

“To Each Their Own”,
Making Democracy Count,
Ismar Volić (2024)

Ismar Volić

MAKING

DEMOCRACY

COUNT

How Mathematics
Improves Voting,
Electoral Maps, and
Representation

To Each Their Own

ARROW'S THEOREM is a bummer: any time we run an election in which voters rank candidates, something could go wrong. A paradoxical outcome is a possibility.

But what if we thought outside the ballot box? These problems are inherent in **ranked elections**. What if we asked voters to evaluate candidates on their own merits without explicitly making comparisons? Voting methods that do this are called **cardinal** (nothing to do with cardinals voting in the conclave or with the St. Louis Cardinals, although St. Louis does use one of these methods). This method contrasts with **ranked methods**, such as instant runoff, the **Borda count**, or the **Copeland method**, which are **ordinal methods**.

You're used to cardinal voting methods. When you choose a few scoops of ice cream, you're using a cardinal method. All you need to know is that you like hazelnut, passion fruit, and cookie dough and don't like any other flavors nearly as much. If you're getting three scoops, you don't need to rank those three or the remaining flavors.

When you rate a product on Amazon or an apartment on Airbnb by giving it anywhere from one to five stars, you're using a cardinal method. In each case, you thought about what you were rating individually without necessarily comparing it to any other alternative. When you give that new TV you bought four stars, you don't first rank all the TVs you've ever owned; you simply judge the new one on its own qualities.

The first way to assess the options, by simply deciding whether you like them or not, is called **approval voting**. The second, in which you



APPROVAL VOTING

RANGE VOTING

APPROVAL VOTING

- Definition: Approval voting allows voters to select ("**approve of**") as many candidates as they like. Each selection counts equally toward the candidate's total score.
- Mechanism: Voters review all candidates **individually** and check a box for each candidate they approve of. The candidate with the highest total approvals wins.

APPROVAL VOTING

- Definition: Approval voting allows voters to select ("**approve of**") as many candidates as they like. Each selection counts equally toward the candidate's total score.
- Mechanism: Voters review all candidates **individually** and check a box for each candidate they approve of. The candidate with the highest total approvals wins.

	5	5	3	2	2	1
Candidate A	✓	✓		✓		✓
Candidate B	✓		✓	✓	✓	
Candidate C		✓	✓	✓		

APPROVAL VOTING

- Definition: Approval voting allows voters to select ("**approve of**") as many candidates as they like. Each selection counts equally toward the candidate's total score.
- Mechanism: Voters review all candidates **individually** and check a box for each candidate they approve of. The candidate with the highest total approvals wins.

	5	5	3	2	2	1
Candidate A	✓	✓		✓		✓
Candidate B	✓		✓	✓	✓	
Candidate C		✓	✓	✓		

A has $5 + 5 + 2 + 1 = 13$ approvals

B has $5 + 3 + 2 + 2 = 12$ approvals

C has $5 + 3 + 2 = 10$ approvals

APPROVAL VOTING

- Definition: Approval voting allows voters to select ("**approve of**") as many candidates as they like. Each selection counts equally toward the candidate's total score.
- Mechanism: Voters review all candidates **individually** and check a box for each candidate they approve of. The candidate with the highest total approvals wins.

	5	5	3	2	2	1
Candidate A	✓	✓		✓		✓
Candidate B	✓		✓	✓	✓	
Candidate C		✓	✓	✓		

A has $5 + 5 + 2 + 1 = 13$ approvals

B has $5 + 3 + 2 + 2 = 12$ approvals

C has $5 + 3 + 2 = 10$ approvals

APPROVAL VOTING

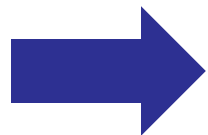
- Definition: Approval voting allows voters to select ("**approve of**") as many candidates as they like. Each selection counts equally toward the candidate's total score.
- Mechanism: Voters review all candidates **individually** and check a box for each candidate they approve of. The candidate with the highest total approvals wins.

	5	5	3	2	2	1
Candidate A	✓	✓		✓		✓
Candidate B	✓		✓	✓	✓	
Candidate C		✓	✓	✓		

A has $5 + 5 + 2 + 1 = 13$ approvals

B has $5 + 3 + 2 + 2 = 12$ approvals

C has $5 + 3 + 2 = 10$ approvals



We do this when picking out **restaurants, movies, ice cream flavors**, etc.

