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ADVENTURES IN DEMOCRATIC DECISION MAKING

BORDA'S RULE

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Let's say people rank candidates:
how should the rankings be
aggregated?

JEAN-CHARLES, CHEVALIER DE BORDA

1733 – 1799

French mathematician, physicist, and Navy officer.

In 1770, he formulated a ranked preferential voting system referred to as the Borda count.





BORDA

Assign points to candidates, from high to low,
depending on their position in people's rankings.

Add up the points.

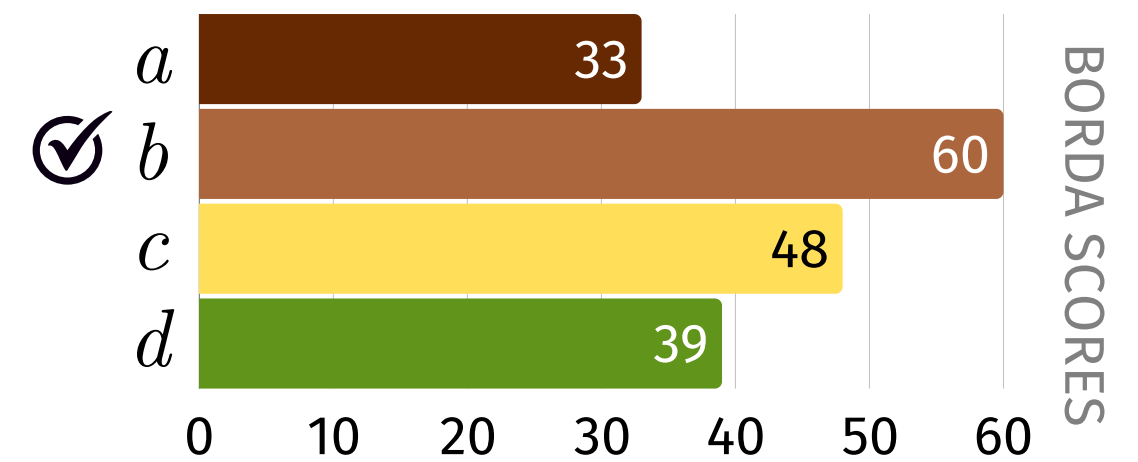
Voila.

BORDA COUNT

Top choice gets 3 points, second-best gets 2 points, third gets 1 point, last gets 0 points.

Borda winner is *b*.

points	11	10	9	
3	<i>a</i>	<i>d</i>	<i>c</i>	PREFERENCES
2	<i>b</i>	<i>b</i>	<i>b</i>	
1	<i>c</i>	<i>c</i>	<i>d</i>	
0	<i>d</i>	<i>a</i>	<i>a</i>	

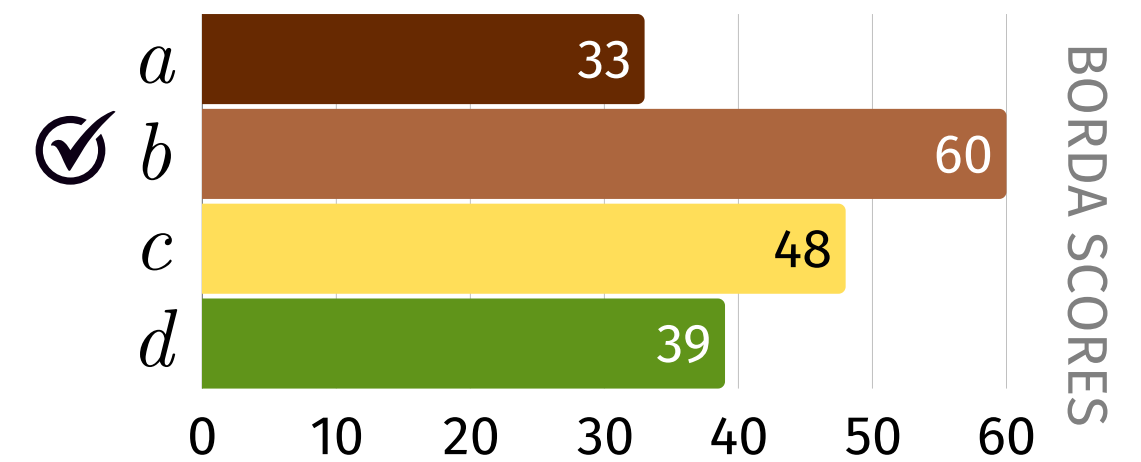


Note that the Borda winner may be a mediocre, or non-exciting alternative.

