LUDWIG-
MAXIMILIANS-
UNIVERSITÄT
MÜNCHEN

JUNE 2, 2025

NETWORKED MINDS:
OPINION DYNAMICS AND COLLECTIVE INTELLIGENCE IN SOCIAL NETWORKS

QUANTIFYING NETWORKS

Adrian Haret
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Florence, the early 1400s.

Florence, the early 1400s. In the shadow of church bells and towers, patrician old-money families jostle for power with upstart, ‘new men’ merchant families.

Rosselli, F. (c. 1470-82). Pianta della Catena, aka Rosselli's *Chain Map*.

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Amid this jostling, one family, the Medicis, rises to prominence.

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Cosimo de' Medici

1389 - 1464

Banker and politician.

Steers the Medicis to dominance in Florence, a position they go on to hold for three centuries to come.

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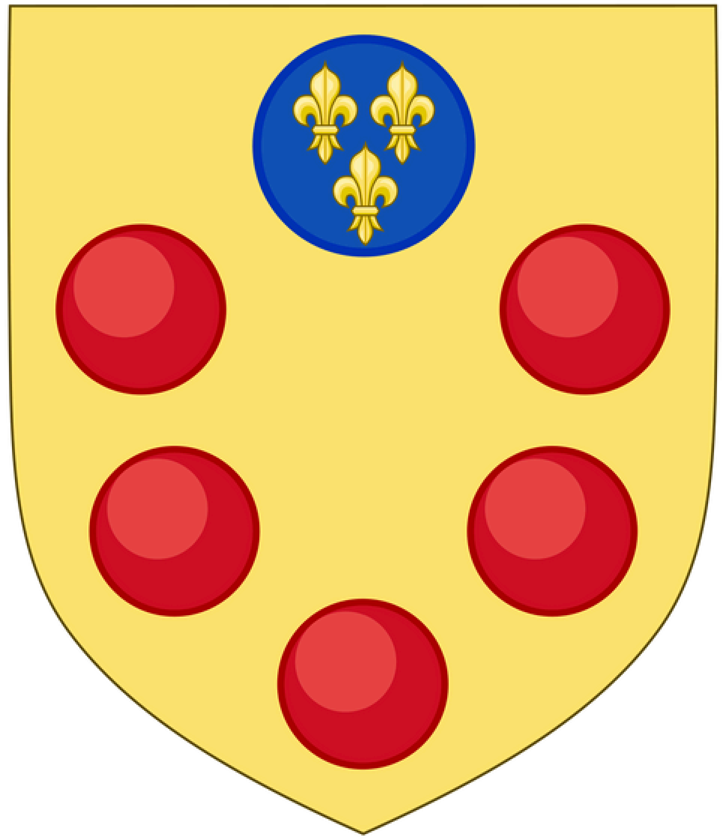
> *Political questions are settled in [Cosimo's] house. The man he chooses holds office… He is who decides peace and war… He is king in all but name.*

Enea Silvio Piccolomini, Bishop of Siena, later Pope Pius II

MEDICI



Originally from the farmlands north of Florence, in Mugello.

Become wealthy through banking.

Cosimo seen as ‘champion of the new men’.

Yet, it seems Cosimo was not an obvious leader.

JOHN F. PADGETT

Cosimo was described by people who knew him as an indecipherable sphinx...

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After passionate pleas by supplicants for action of some sort, Cosimo typically would terminate a meeting graciously but icily...

...with little more commitment than “Yes my son, I shall look into that.”

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We have a puzzle on our hands.

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Cosimo was multiply embedded in complicated and sprawling Florentine marriage, economic, and patronage elite networks.

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He engineered himself into a very important position in the network of early 15th century Florentine elites.

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So what *is* a network?

MODEL

We represent a *social network* as a graph $G = (V, E)$, where V is the set of *vertices* (agents) and E is the set of *edges* (relationships).

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The graph can be *directed* or *undirected*, depending on whether relationships are one-way or two-way.

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Like, who married who.

FLORENTINE FAMILIES GRAPH

A link represents a marriage between the two families.

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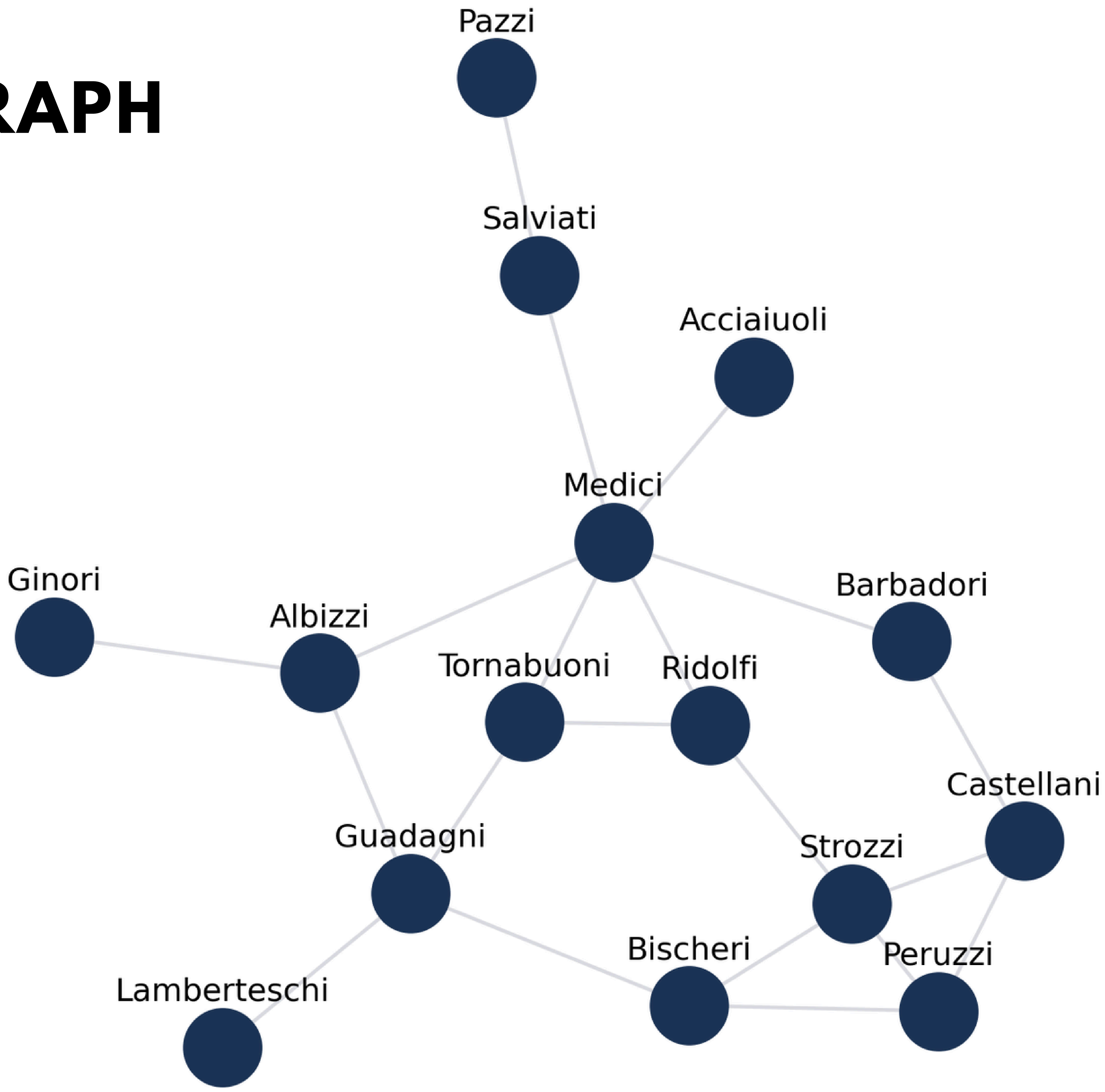

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Considered as *strong* ties, in Granovetter's sense.

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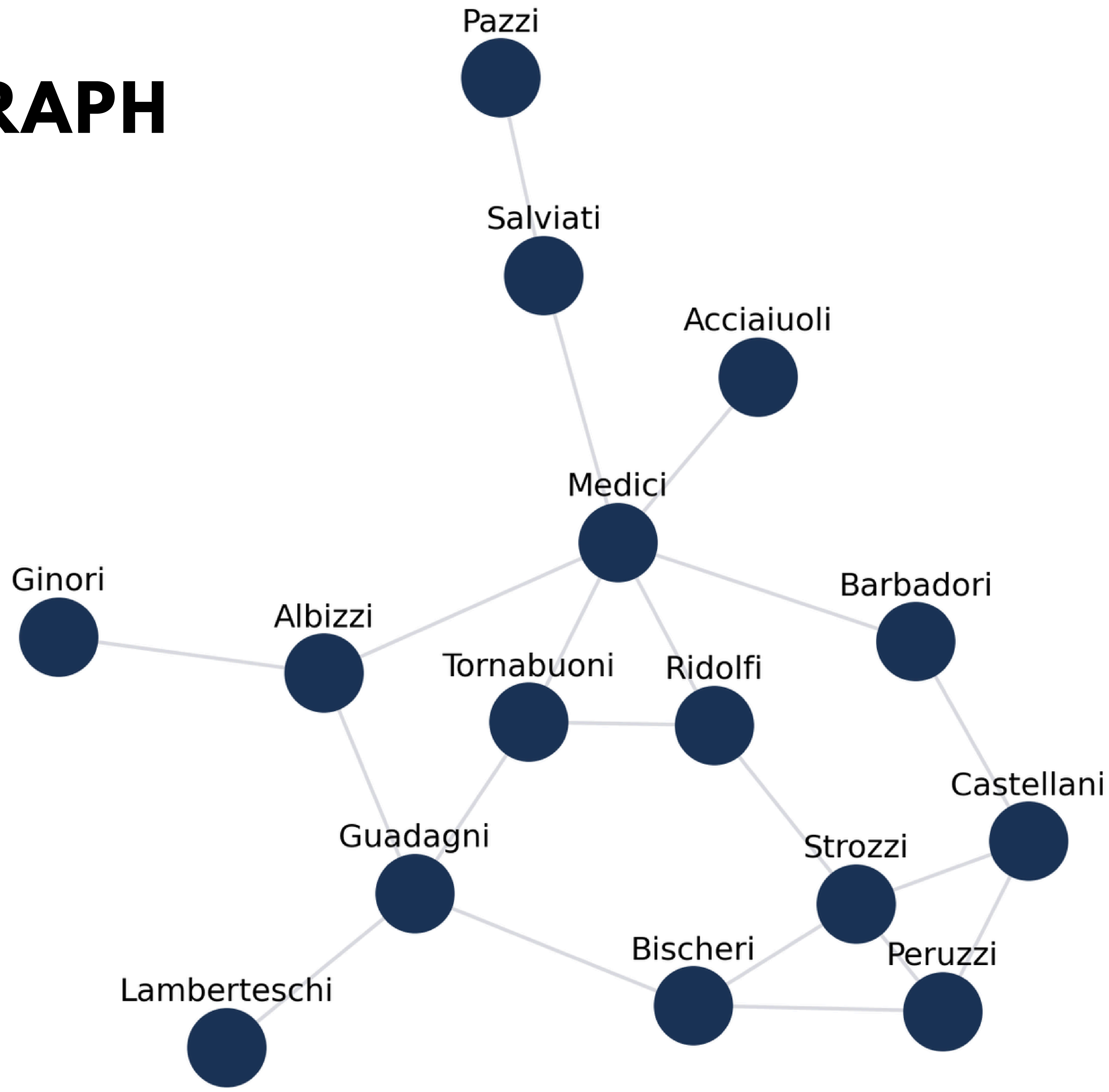
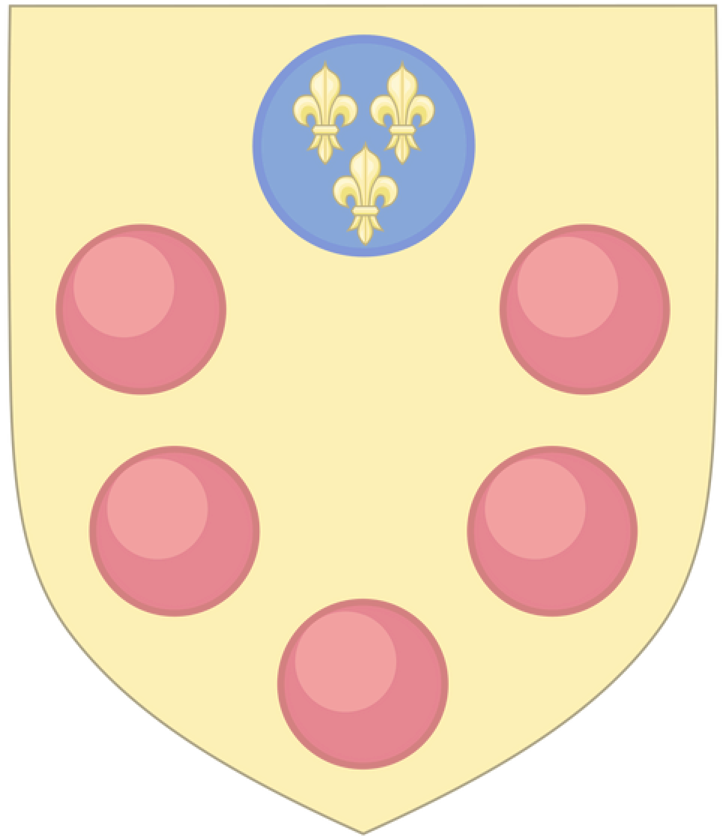


MEDICI



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ALBIZZI

Old-money wool-merchant dynasty.

Maso, and then Rinaldo degli Albizzi basically *were* the Florentine state until Cosimo showed them the door in 1434.

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Check out out paper!

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What do we want to keep track of with networks?

DEFINITION (NEIGHBORHOODS)

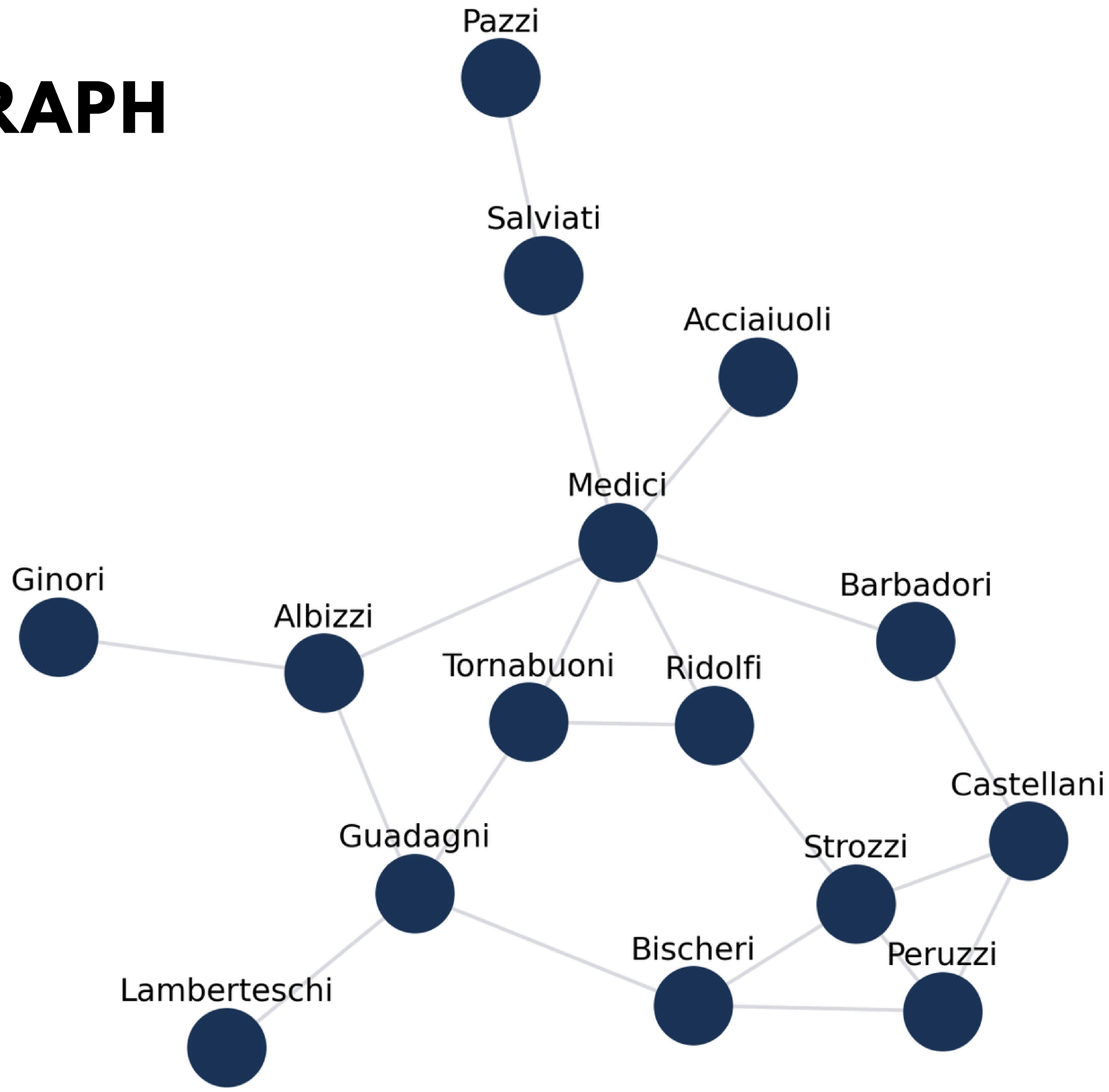
The *neighborhood* $N(i)$ of a node i is the set of nodes that are directly connected to i:

$$N(i) = \{j \in V \mid (i, j) \in E\}.$$

FLORENTINE FAMILIES GRAPH

Neighborhoods

The neighborhood of the Medici consists of the Salviati, Acciaiuoli, Barbadori, Ridolfi, Tornabuoni and Albizzi.



DEFINITION (DEGREE)

The *degree* d_i of a node i is the size of i's neighborhood:

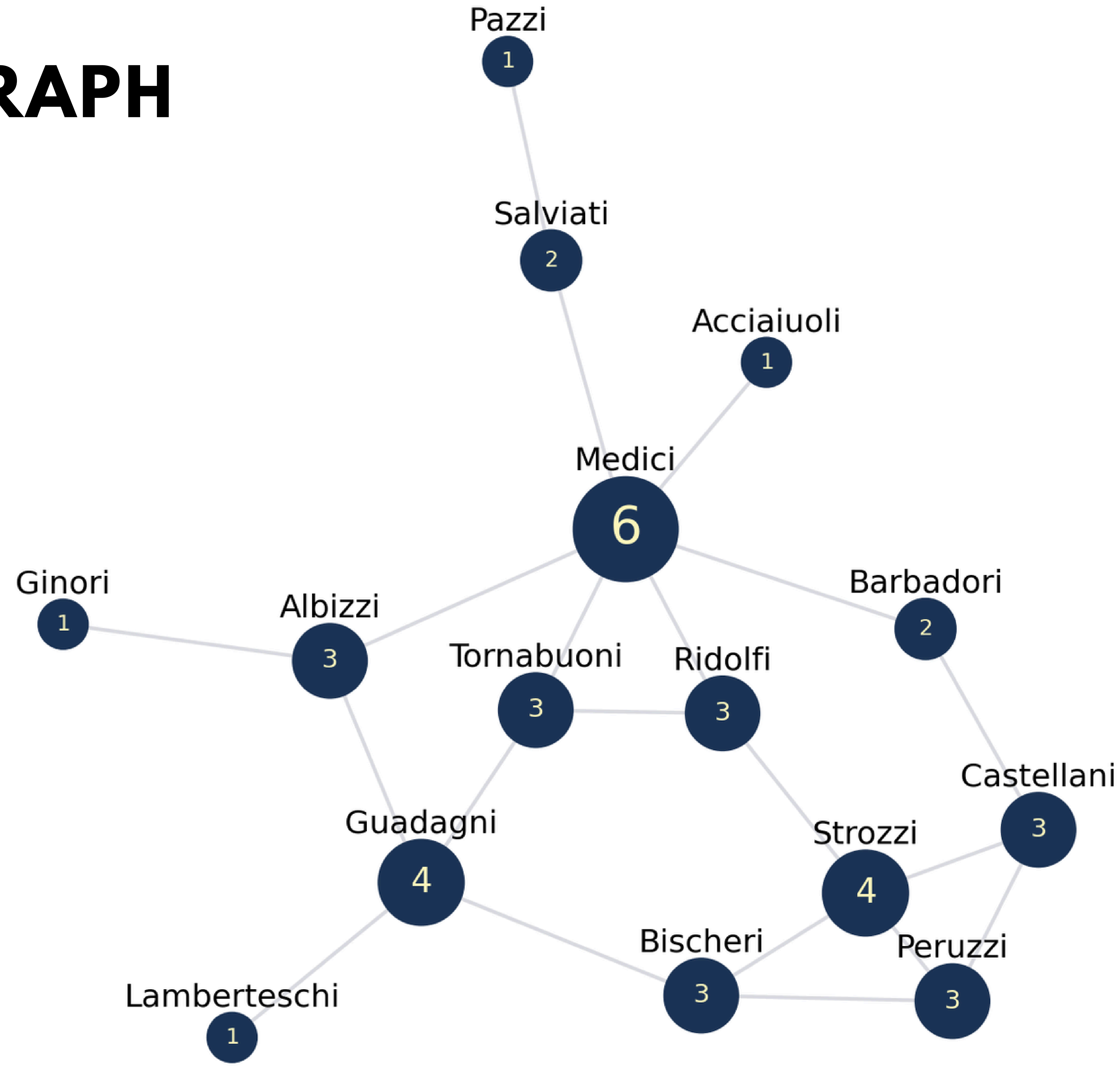
$$d_i = \left|N(i)\right|,$$

i.e., the number of nodes directly connected to i.