APPROVAL VOTING



APPROVAL VOTING

- + Simple
- + Prevents vote splitting
- + Promotes positive campaigning
- + Encourages honest voting (no “wasted” votes)
- + Benefits both major and minor parties



APPROVAL VOTING

- + Simple
- + Prevents vote splitting
- + Promotes positive campaigning
- + Encourages honest voting (no “wasted” votes)
- + Benefits both major and minor parties



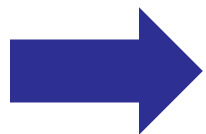
- Ties are possible
- If voter only votes once, same as standard plurality
- If voter approves of all, then vote useless
- No information on the degree of approval

APPROVAL VOTING

- + Simple
- + Prevents vote splitting
- + Promotes positive campaigning
- + Encourages honest voting (no “wasted” votes)
- + Benefits both major and minor parties



- Ties are possible
- If voter only votes once, same as standard plurality
- If voter approves of all, then vote useless
- No information on the degree of approval



Complies with anonymity, neutrality, monotonicity, and IIA, as not a ranked voting system

APPROVAL VOTING: IN ACTION

Works well in...

APPROVAL VOTING: IN ACTION

Works well in...



Not opposed to anyone being elected...

APPROVAL VOTING: IN ACTION

Works well in...



Not opposed to anyone being elected...



No limit of candidates to be elected...

APPROVAL VOTING: IN ACTION

Where it can go wrong...

APPROVAL VOTING: IN ACTION

Where it can go wrong...

- Suppose 10 voters participate
 - 9 *approve* Candidates A and B; they *like* A more but think B is ok too.
 - 1 approves only Candidate B.

APPROVAL VOTING: IN ACTION

Where it can go wrong...

- Suppose 10 voters participate
 - 9 *approve* Candidates A and B; they *like* A more but think B is ok too.
 - 1 approves only Candidate B.
- Results:
 - A = 9 approvals
 - B = 10 approvals

APPROVAL VOTING: IN ACTION

Where it can go wrong...

- Suppose 10 voters participate
 - 9 *approve* Candidates A and B; they *like* A more but think B is ok too.
 - 1 approves only Candidate B.
- Results:
 - A = 9 approvals
 - B = 10 approvals
- Winner: Candidate B, even though 90% of voters preferred A over B

APPROVAL VOTING: IN ACTION

Where it can go wrong...

- Suppose 10 voters participate
 - 9 *approve* Candidates A and B; they *like* A more but think B is ok too.
 - 1 approves only Candidate B.
- Results:
 - A = 9 approvals
 - B = 10 approvals
- **Winner: Candidate B, even though 90% of voters preferred A over B**

To Each Their Own

ARROW'S THEOREM is a bummer: any time we run an election in which voters rank candidates, something could go wrong. A paradoxical outcome is a possibility.

But what if we thought outside the ballot box? These problems are inherent in **ranked elections**. What if we asked voters to evaluate candidates on their own merits without explicitly making comparisons? Voting methods that do this are called **cardinal** (nothing to do with cardinals voting in the conclave or with the St. Louis Cardinals, although St. Louis does use one of these methods). This method contrasts with **ranked methods**, such as instant runoff, the **Borda count**, or the **Copeland method**, which are **ordinal methods**.

You're used to cardinal voting methods. When you choose a few scoops of ice cream, you're using a cardinal method. All you need to know is that you like hazelnut, passion fruit, and cookie dough and don't like any other flavors nearly as much. If you're getting three scoops, you don't need to rank those three or the remaining flavors.

When you rate a product on Amazon or an apartment on Airbnb by giving it anywhere from one to five stars, you're using a cardinal method. In each case, you thought about what you were rating individually without necessarily comparing it to any other alternative. When you give that new TV you bought four stars, you don't first rank all the TVs you've ever owned; you simply judge the new one on its own qualities.