BORDA COUNT

No one thinks *b* is the best choice!

points	11	10	9	
3	<i>a</i>	<i>d</i>	<i>c</i>	PREFERENCES
2	<i>b</i>	<i>b</i>	<i>b</i>	
1	<i>c</i>	<i>c</i>	<i>d</i>	
0	<i>d</i>	<i>a</i>	<i>a</i>	

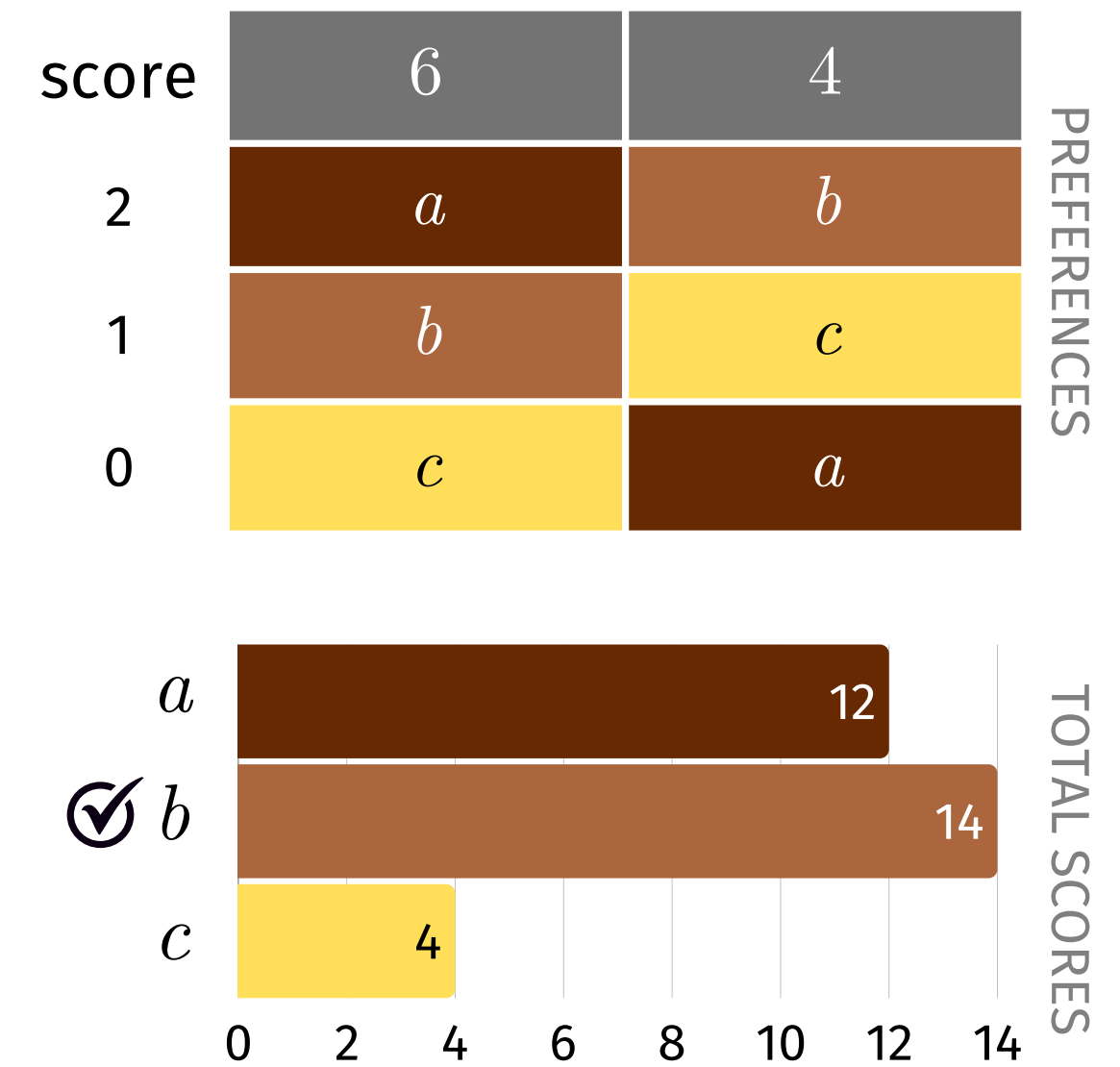


It gets worse: the Borda winner can be dominated by another alternative in a majority of the rankings.