FLORENTINE FAMILIES GRAPH

Degrees

Highest degrees: the Medicis (6), the Guadagnis (4), the Strozzis (4).



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The *degree distribution* $P(d)$ of a network is a description of the relative frequencies of nodes that have different degrees.

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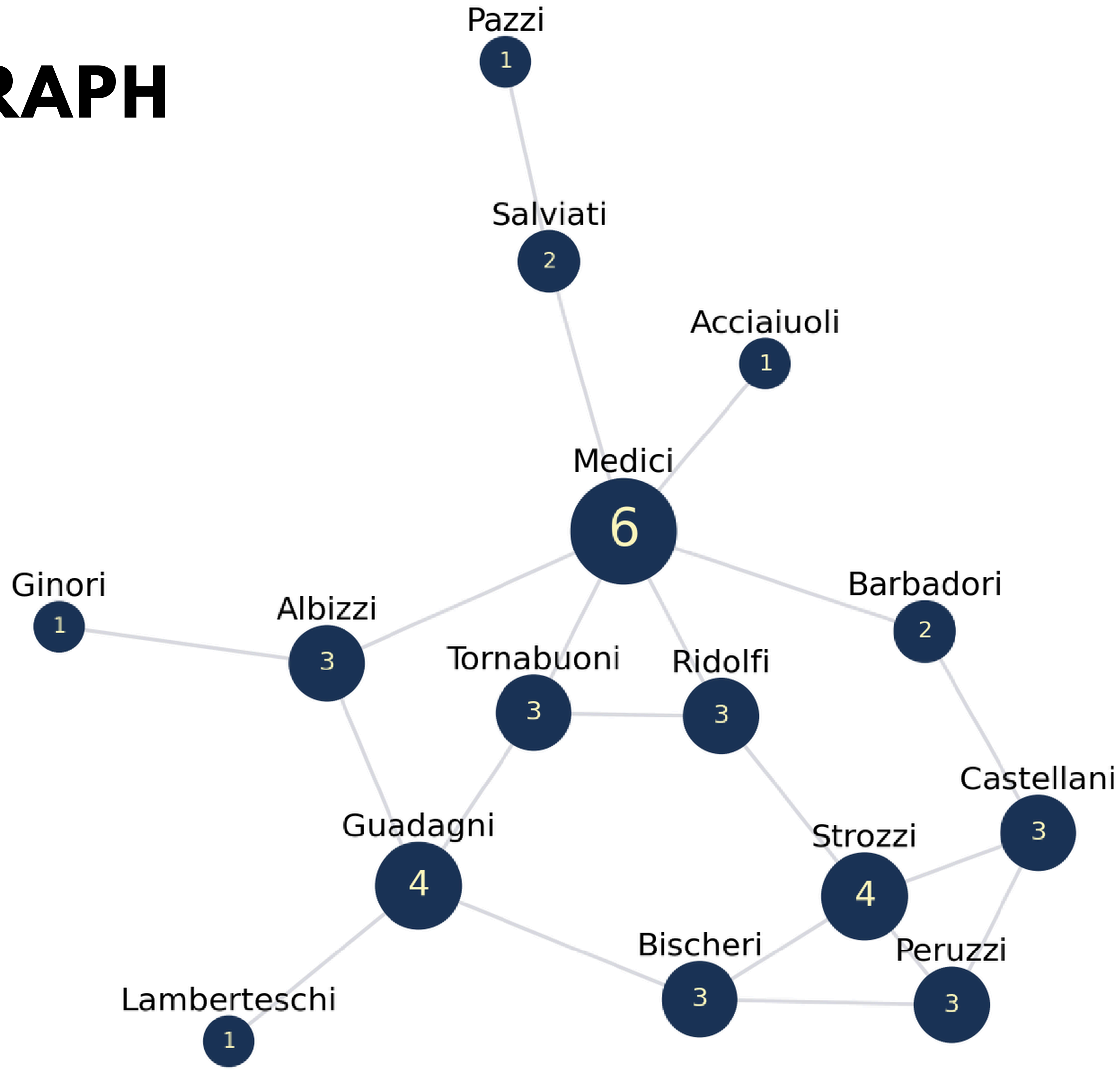
The function P can be a frequency distribution, if we are describing a specific network; or a probability distribution, if we are working with random networks.

FLORENTINE FAMILIES GRAPH

Degree distribution

Degree	Family
6	Medici
5	-
4	Guadagni, Strozzi
3	Tornabuoni, Ridolfi, Albizzi, Bischeri, Peruzzi, Castellani
2	Salviati, Barbadori
1	Pazzi, Ginori, Lamberteschi, Acciaiuoli



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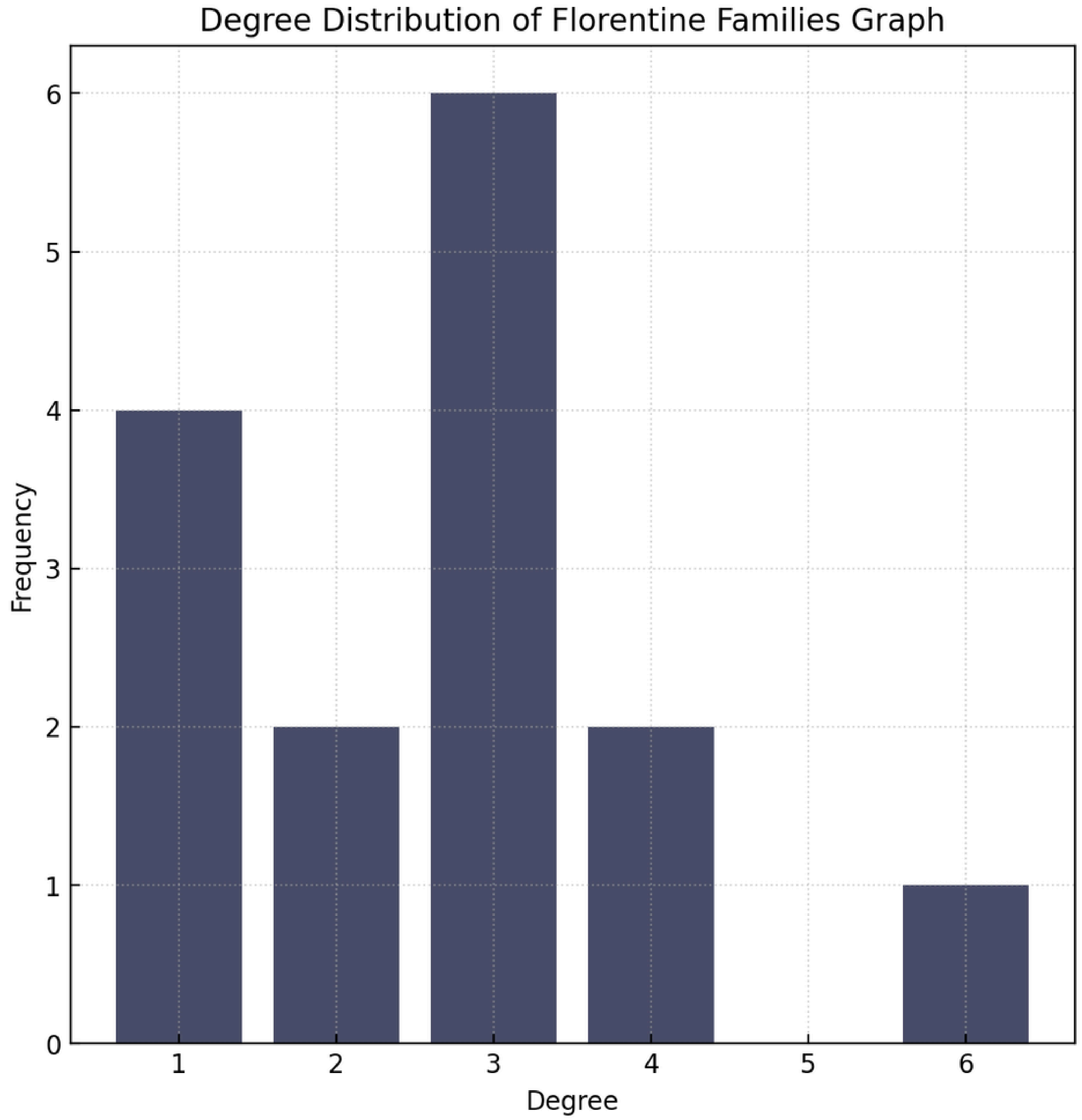


What kind of degree distributions are there?

What kind of degree distributions are there? Depends on the type of graph.

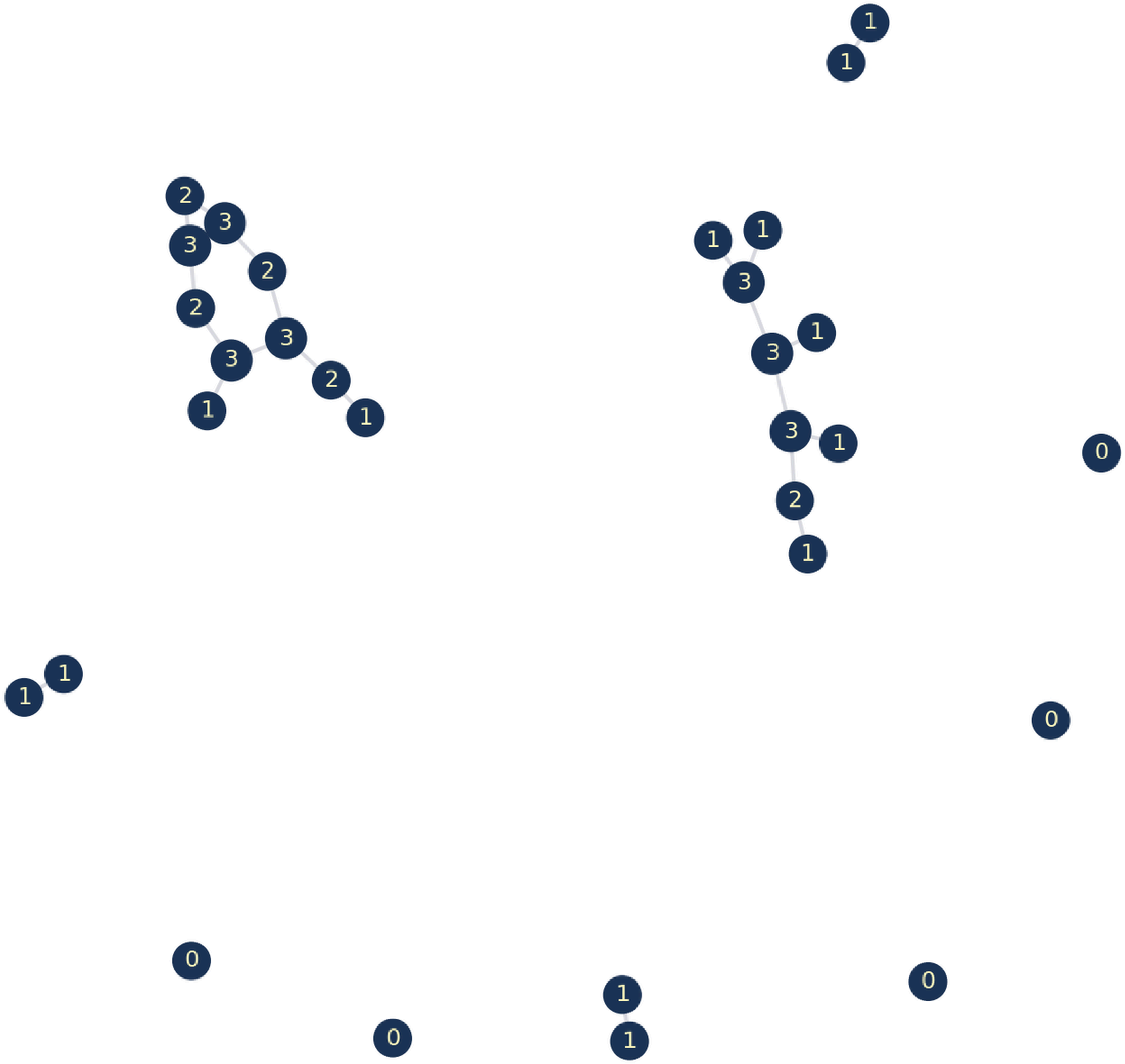
What kind of degree distributions are there? Depends on the type of graph. Here's one popular model.

DEFINITION (ERDŐS–RÉNYI MODEL)

For a set of n nodes and a probability p, the *Erdős-Rényi model* generates a random graph $G(n, p)$ by going through all possible pairs of nodes and adding an edge between them with probability p.

$G(n, p)$ EXAMPLE

$$n = 30, p = 0.05$$



*Node labels are the degrees.

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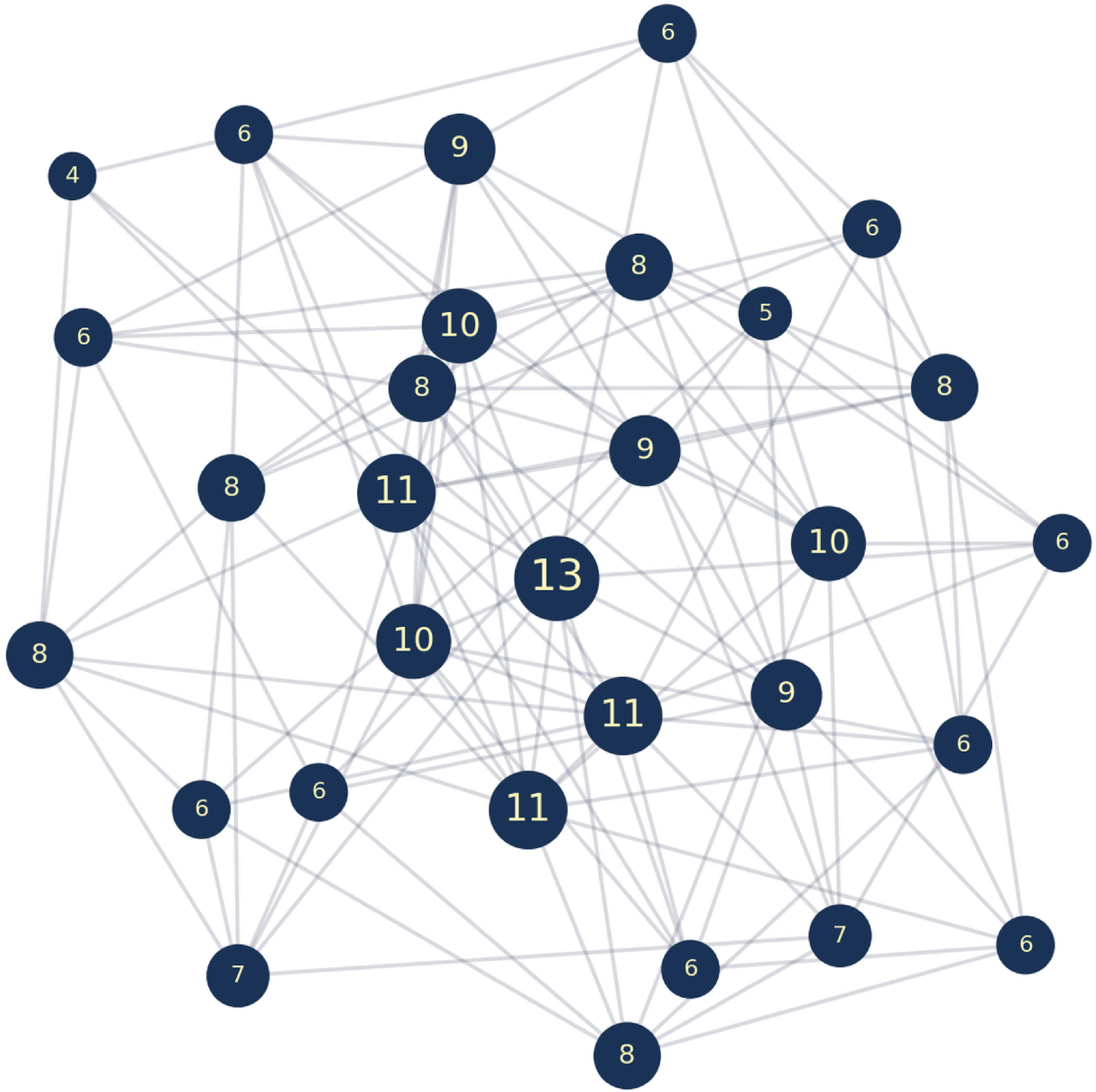

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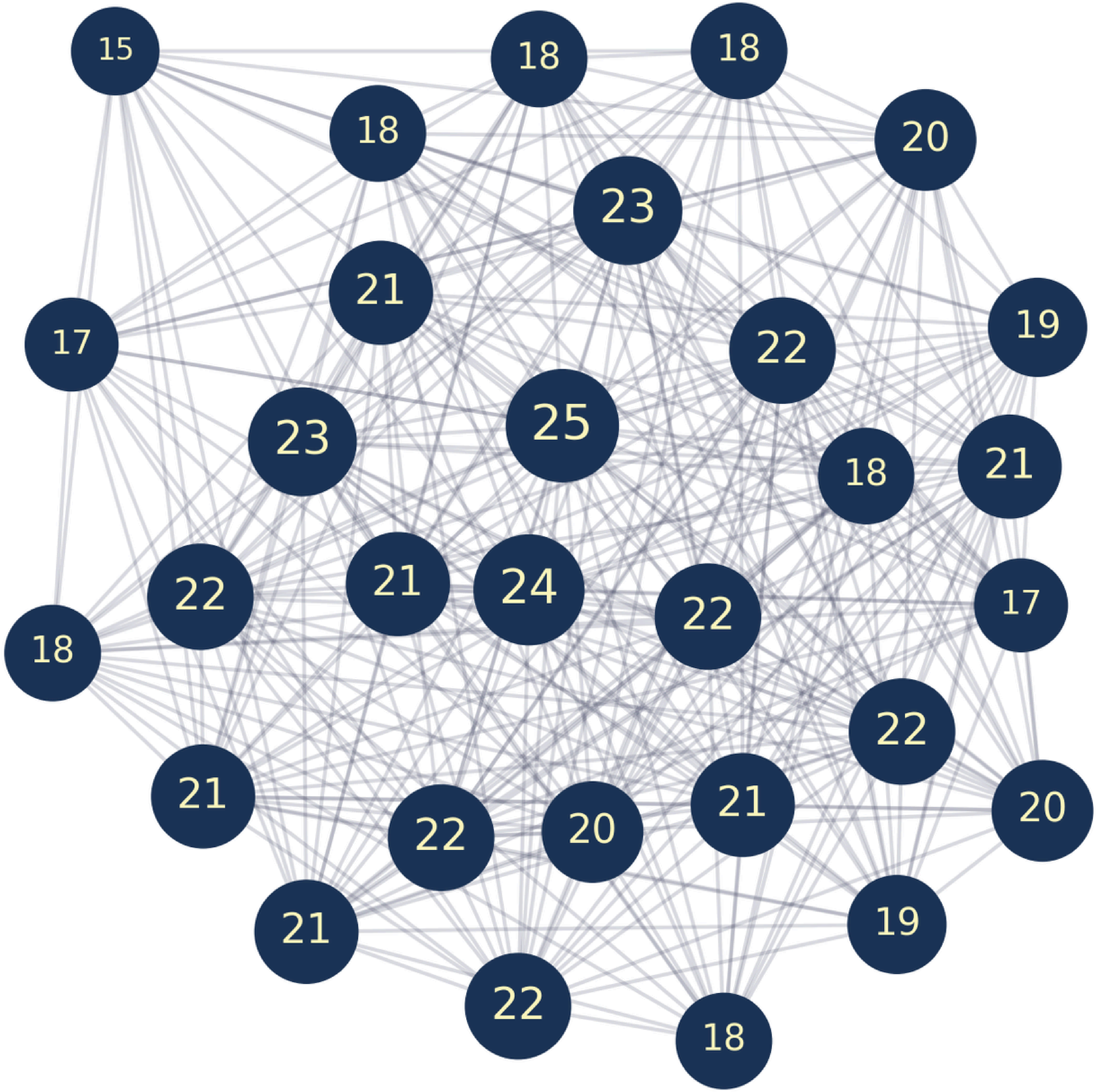
$G(n, p)$ EXAMPLE

$$n = 30, p = 0.7$$



*Node labels are the degrees.

