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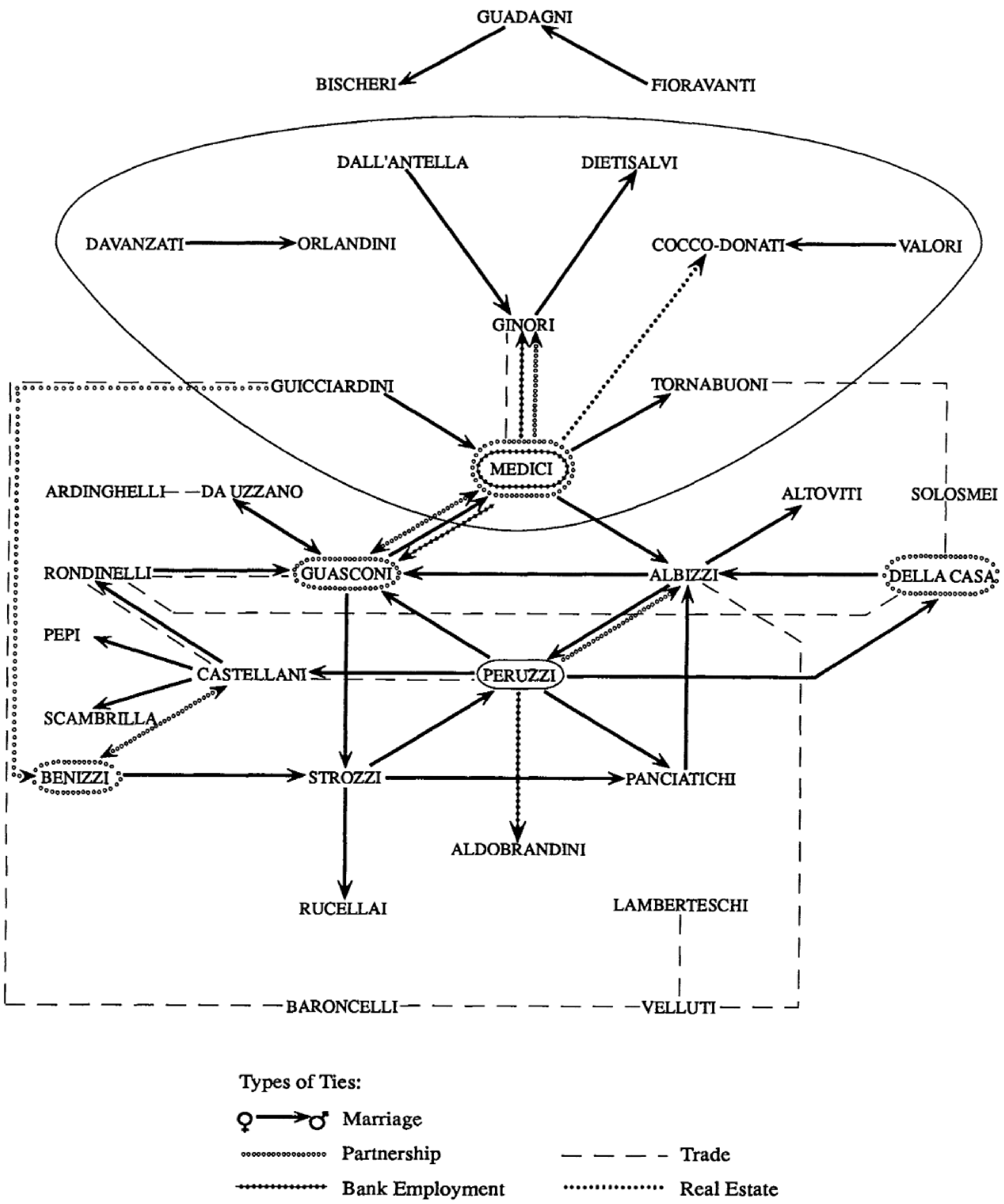

We can see this in the Medici's high level of centrality, on pretty much all measures.