The first way to assess the options, by simply deciding whether you like them or not, is called **approval voting**. The second, in which you



APPROVAL VOTING

RANGE VOTING

RANGE VOTING

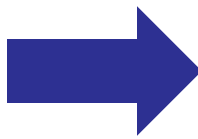
Governor candidates		Score <i>each</i> candidate by filling a number (0 is worst; 9 is best)
1: Candidate A	→	<input checked="" type="radio"/> 0 <input type="radio"/> 1 <input type="radio"/> 2 <input type="radio"/> 3 <input type="radio"/> 4 <input type="radio"/> 5 <input type="radio"/> 6 <input type="radio"/> 7 <input type="radio"/> 8 <input type="radio"/> 9
2: Candidate B	→	<input type="radio"/> 0 <input type="radio"/> 1 <input type="radio"/> 2 <input type="radio"/> 3 <input type="radio"/> 4 <input type="radio"/> 5 <input type="radio"/> 6 <input type="radio"/> 7 <input type="radio"/> 8 <input checked="" type="radio"/> 9
3: Candidate C	→	<input type="radio"/> 0 <input type="radio"/> 1 <input type="radio"/> 2 <input type="radio"/> 3 <input type="radio"/> 4 <input type="radio"/> 5 <input type="radio"/> 6 <input checked="" type="radio"/> 7 <input type="radio"/> 8 <input type="radio"/> 9

RANGE VOTING

Governor candidates		Score <i>each</i> candidate by filling a number (0 is worst; 9 is best)
1: Candidate A	→	<input type="radio"/> 0 <input type="radio"/> 1 <input type="radio"/> 2 <input type="radio"/> 3 <input type="radio"/> 4 <input type="radio"/> 5 <input type="radio"/> 6 <input type="radio"/> 7 <input type="radio"/> 8 <input checked="" type="radio"/> 9
2: Candidate B	→	<input checked="" type="radio"/> 0 <input type="radio"/> 1 <input type="radio"/> 2 <input type="radio"/> 3 <input type="radio"/> 4 <input type="radio"/> 5 <input type="radio"/> 6 <input type="radio"/> 7 <input type="radio"/> 8 <input type="radio"/> 9
3: Candidate C	→	<input checked="" type="radio"/> 0 <input type="radio"/> 1 <input type="radio"/> 2 <input type="radio"/> 3 <input type="radio"/> 4 <input type="radio"/> 5 <input type="radio"/> 6 <input type="radio"/> 7 <input type="radio"/> 8 <input type="radio"/> 9

RANGE VOTING

Governor candidates		Score <i>each</i> candidate by filling a number (0 is worst; 9 is best)
1: Candidate A	→	0 1 2 3 4 5 6 7 8 9
2: Candidate B	→	0 1 2 3 4 5 6 7 8 9
3: Candidate C	→	0 1 2 3 4 5 6 7 8 9

 The range is mathematically unimportant; the main consideration is the voter's psychology

RANGE VOTING: IN ACTION



RANGE VOTING: IN ACTION



RANGE VOTING: IN ACTION



RANGE VOTING: IN ACTION

Hold up: where it's not so good...

RANGE VOTING: IN ACTION

Hold up: where it's not so good...

- The scores can mean different things to different people
- Computing an average

RANGE VOTING: IN ACTION

Hold up: where it's not so good...

- The scores can mean different things to different people
- Computing an average

Candidate	Voter 1 Score	Voter 2 Score	Total Score	Average Score
A	2	3	5	2.50
B	4	5	9	4.50
C	6	7	13	6.50

RANGE VOTING: IN ACTION

Hold up: where it's not so good...

- The scores can mean different things to different people
- Computing an average

Candidate	Voter 1 Score	Voter 2 Score	Total Score	Average Score
A	2	3	5	2.50
B	4	5	9	4.50
C	6	7	13	6.50

RANGE VOTING: IN ACTION

Hold up: where it's not so good...

- The scores can mean different things to different people
- Computing an average

Candidate	Voter 1 Score	Voter 2 Score	Voter 3 Score	Total Score	Average Score
A	2	3	9	14	4.66
B	4	5	0	9	3.00
C	6	7	0	13	4.33

RANGE VOTING: IN ACTION

Hold up: where it's not so good...