What does the degree distribution for random graphs look like?

$G(n, p)$

Degree distribution

The degree distribution of an Erdős-Rényi random graph $G(n, p)$ is given by the *binomial distribution*:

$$\Pr\left[d = k\right] = \binom{n-1}{k} p^k (1-p)^{n-1-k}.$$

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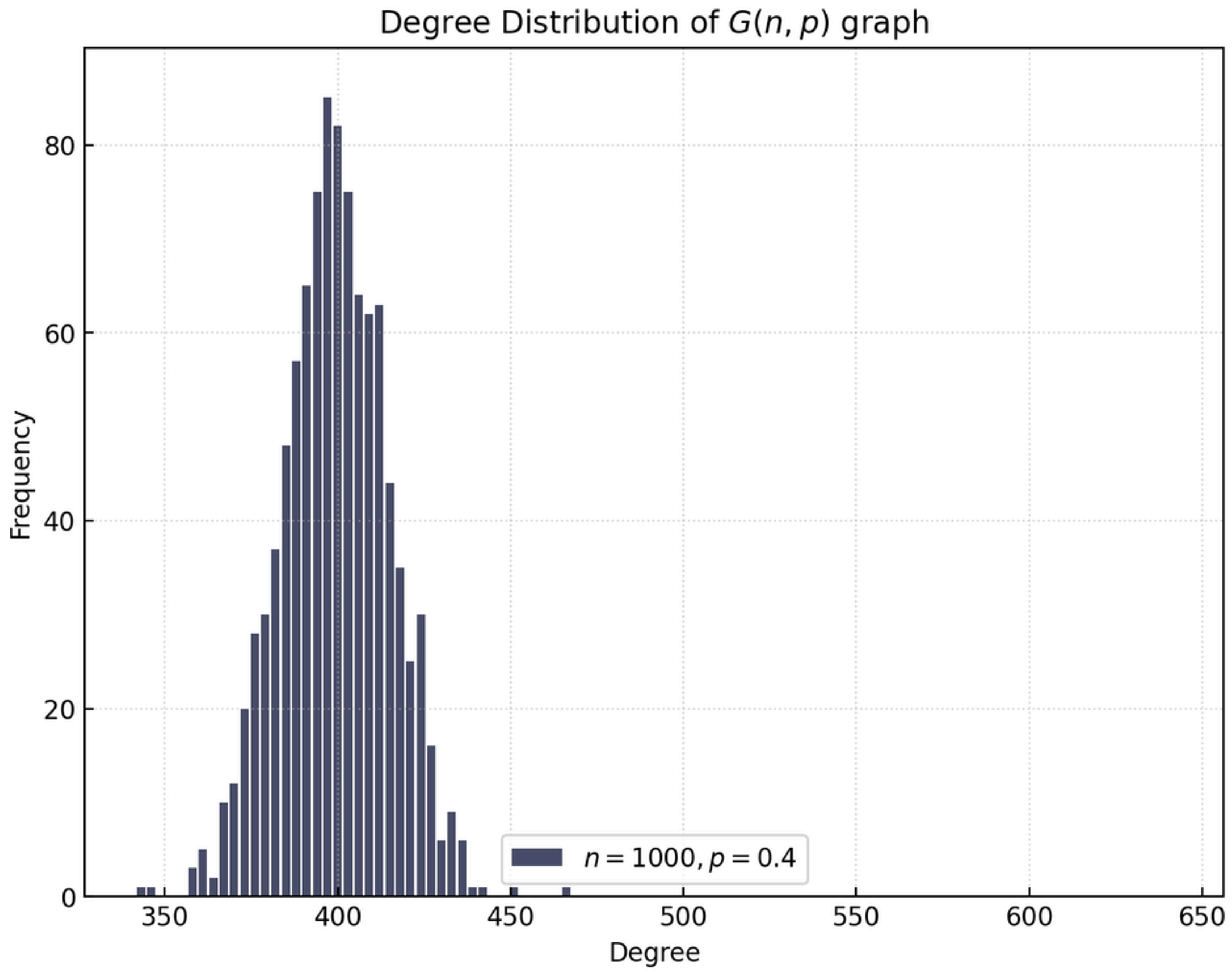
The ‘typical’ node has degree $(n-1)p$.



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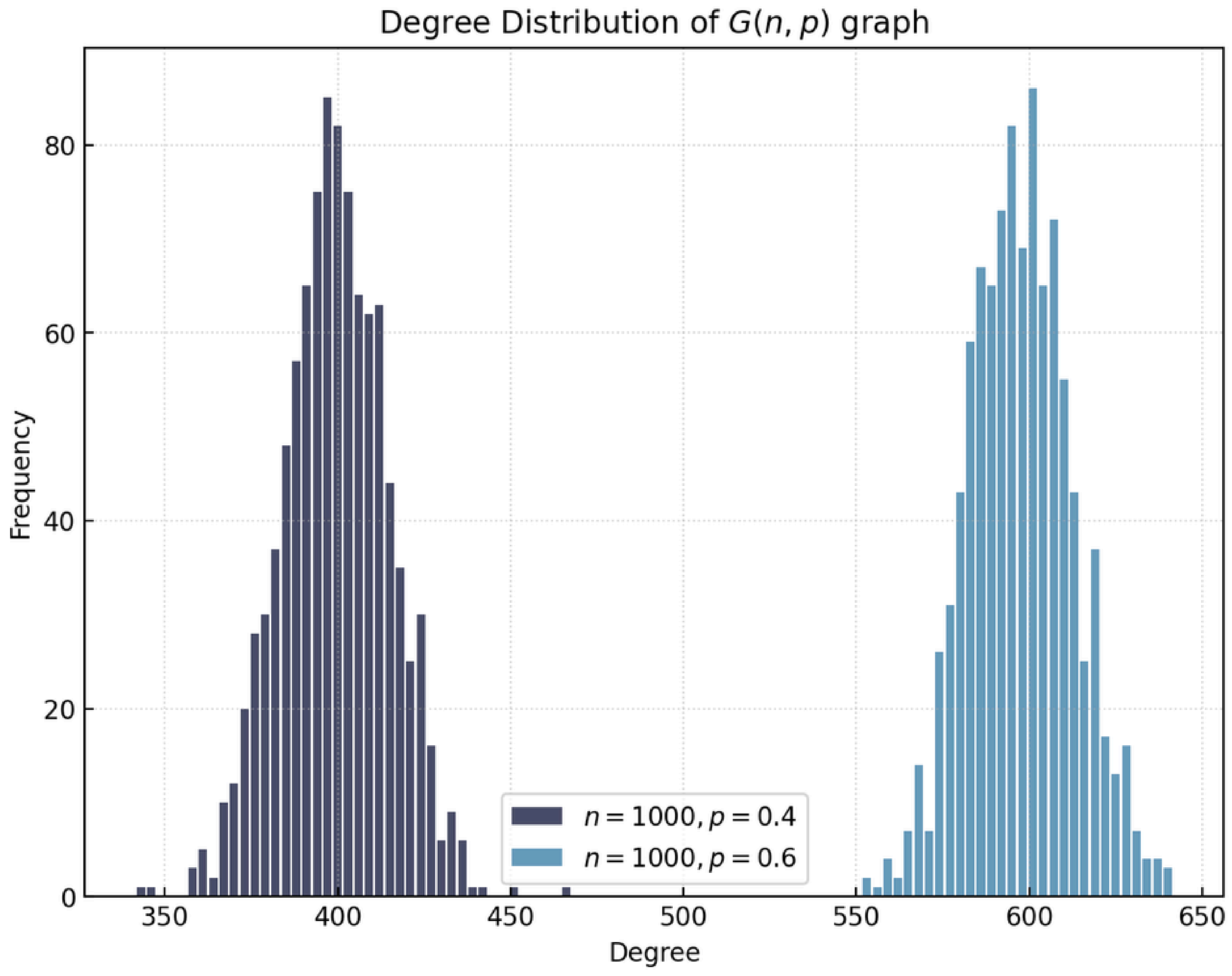
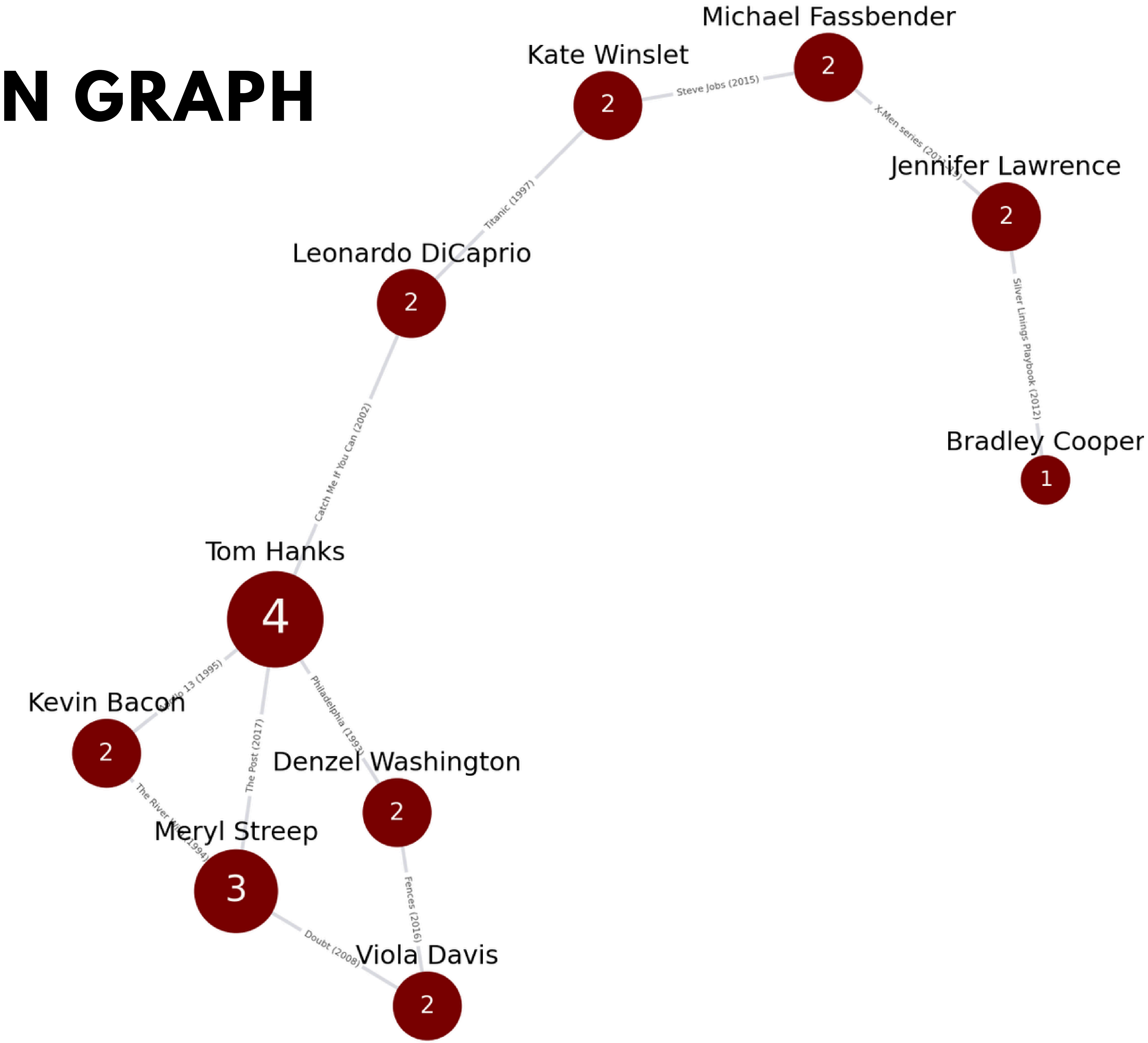


Unfortunately, many real world social networks do not look like random graphs.

ACTOR COLLABORATION GRAPH

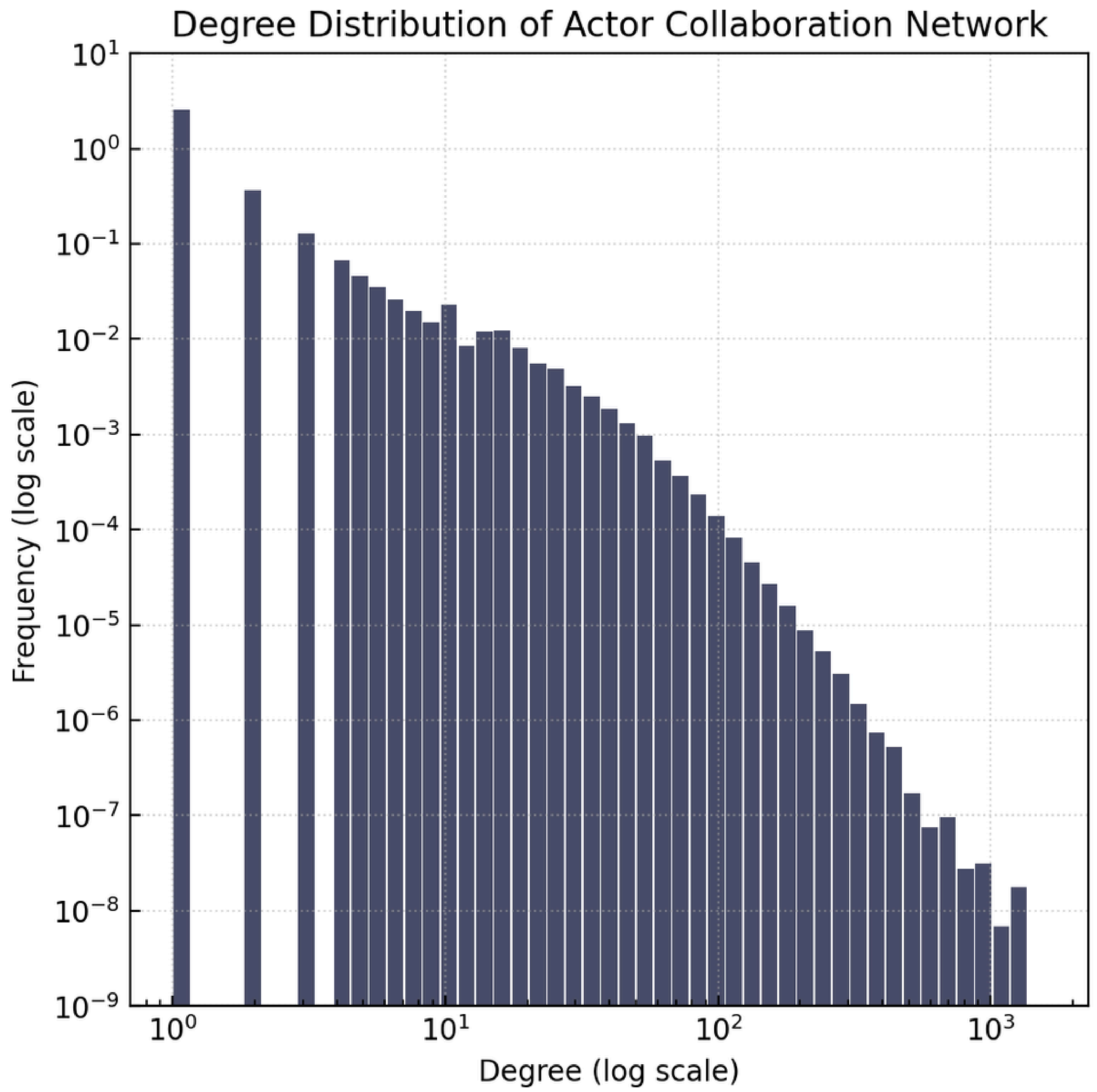
Actors are connected if they acted together in a movie.



ACTOR COLLABORATION GRAPH

Degree distribution

Many nodes of low degree, some of middling degree, a couple with high degree.



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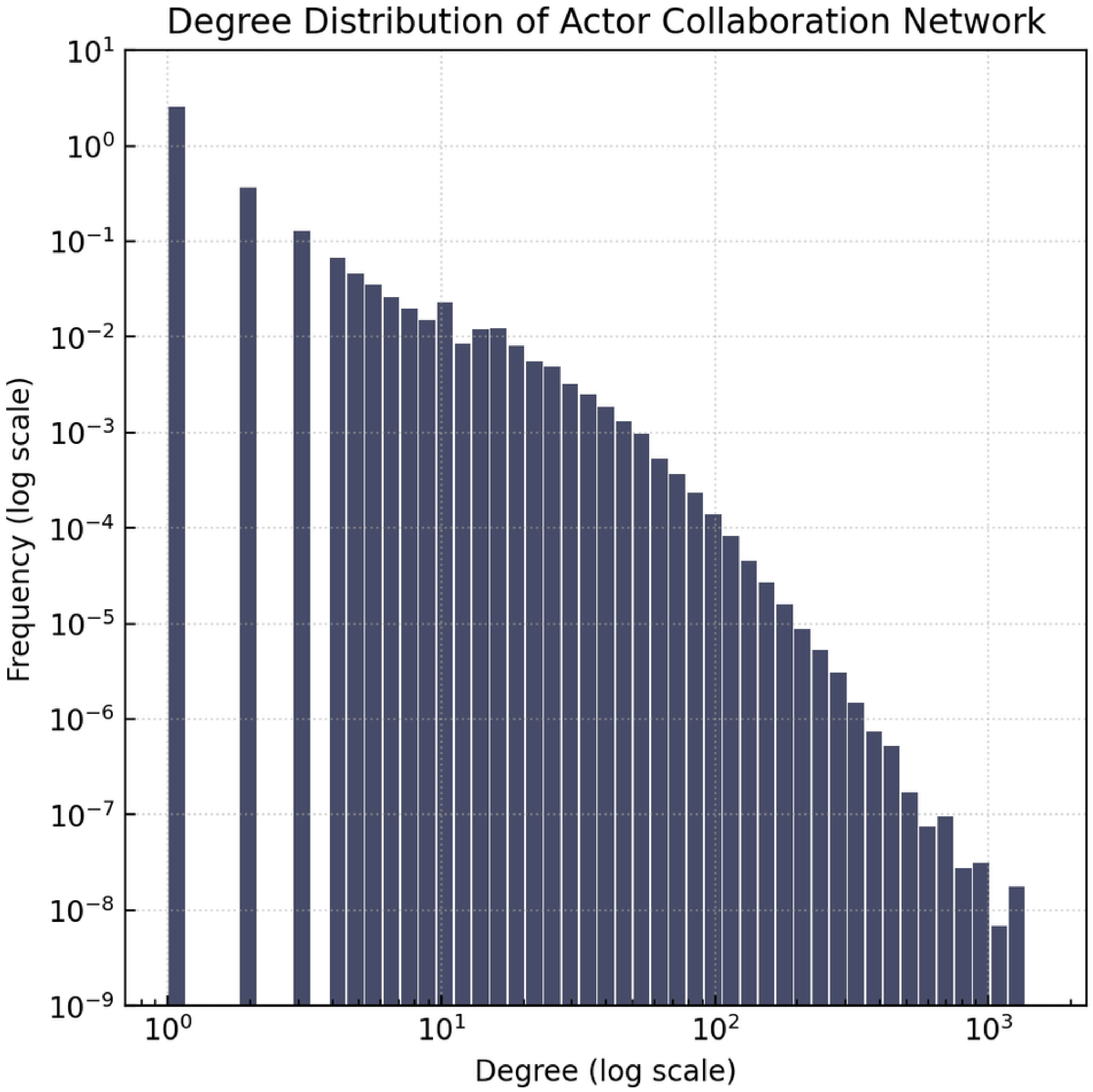
No ‘typical’ node.



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DEFINITION (SCALE FREE NETWORK)

A *scale-free network* is a type of network whose degree distribution follows a power law:

$$P(d) \sim c \cdot d^{-\gamma},$$

where $c > 0$ and γ is a constant typically in the range $2 < \gamma < 3$.