BORDA WINNER IS MAJORITY-DOMINATED BY ANOTHER CANDIDATE

Alternative b is the Borda winner.

But a majority prefers a to b .



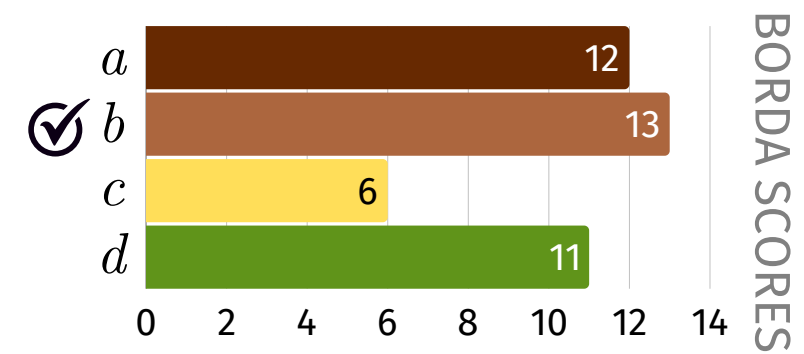
There is another type of property
that Borda fails.

BORDA RULE CHANGE OF WINNER

b is the Borda winner in the first profile.

points	3	2	2
3	<i>d</i>	<i>a</i>	<i>b</i>
2	<i>a</i>	<i>b</i>	<i>c</i>
1	<i>b</i>	<i>c</i>	<i>d</i>
0	<i>c</i>	<i>d</i>	<i>a</i>

PREFERENCES



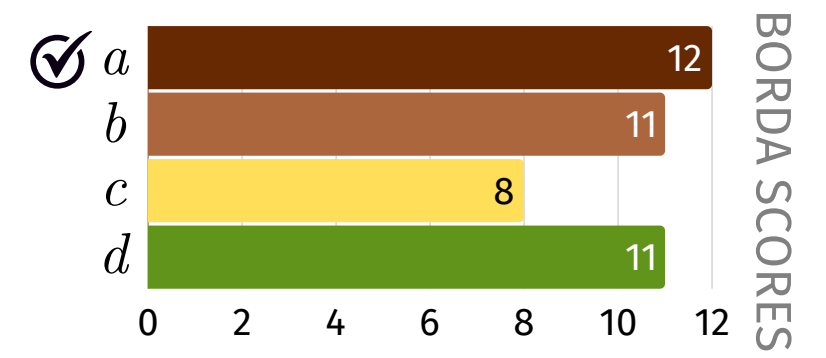
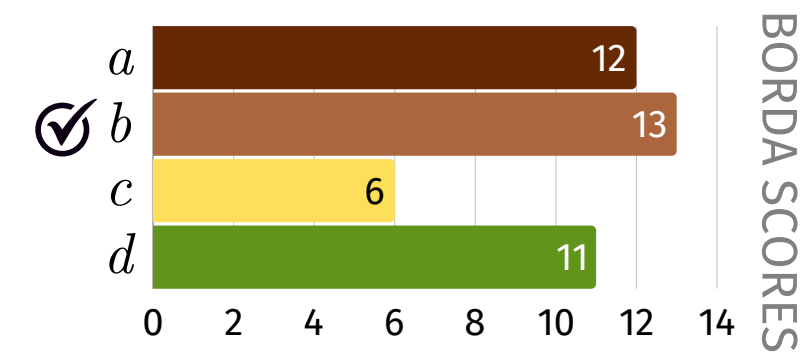
BORDA RULE CHANGE OF WINNER

b is the Borda winner in the first profile.

a is the Borda winner in the second profile.

points	3	2	2	
3	<i>d</i>	<i>a</i>	<i>b</i>	PREFERENCES
2	<i>a</i>	<i>b</i>	<i>c</i>	
1	<i>b</i>	<i>c</i>	<i>d</i>	
0	<i>c</i>	<i>d</i>	<i>a</i>	

	3	2	2	
	<i>d</i>	<i>a</i>	<i>b</i>	PREFERENCES
	<i>a</i>	<i>c</i>	<i>c</i>	
	<i>b</i>	<i>b</i>	<i>d</i>	
	<i>c</i>	<i>d</i>	<i>a</i>	