- The scores can mean different things to different people
- Computing an average

Candidate	Voter 1 Score	Voter 2 Score	Voter 3 Score	Total Score	Average Score
A	2	3	9	14	4.66
B	4	5	0	9	3.00
C	6	7	0	13	4.33

RANGE VOTING: IN ACTION

Hold up: where it's not so good...

- The scores can mean different things to different people
- Computing an average

Candidate	Voter 1 Score	Voter 2 Score	Voter 3 Score	Total Score	Average Score
A	2	3	9	14	4.66
B	4	5	0	9	3.00
C	6	7	0	13	4.33

Strategic / tactical voting (!)

RANGE VOTING: IN ACTION

Hold up: where it's not so good...

- The scores can mean different things to different people
- Computing an average → mitigated by systems that use the median instead

Candidate	Voter 1 Score	Voter 2 Score	Voter 3 Score	Total Score	Average Score
A	2	3	9	14	4.66
B	4	5	0	9	3.00
C	6	7	0	13	4.33

To Each Their Own

ARROW'S THEOREM is a bummer: any time we run an election in which voters rank candidates, something could go wrong. A paradoxical outcome is a possibility.

But what if we thought outside the ballot box? These problems are inherent in **ranked elections**. What if we asked voters to evaluate candidates on their own merits without explicitly making comparisons? Voting methods that do this are called **cardinal** (nothing to do with cardinals voting in the conclave or with the St. Louis Cardinals, although St. Louis does use one of these methods). This method contrasts with **ranked methods**, such as instant runoff, the **Borda count**, or the **Copeland method**, which are **ordinal methods**.

You're used to cardinal voting methods. When you choose a few scoops of ice cream, you're using a cardinal method. All you need to know is that you like hazelnut, passion fruit, and cookie dough and don't like any other flavors nearly as much. If you're getting three scoops, you don't need to rank those three or the remaining flavors.

When you rate a product on Amazon or an apartment on Airbnb by giving it anywhere from one to five stars, you're using a cardinal method. In each case, you thought about what you were rating individually without necessarily comparing it to any other alternative. When you give that new TV you bought four stars, you don't first rank all the TVs you've ever owned; you simply judge the new one on its own qualities.

The first way to assess the options, by simply deciding whether you like them or not, is called **approval voting**. The second, in which you



APPROVAL VOTING

RANGE VOTING

To Each Their Own

ARROW'S THEOREM is a bummer: any time we run an election in which voters rank candidates, something could go wrong. A paradoxical outcome is a possibility.

But what if we thought outside the ballot box? These problems are inherent in **ranked elections**. What if we asked voters to evaluate candidates on their own merits without explicitly making comparisons? Voting methods that do this are called **cardinal** (nothing to do with cardinals voting in the conclave or with the St. Louis Cardinals, although St. Louis does use one of these methods). This method contrasts with **ranked methods**, such as instant runoff, the Borda count, or the Copeland method, which are **ordinal methods**.

You're used to cardinal voting methods. When you choose a few scoops of ice cream, you're using a cardinal method. All you need to know is that you like hazelnut, passion fruit, and cookie dough and don't like any other flavors nearly as much. If you're getting three scoops, you don't need to rank those three or the remaining flavors.

When you rate a product on Amazon or an apartment on Airbnb by giving it anywhere from one to five stars, you're using a cardinal method. In each case, you thought about what you were rating individually without necessarily comparing it to any other alternative. When you give that new TV you bought four stars, you don't first rank all the TVs you've ever owned; you simply judge the new one on its own qualities.

The first way to assess the options, by simply deciding whether you like them or not, is called **approval voting**. The second, in which you

APPROVAL VOTING

	5	5	3	2	2	1
Candidate A	✓	✓		✓		✓
Candidate B	✓		✓	✓	✓	
Candidate C		✓	✓	✓		

RANGE VOTING

Governor candidates	Score each candidate by filling a number (0 is worst; 9 is best)
1: Candidate A	→ (0) (1) (2) (3) (4) (5) (6) (7) (8) (9)
2: Candidate B	→ (0) (1) (2) (3) (4) (5) (6) (7) (8) (9)
3: Candidate C	→ (0) (1) (2) (3) (4) (5) (6) (7) (8) (9)

Thank you for your attention!
Any questions?