SCALE FREE NETWORKS

Intuition

The relative frequencies stay constant as the degree grows from d to kd:

$$\begin{aligned}\frac{P(kd)}{P(d)} &= \frac{c \cdot k^{-\gamma} d^{-\gamma}}{c \cdot d^{-\gamma}} \\ &= k^{-\gamma}.\end{aligned}$$

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This is true regardless of the degree d we start with:

$$\frac{P(2)}{P(1)} = \frac{P(20)}{P(10)} = \frac{P(200)}{P(100)} = \dots$$

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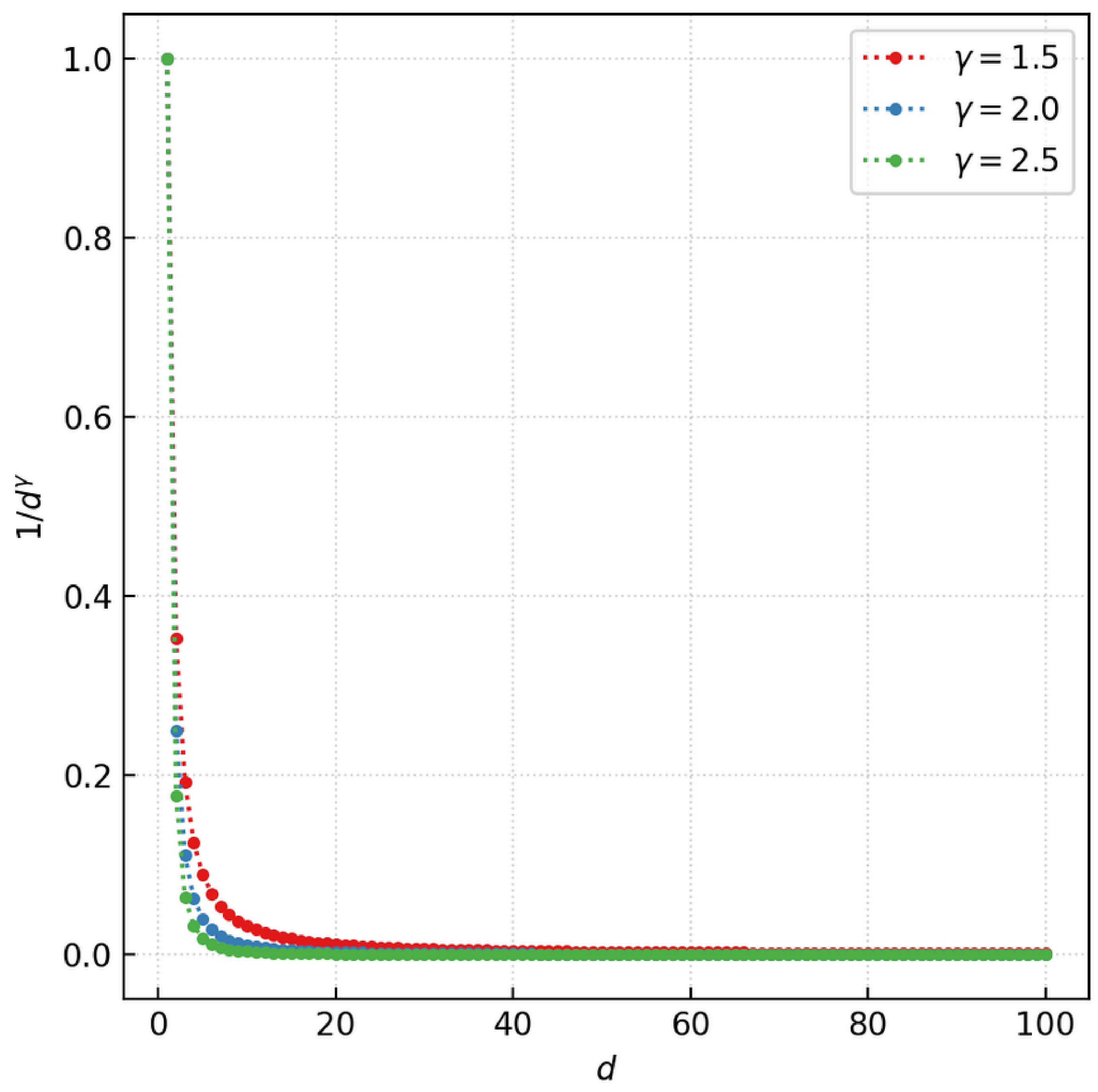
Best seen when graphing the degree distribution…



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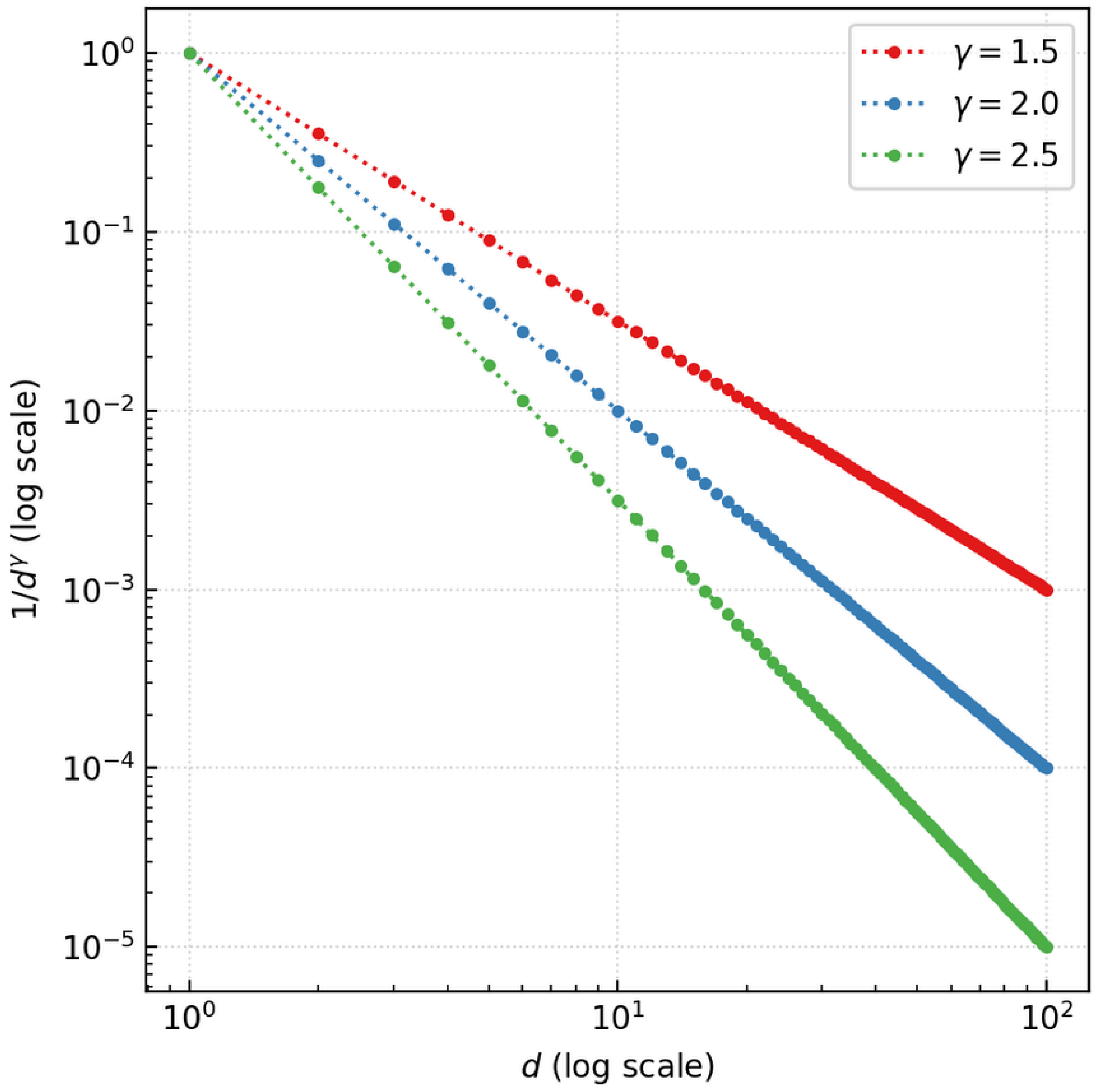
...on a *log-log scale*, for readability: the degree distribution of a scale-free network appears as a straight line with slope $-\gamma$.



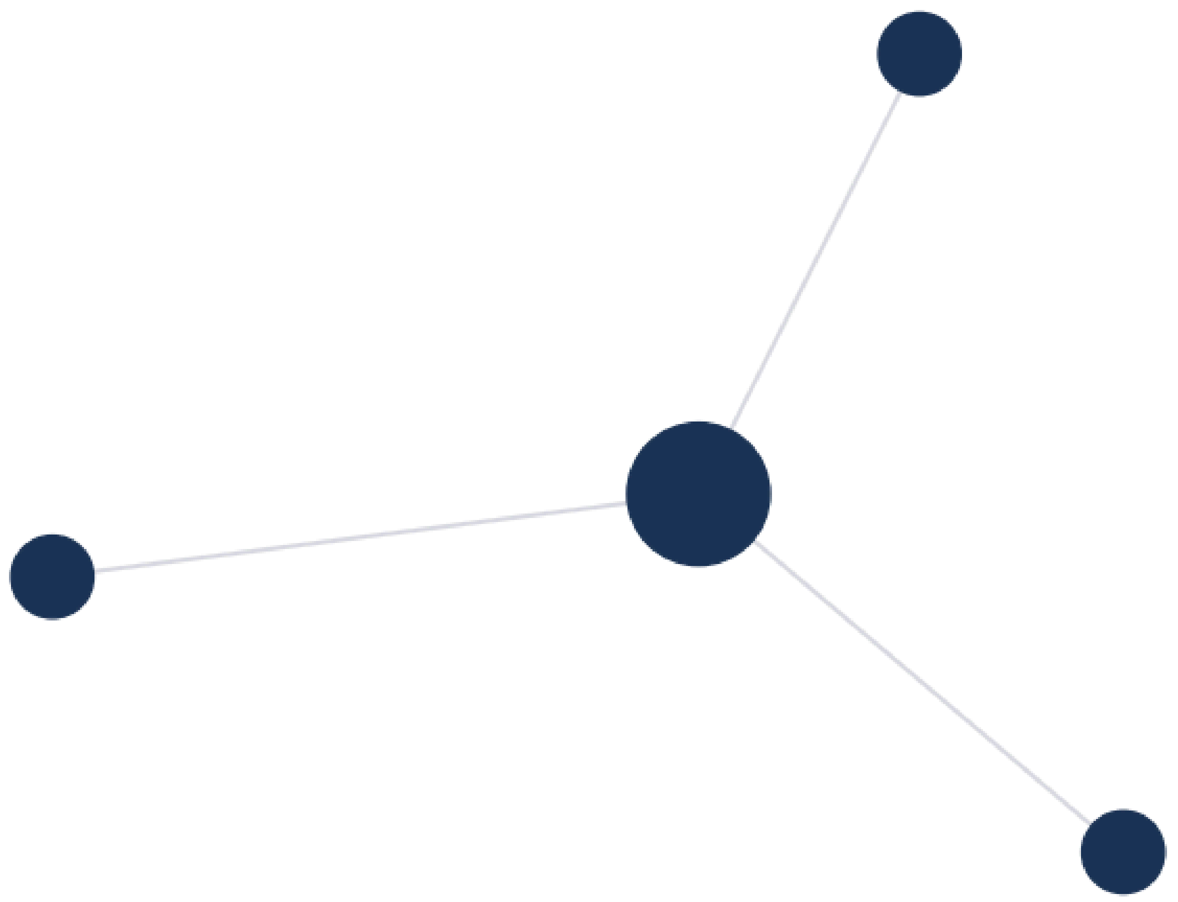
How can we generate scale-free networks?

DEFINITION (BARABÁSI–ALBERT MODEL)
Start with m_0 nodes. New nodes are added, one at a time. Each new node connects to an existing node i with a probability p_i proportional to i's degree:

$$p_i = \frac{d_i}{\sum_j d_j}.$$

BARABÁSI–ALBERT MODEL
Intuition

In this process, nodes that are already well-connected get more connections.



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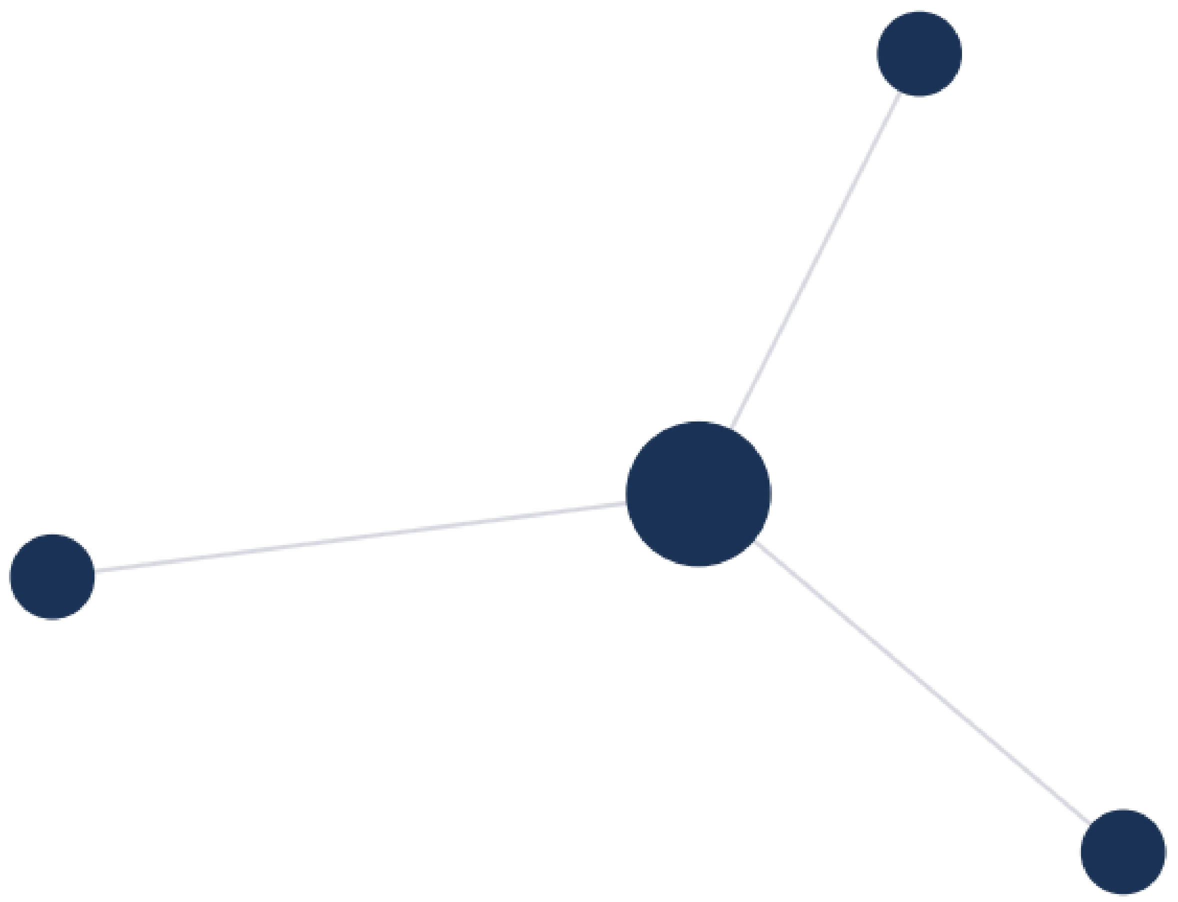
In this process, nodes that are already well-connected get more connections.

A richer-get-richer type of effect.



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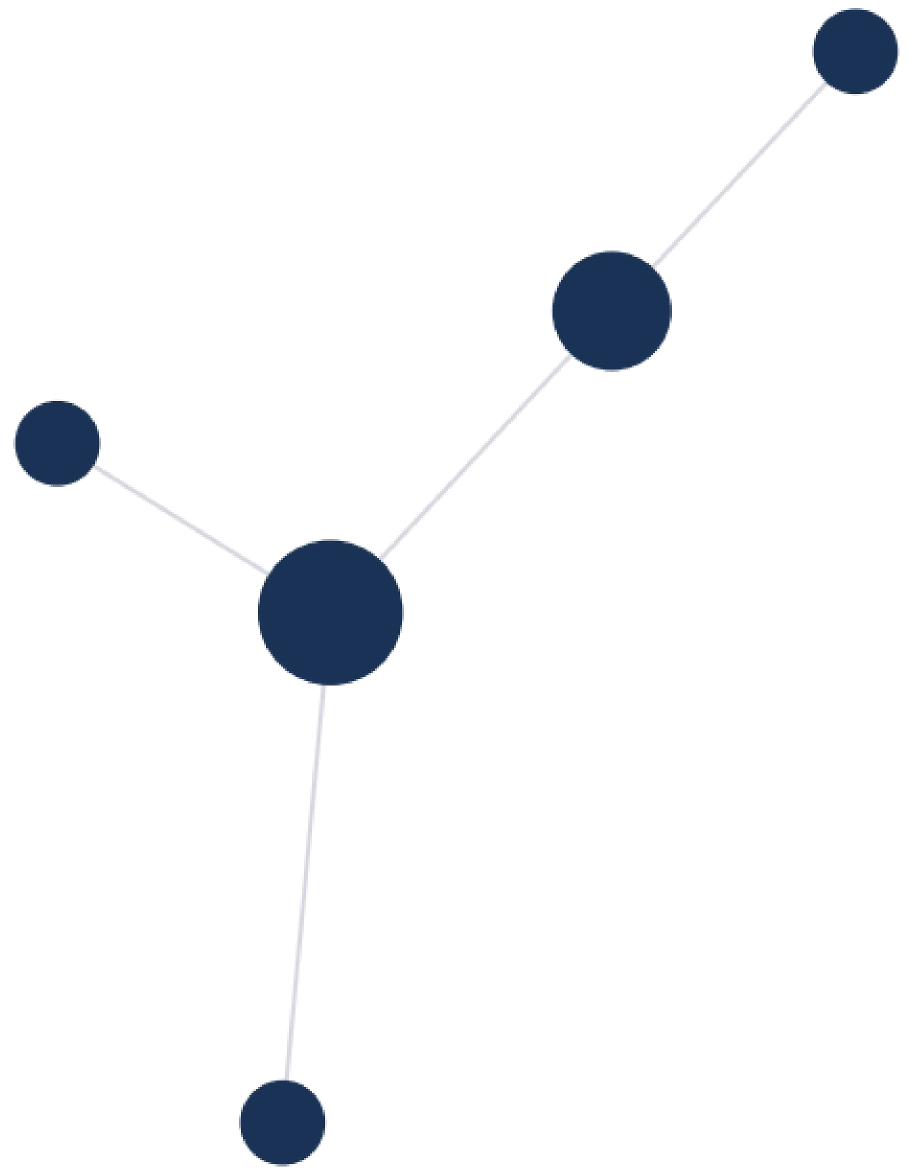

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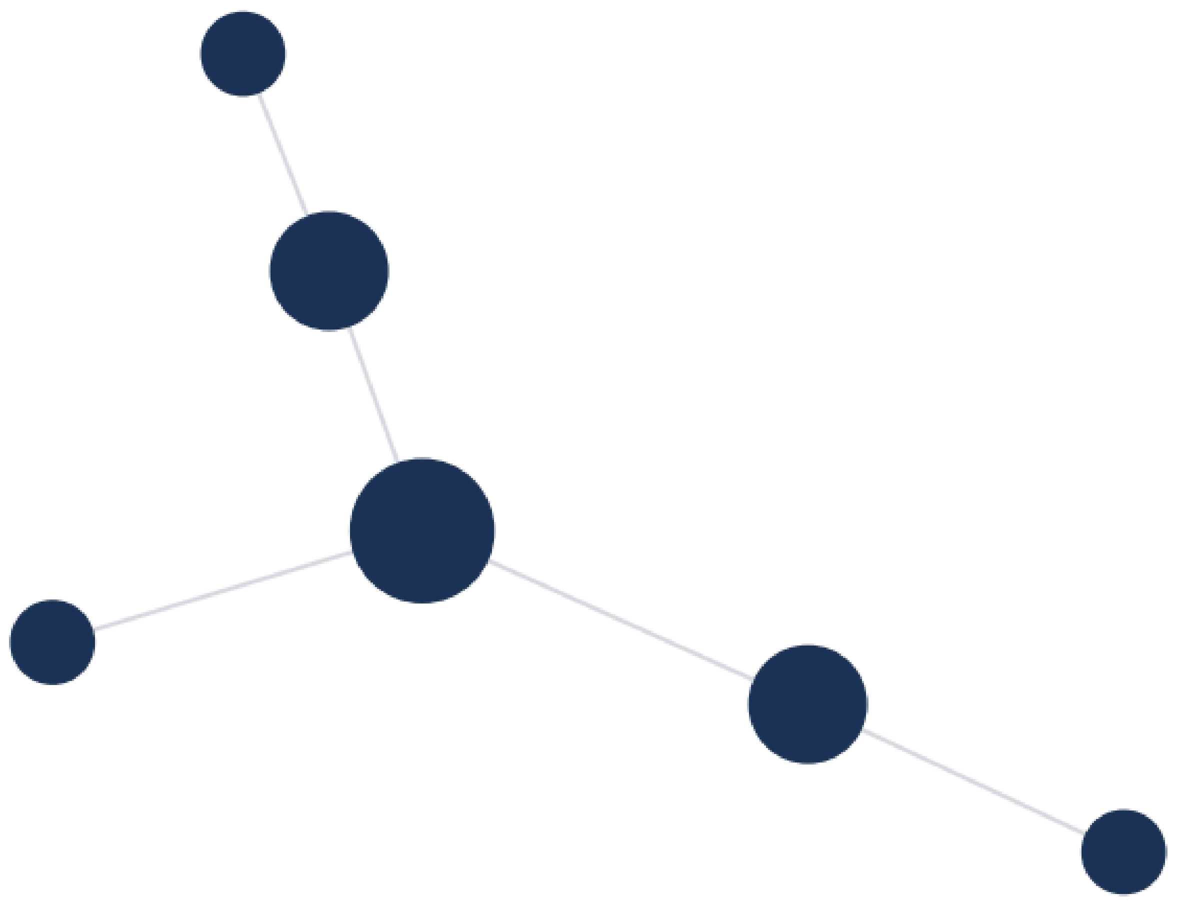
Leads to the emergence of hubs.



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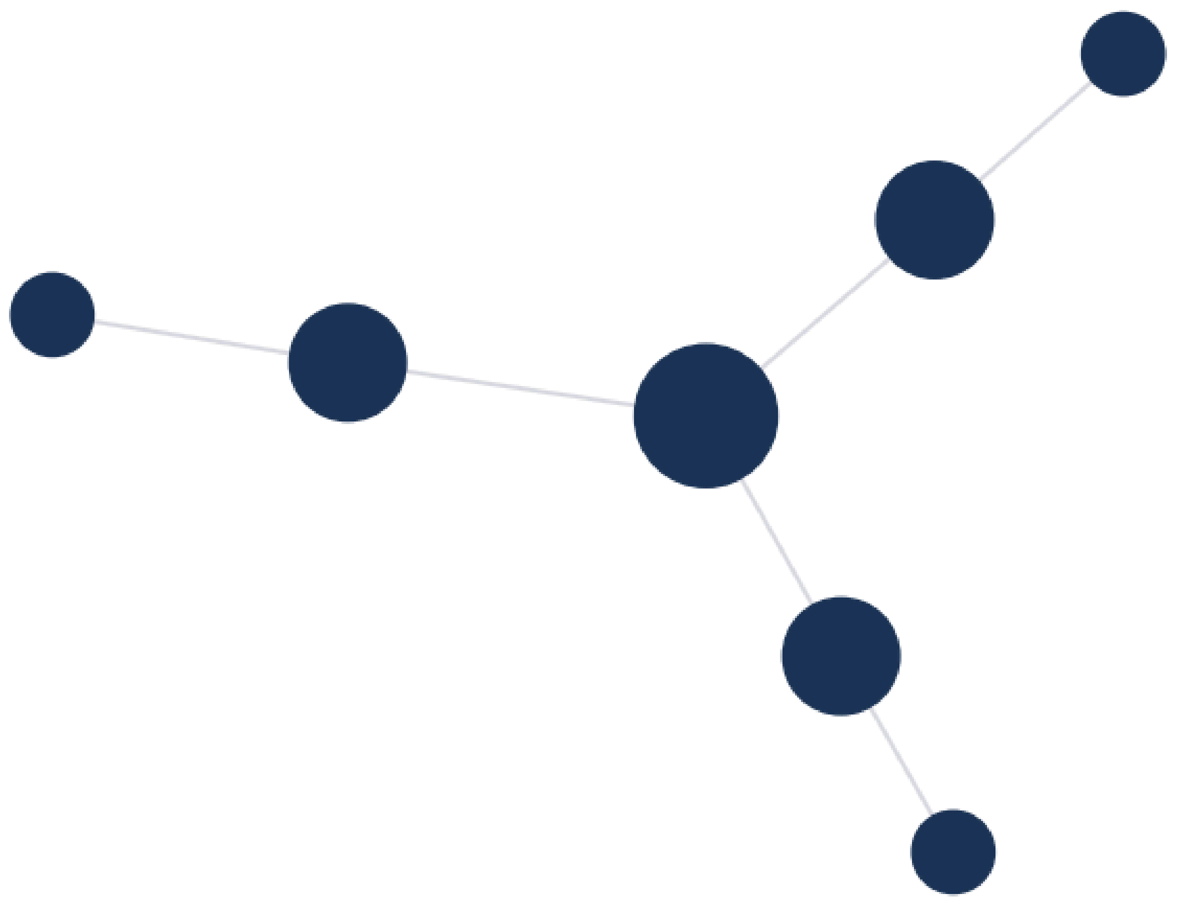

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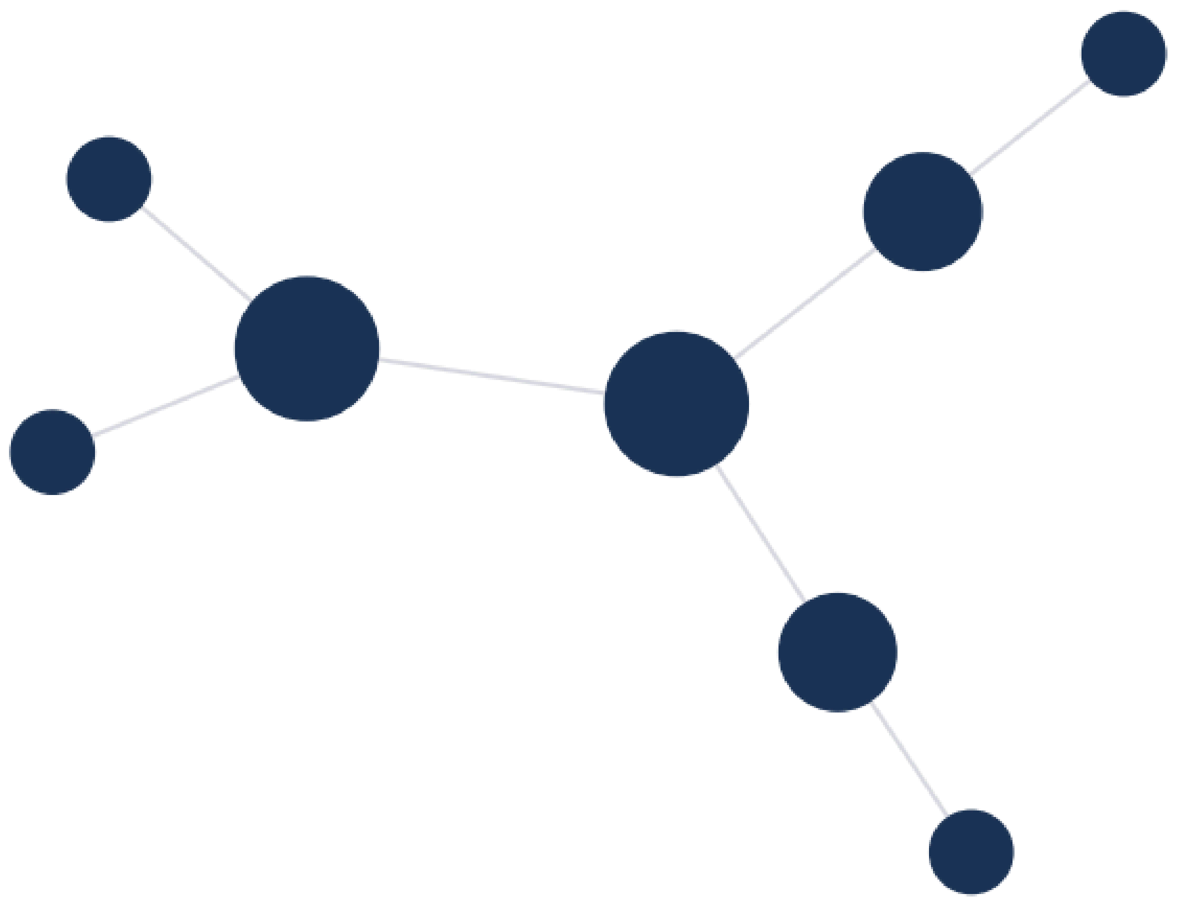

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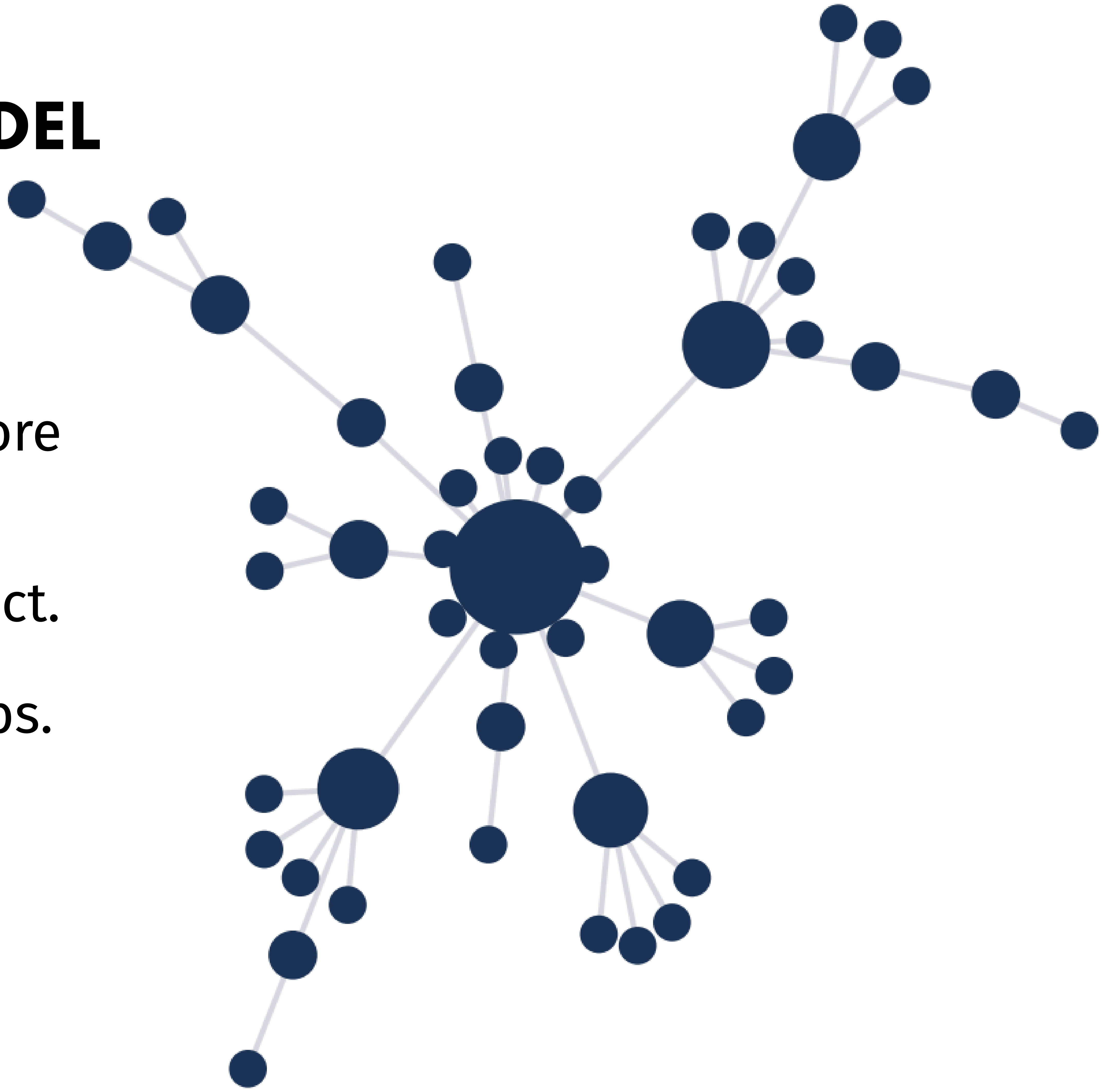
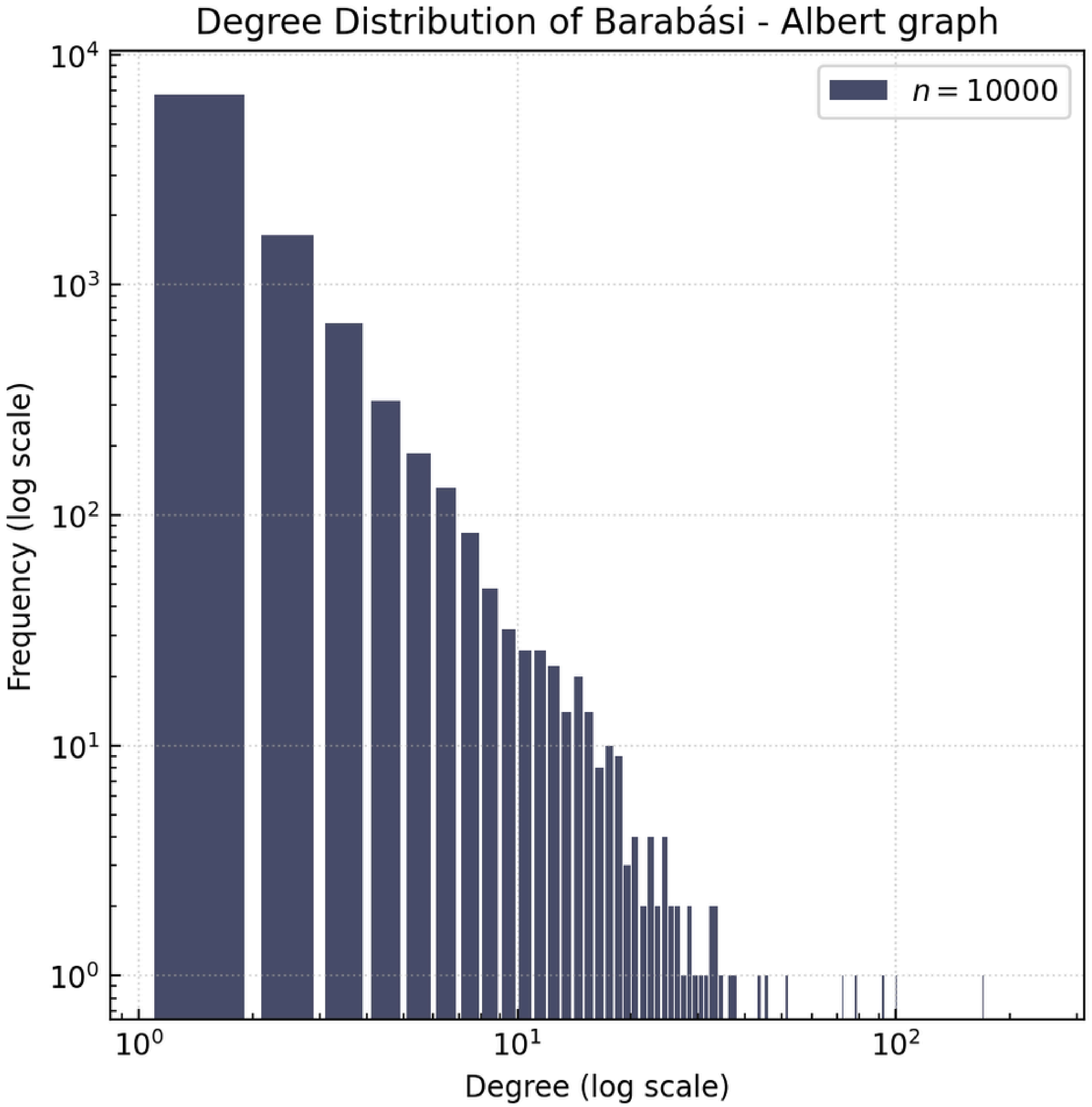


BARABÁSI–ALBERT MODEL
Degree distribution

We see the expected pattern in the degree distribution.



Another thing we want to look at are the distances between the nodes.

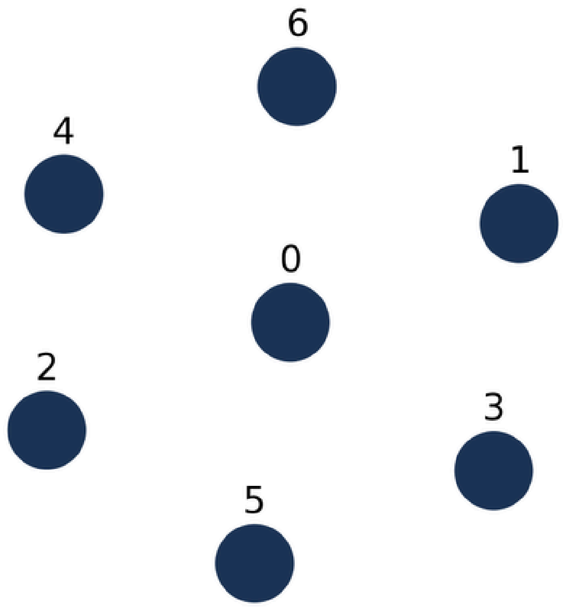
DEFINITION (DISTANCE BETWEEN NODES)

The *distance* $d(i, j)$ between two nodes i and j in a network is the length of the shortest path between i and j.

PATHS AND STARS AND TREES

Distances

G_{empty}



$d(i, j) = \infty, \text{for all } i, j$

PATHS AND STARS AND TREES

Distances

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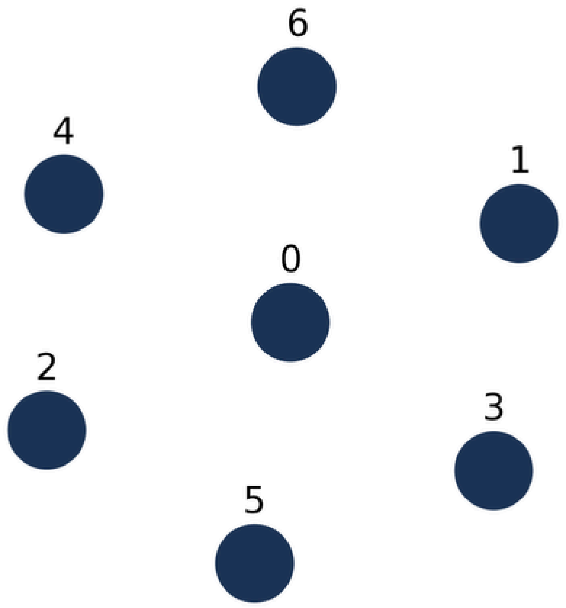

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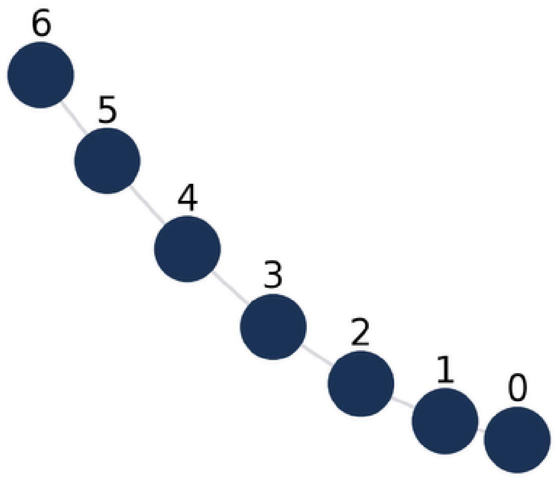
G_{path}



$d(1,4) = 3$

PATHS AND STARS AND TREES

Distances

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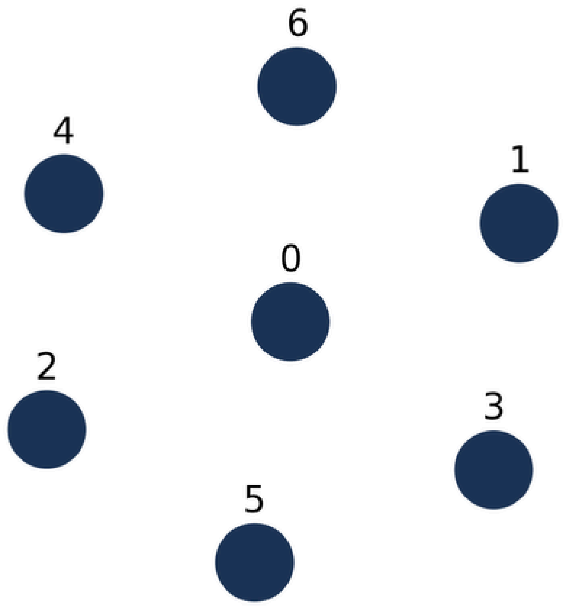

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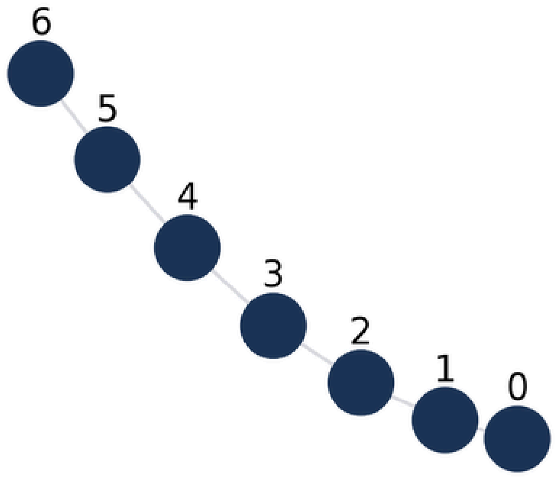
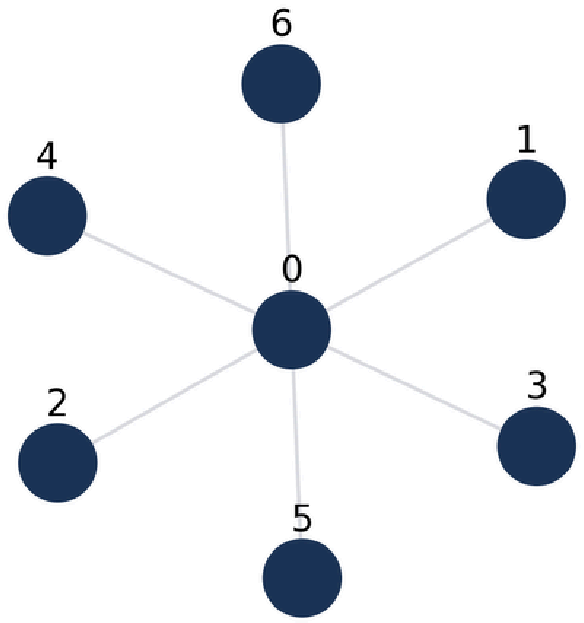


$d(1, 4) = 3$

G_{star}

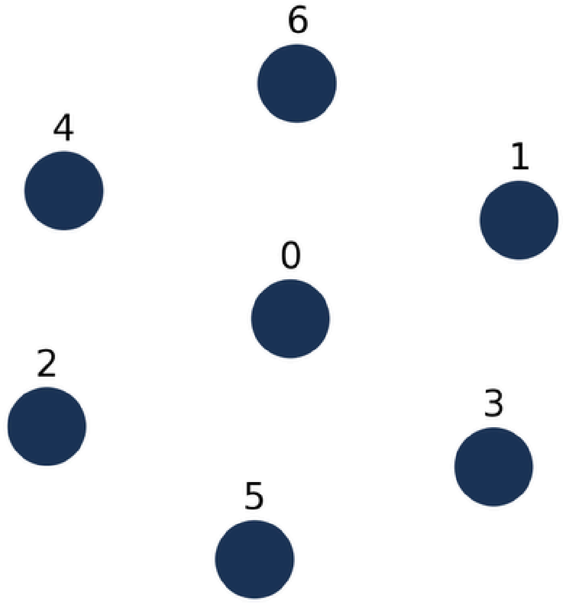


$d(1, 2) = 2$

PATHS AND STARS AND TREES

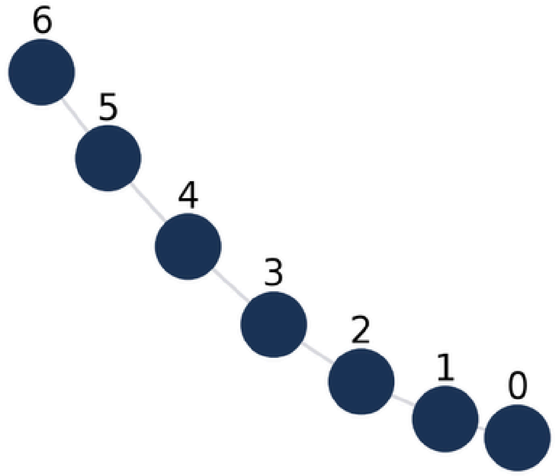
Distances

G_{empty}



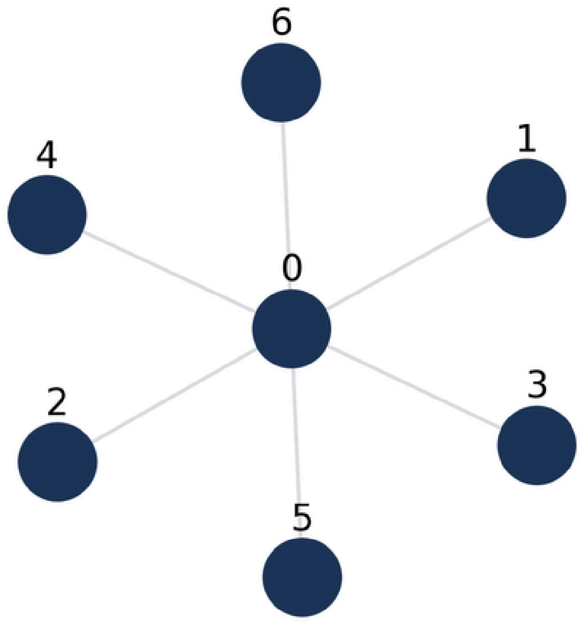
$d(i, j) = \infty, \text{for all } i, j$

G_{path}



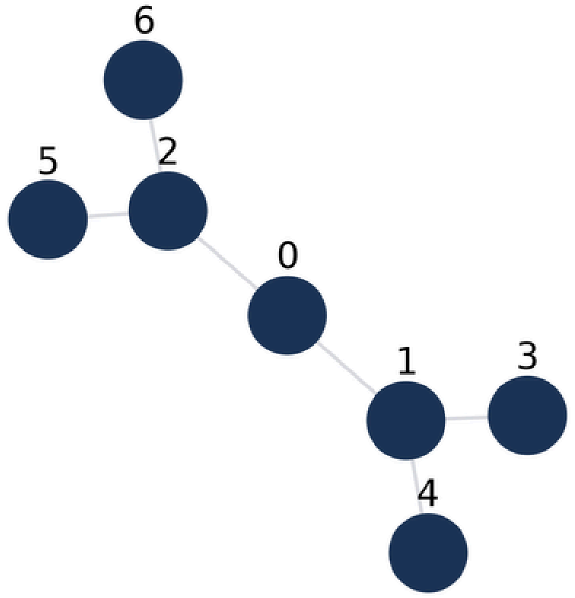
$d(1, 4) = 3$

G_{star}



$d(1, 2) = 2$

G_{tree}

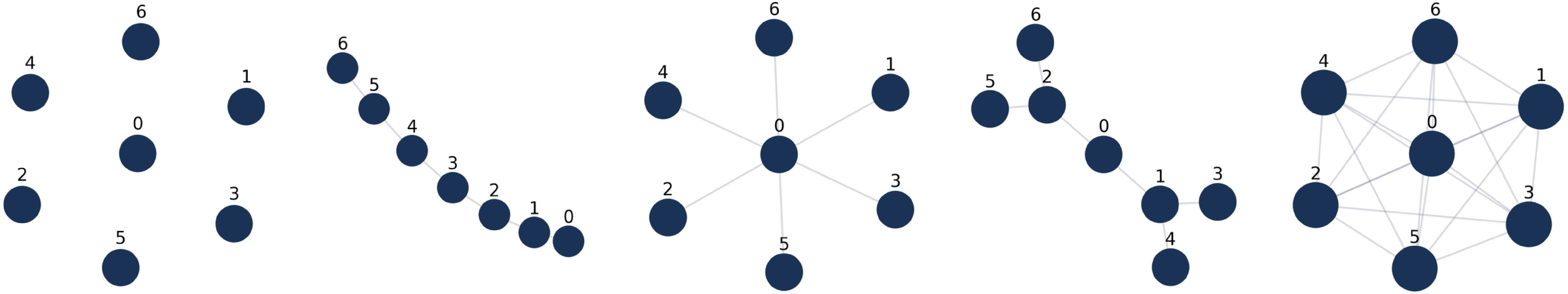


$d(4, 6) = 4$

PATHS AND STARS AND TREES

Distances

G_{empty} G_{path} G_{star} G_{tree} G_{complete}



$d(i,j) = \infty, \text{for all } i, j$ $d(1,4) = 3$ $d(1,2) = 2$ $d(4,6) = 4$ $d(i,j) = 1, \text{for any } i, j$

DEFINITION (DIAMETER)

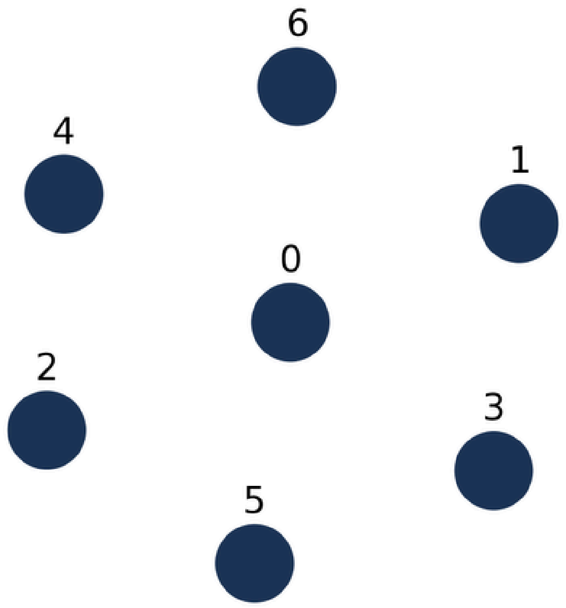
The *diameter* of a network G is the maximum distance between any two nodes:

$$\text{diam}(G) = \max_{i,j \in V} d(i,j).$$

PATHS AND STARS AND TREES

Diameters

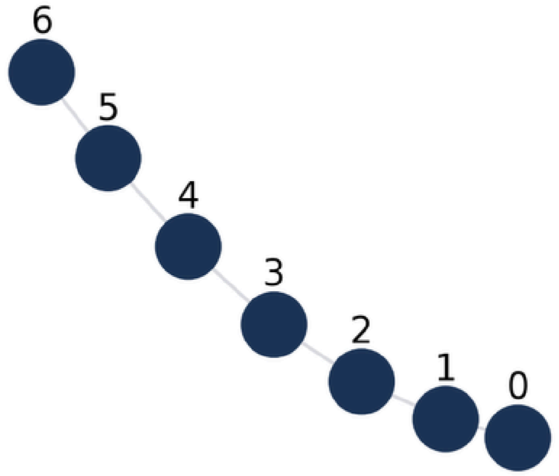
G_{empty}



$d(i,j) = \infty$, for all i, j

$\text{diam}(G_{\text{empty}}) = \infty$

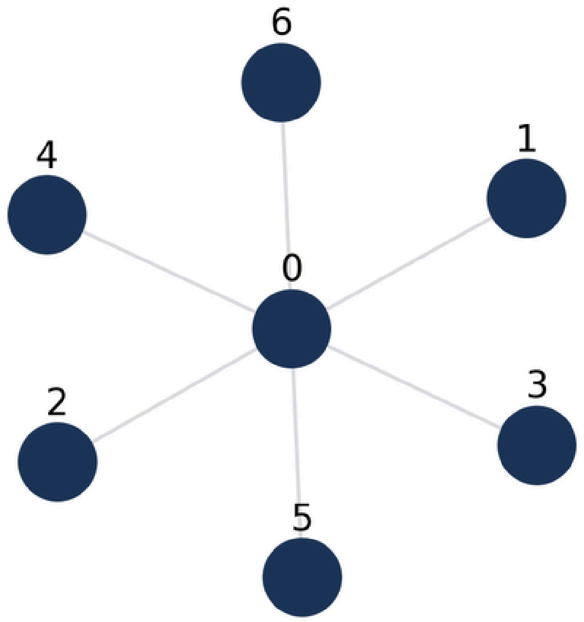
G_{path}



$d(1,4) = 3$

$\text{diam}(G_{\text{path}}) = 5$

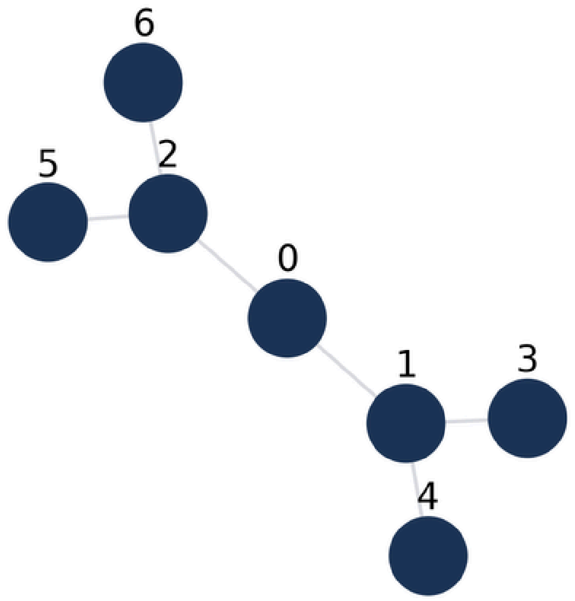
G_{star}



$d(1,2) = 2$

$\text{diam}(G_{\text{star}}) = 2$

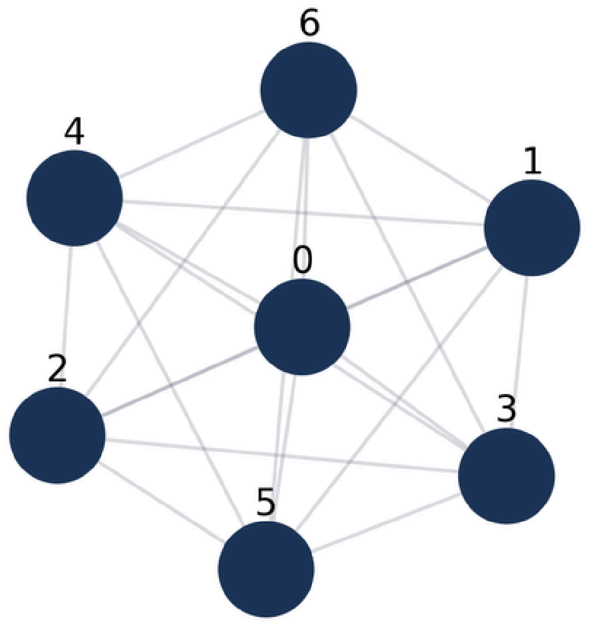
G_{tree}



$d(4,6) = 4$

$\text{diam}(G_{\text{tree}}) = 4$

G_{complete}



$d(i,j) = 1$, for any i, j

$\text{diam}(G_{\text{complete}}) = 1$

DEFINITION (AVERAGE SHORTEST PATH LENGTH)

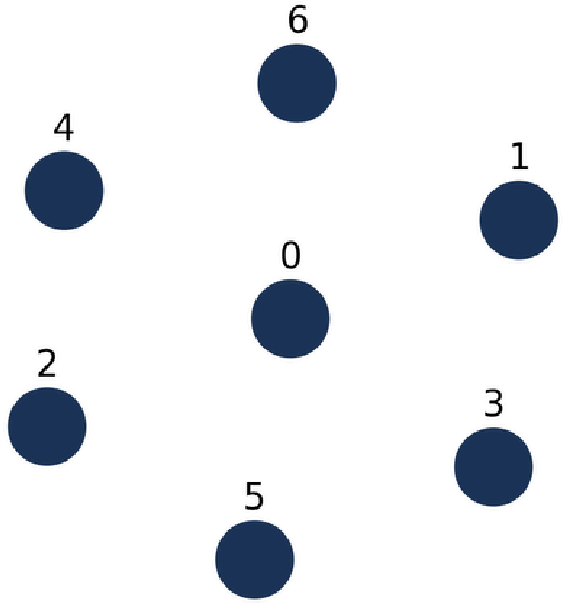
The *average shortest path length* $L(G)$ of a network G is the average distance between all pairs of nodes:

$$L(G) = \frac{1}{|V|(|V|-1)} \sum_{i,j \in V} d(i,j).$$

PATHS AND STARS AND TREES

Average shortest path lengths

G_{empty}

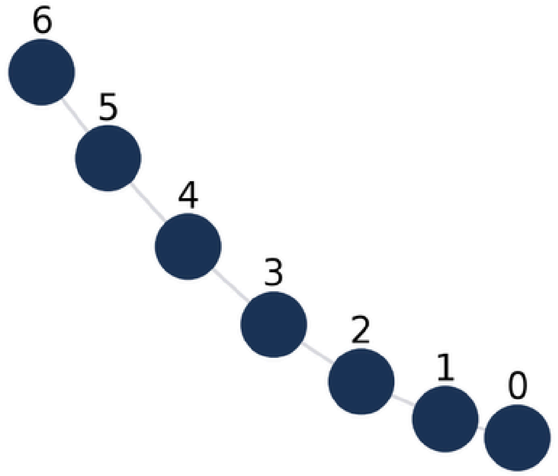


$d(i,j) = \infty$, for all i, j

$\text{diam}(G_{\text{empty}}) = \infty$

$L(G_{\text{empty}}) = \infty$

G_{path}

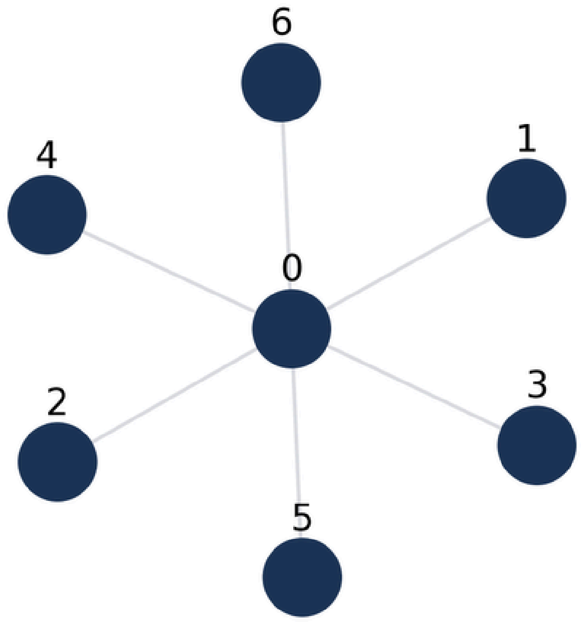


$d(1,4) = 3$

$\text{diam}(G_{\text{path}}) = 5$

$L(G_{\text{path}}) = 2.33$

G_{star}

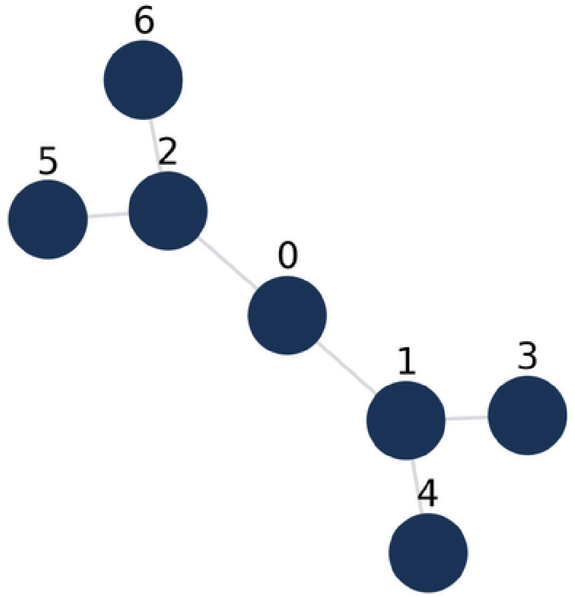


$d(1,2) = 2$

$\text{diam}(G_{\text{star}}) = 2$

$L(G_{\text{star}}) = 1.71$

G_{tree}

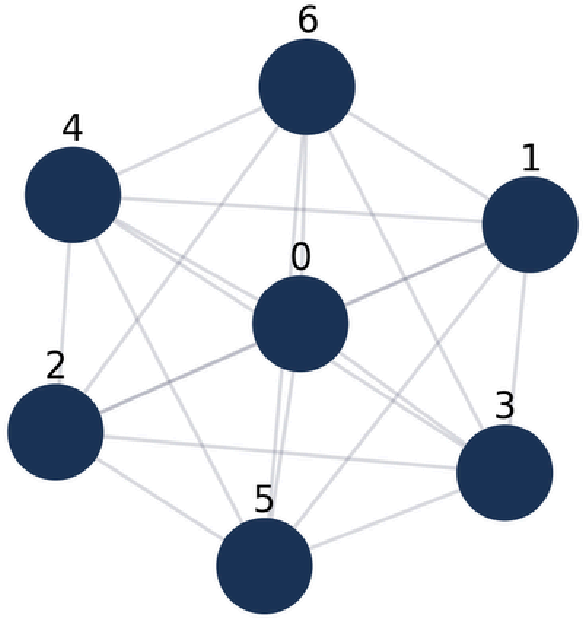


$d(4,6) = 4$

$\text{diam}(G_{\text{tree}}) = 4$

$L(G_{\text{tree}}) = 2.28$

G_{complete}



$d(i,j) = 1$, for any i, j

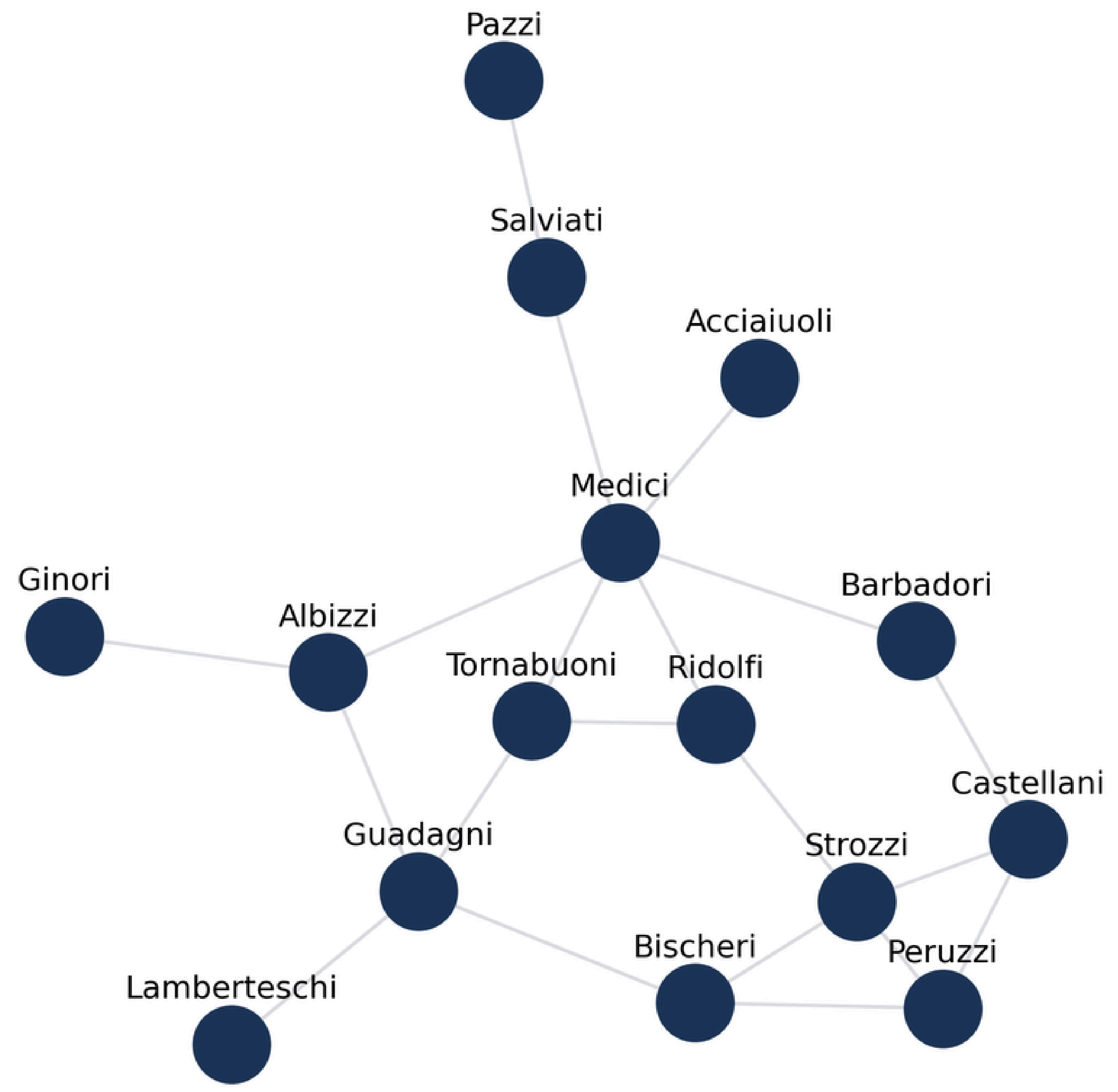
$\text{diam}(G_{\text{complete}}) = 1$

$L(G_{\text{complete}}) = 1$

FLORENTINE FAMILIES GRAPH

Diameter & average shortest path length

If the graph was a line with 15 elements, the diameter would be 14 and the average shortest path length would be 5.3.



$$\text{diam}(G_{\text{Florentine Families}}) = 5$$
$$L(G_{\text{Florentine Families}}) = 2.49$$

Another important factor is the clustering, i.e., how close a graph is to being a clique.

DEFINITION (CLUSTERING COEFFICIENT OF INDIVIDUAL NODES)

The *clustering coefficient* $Cl(i)$ of a node i is the fraction of pairs of its neighbors connected to each other:

$$Cl(i) = \frac{\text{number of edges between neighbors of } i}{\text{number of pairs of neighbors of } i} = \frac{\left|\left\{\{j,k\} \subseteq N(i) \mid (j,k) \in E\right\}\right|}{\left|\left\{\{j,k\} \subseteq N(i)\right\}\right|}$$

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$$= \frac{\left|\Big\{\{j,k\} \subseteq N(i) \mid (j,k) \in E\Big\}\right|}{\binom{d_i}{2}}$$

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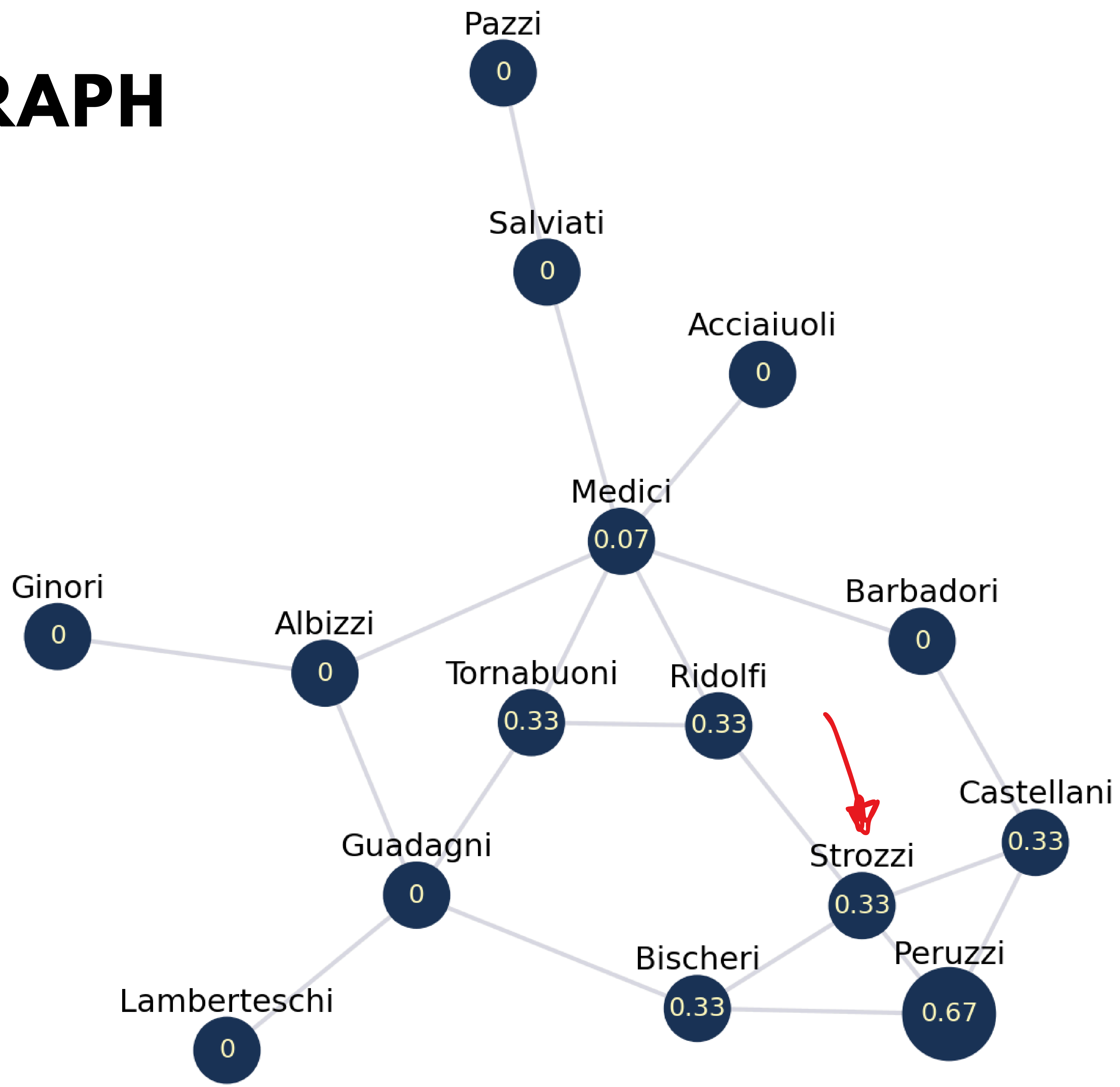
$$= \frac{\left|\Big\{\{j,k\} \subseteq N(i) \mid (j,k) \in E\Big\}\right|}{\binom{d_i}{2}}$$

$$= 2 \cdot \frac{\left|\Big\{\{j,k\} \subseteq N(i) \mid (j,k) \in E\Big\}\right|}{d_i(d_i - 1)}.$$

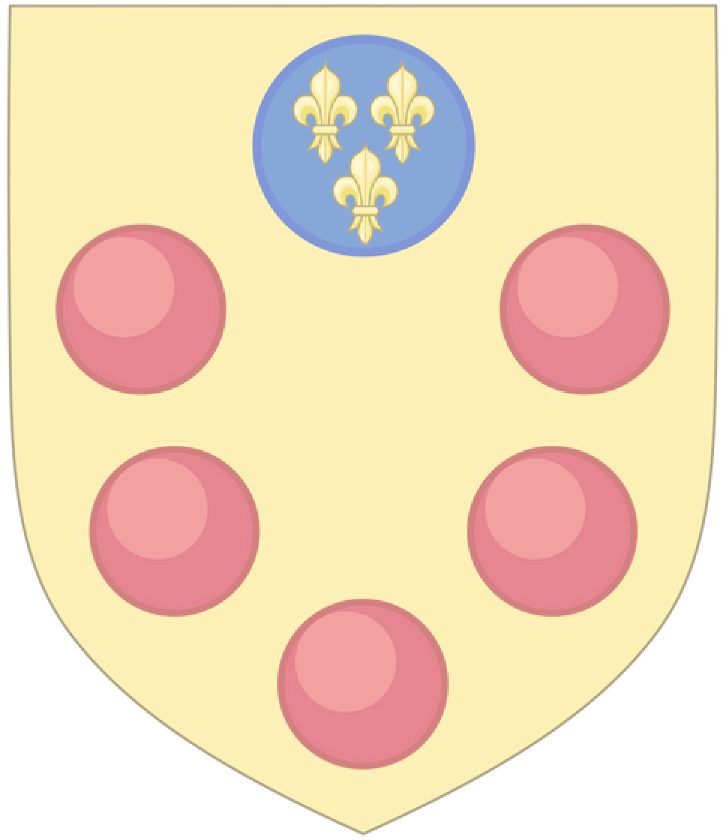
FLORENTINE FAMILIES GRAPH
Clustering coefficients of nodes

The Strozzi have six pairs of neighbors, two of which are connected.

Their clustering coefficient is thus ⅓.



MEDICI



Originally from the farmlands north of Florence, in Mugello.

Become wealthy through banking.

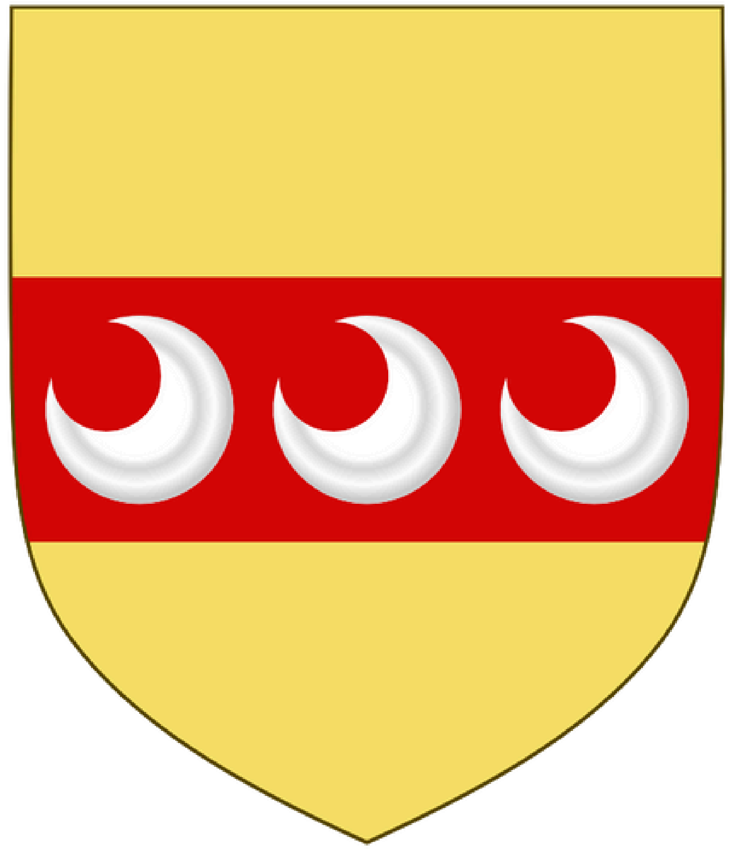
Cosimo seen as 'champion of the new men'.

ALBIZZI

Old-money wool-merchant dynasty.

Maso, and then Rinaldo degli Albizzi basically *were* the Florentine state until Cosimo showed them the door in 1434.

STROZZI



Biggest balance-sheet in Florence until 1434.

Bankers, art patrons, ringleaders of one-year exile for Cosimo.

Line goes on, with living descendants to this day.

NATALIA GUICCIARDINI STROZZI

The Strozzi's were enemies of the Medici family.

The Medici's kept us in exile, and throughout the Renaissance the two clans were constantly involved in a power struggle.

Yet today we have portraits of the Medici in our house.

As a reminder that we're still here and they're not…

An interview with Natalia Guicciardini Strozzi. (2007, May 16). The Florentine.

DEFINITION (CLUSTERING COEFFICIENT OF NETWORK)

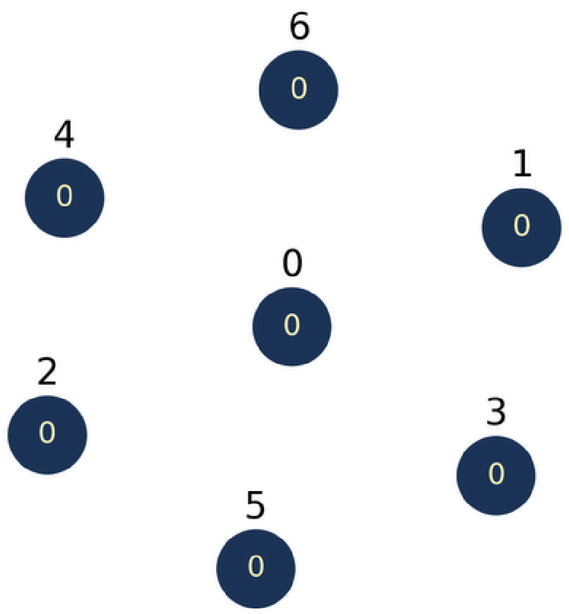
The *clustering coefficient* $Cl(G)$ of network G is the average clustering coefficient of its nodes:

$$Cl(G) = \frac{1}{|V|} \sum_{i \in V} Cl(i).$$

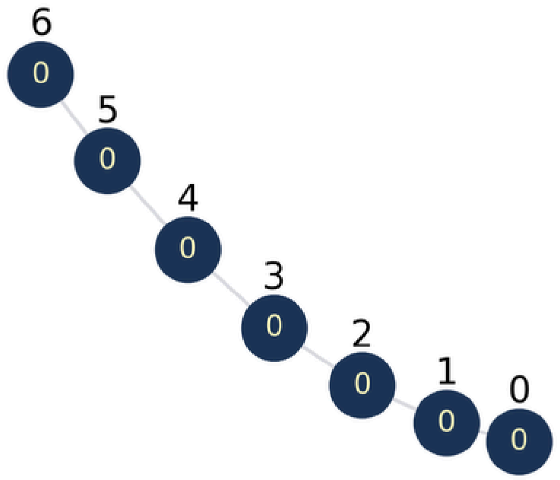
Note that the clustering coefficient is a number between 0 and 1.

PATHS AND STARS AND TREES

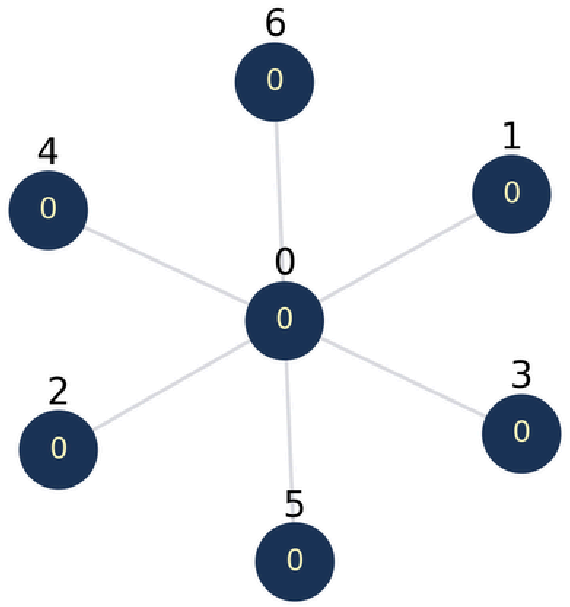
Average clustering



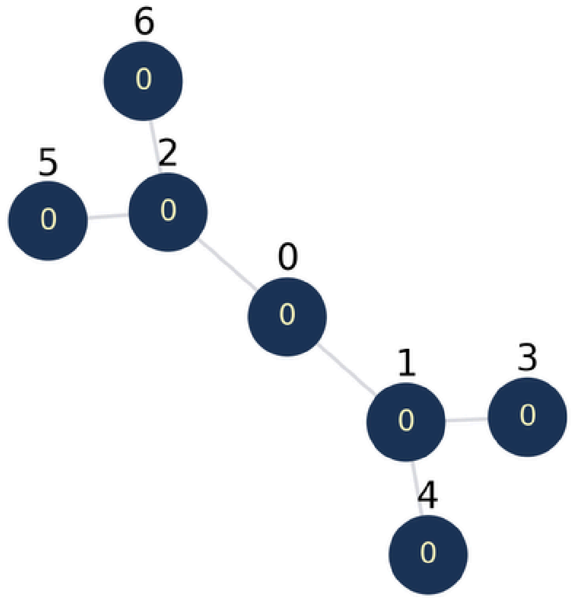
$Cl(G_{\text{empty}}) = 0$



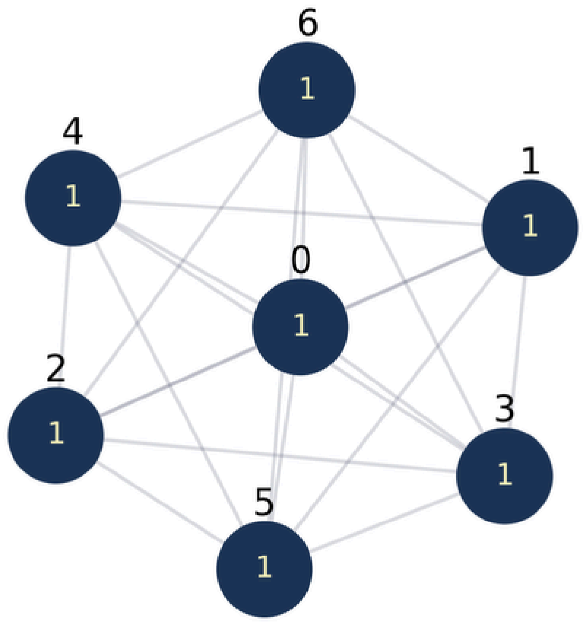
$Cl(G_{\text{path}}) = 0$



$Cl(G_{\text{star}}) = 0$



$Cl(G_{\text{tree}}) = 0$

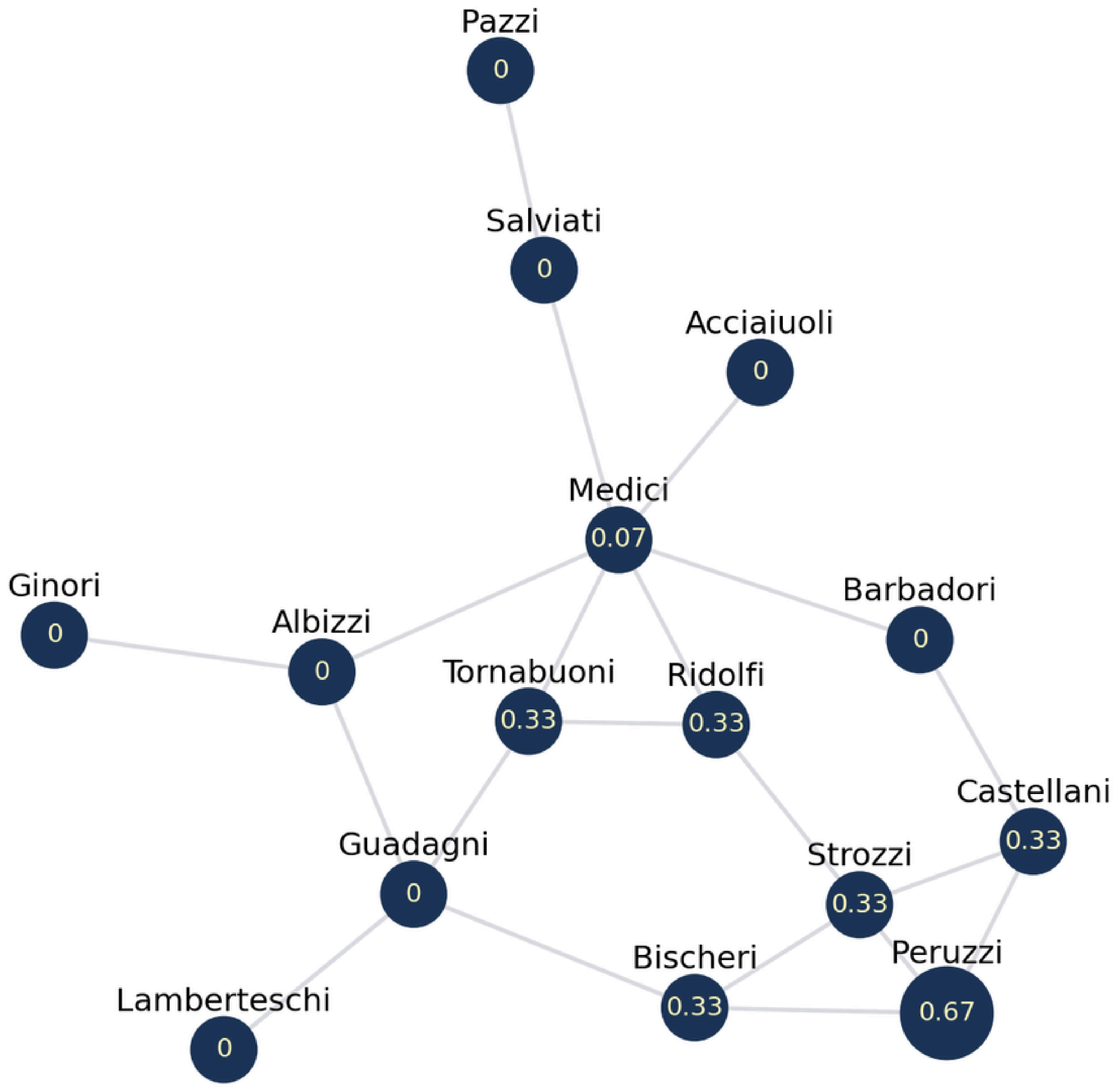


$Cl(G_{\text{complete}}) = 1$

FLORENTINE FAMILIES GRAPH

Clustering coefficient

Average clustering for Florentine families.



$$Cl(G_{\text{Florentine-Families}}) = 0.16$$

An important class of networks are *small-world* networks.

DEFINITION (SMALL WORLD NETWORK)

A *small-world network* is a type of network characterized by a *high clustering coefficient* and *short average path length*:

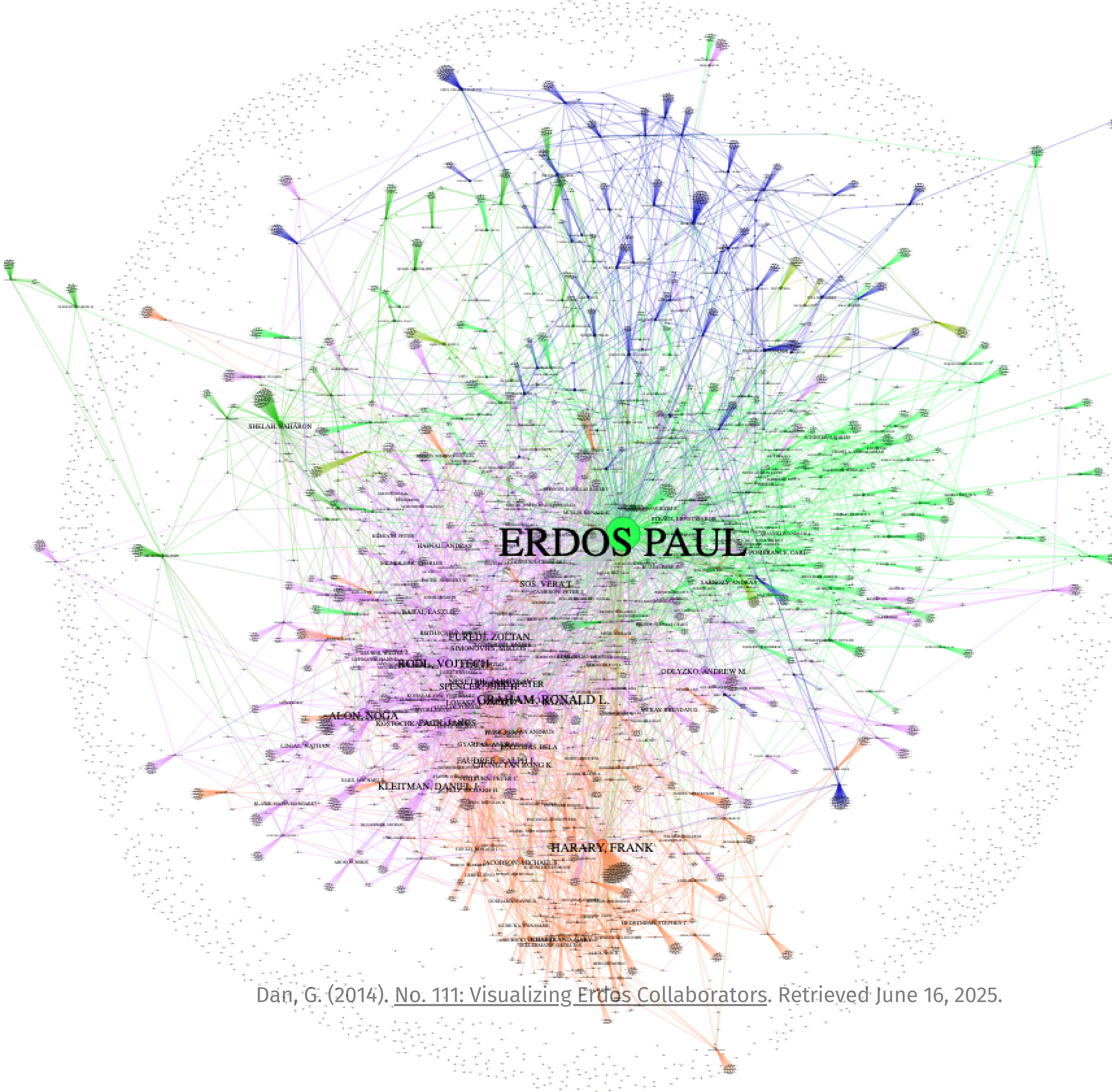
$$L(G) \sim \log(|V|),$$

i.e., the average distance between any two nodes is proportional to the logarithm of the number of nodes.

The idea of a small world network also underlies things like the *six degrees of separation* factoid.

PAUL ERDŐS

And the Erdős number!



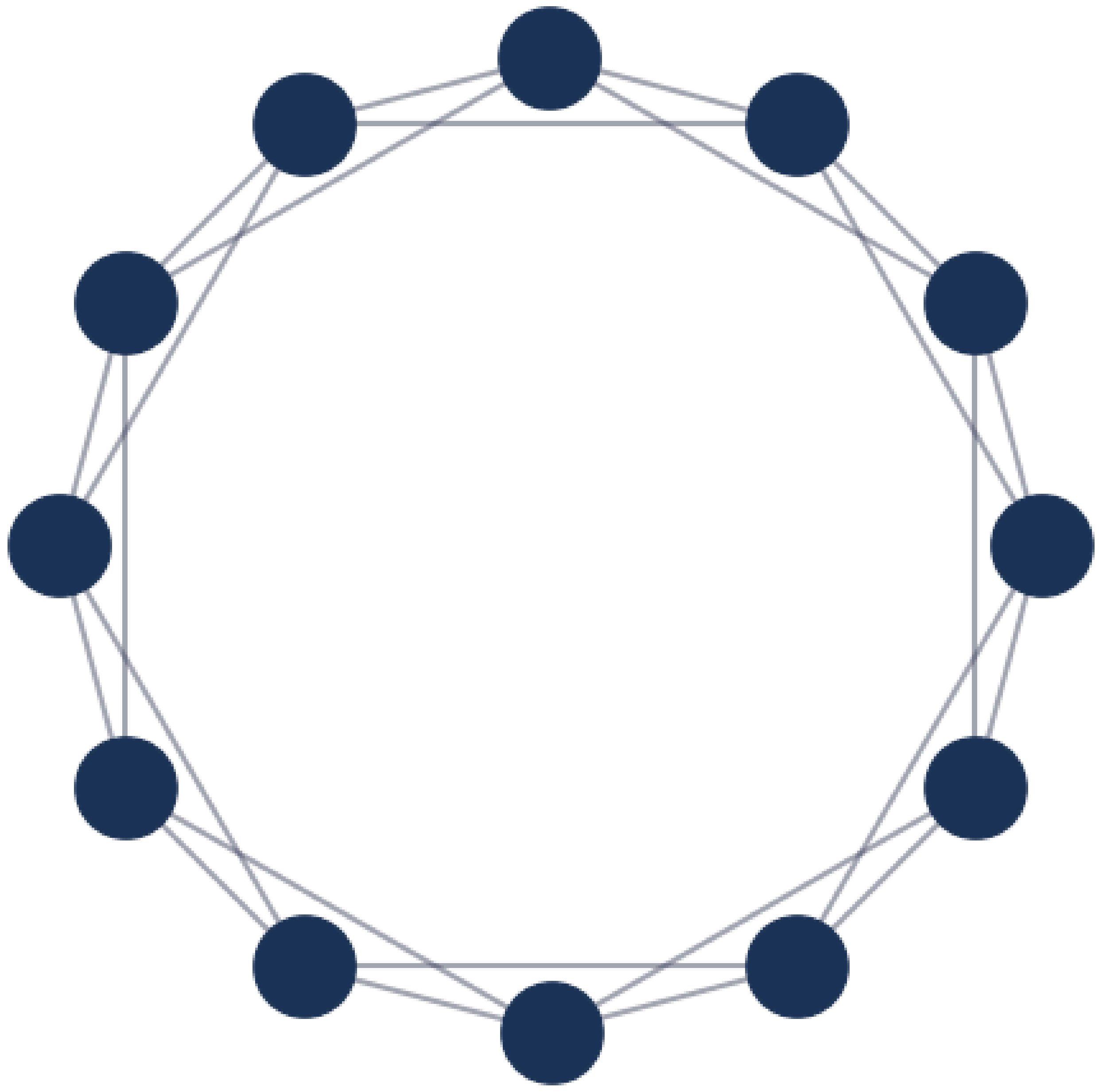
Dan, G. (2014). No. 111: Visualizing Erdos Collaborators. Retrieved June 16, 2025.

DEFINITION (WATTS-STROGATZ MODEL)

Start with a graph where each node is connected to exactly k other nodes. Then, at every time step, randomly rewire each edge with probability p to an unattached node.

WATTS-STROGATZ MODEL
Intuition

Take $k = 4$ and $p = 0.2$.



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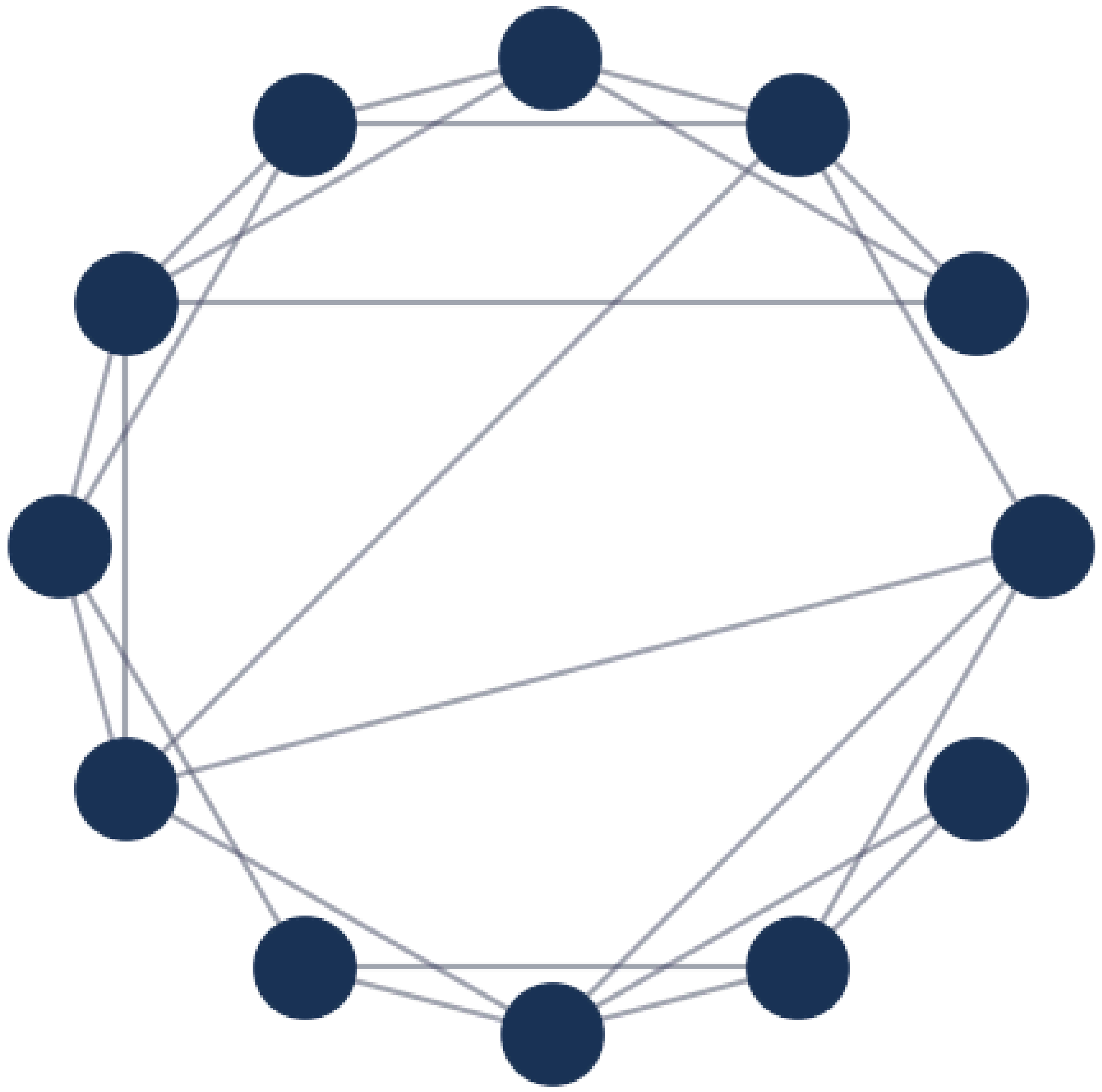

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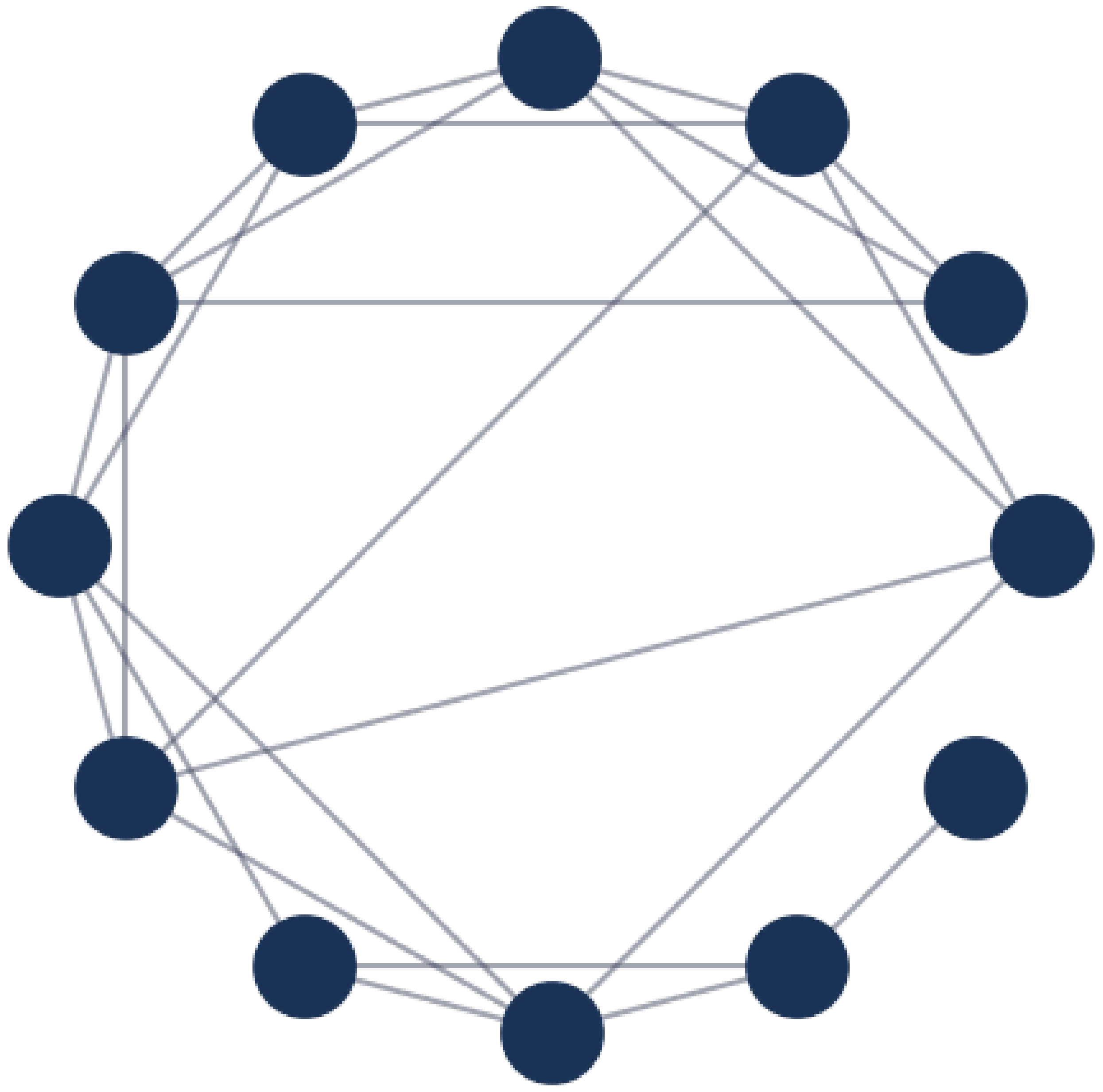

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DUNCAN WATTS

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DUNCAN WATTS

Our model tries to fix that.

Watts, D. J., & Strogatz, S. H. (1998). Collective dynamics of “small-world” networks. *Nature*, 393(6684), 440–442.

Ok, but we’re no closer to figuring out what made Cosimo so effective.