BORDA RULE CHANGE OF WINNER

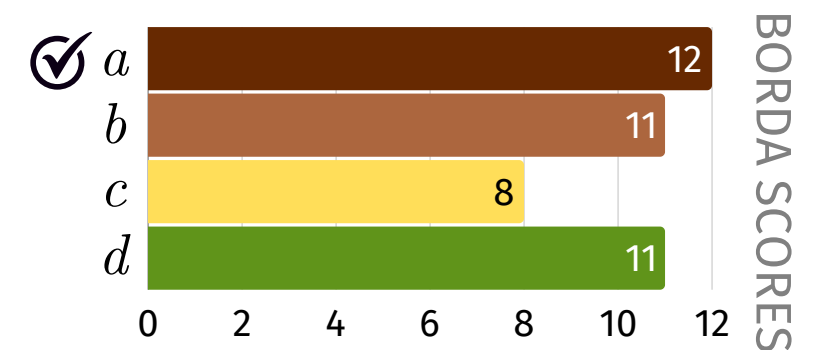
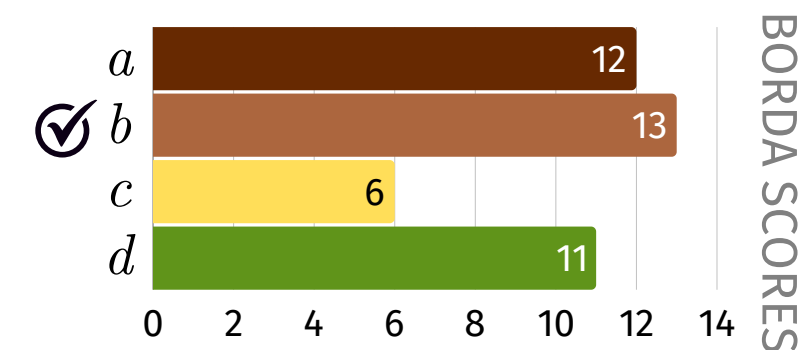
b is the Borda winner in the first profile.

a is the Borda winner in the second profile.

But the profiles do not differ in the way voters rank *a* and *b*!

points	3	2	2	
3	<i>d</i>	<i>a</i>	<i>b</i>	PREFERENCES
2	<i>a</i>	<i>b</i>	<i>c</i>	
1	<i>b</i>	<i>c</i>	<i>d</i>	
0	<i>c</i>	<i>d</i>	<i>a</i>	

points	3	2	2	
3	<i>d</i>	<i>a</i>	<i>b</i>	PREFERENCES
2	<i>a</i>	<i>c</i>	<i>c</i>	
1	<i>b</i>	<i>b</i>	<i>d</i>	
0	<i>c</i>	<i>d</i>	<i>a</i>	



DEFINITION

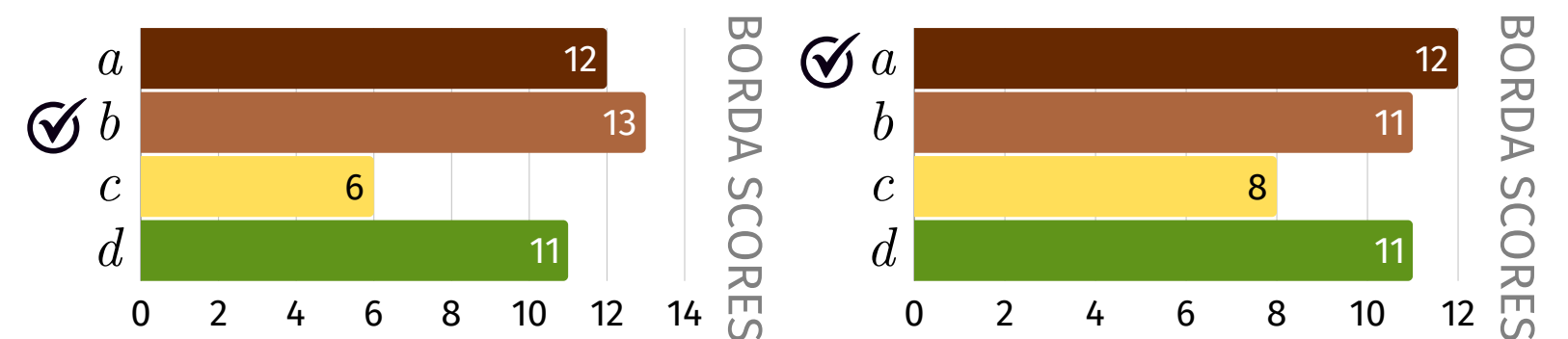
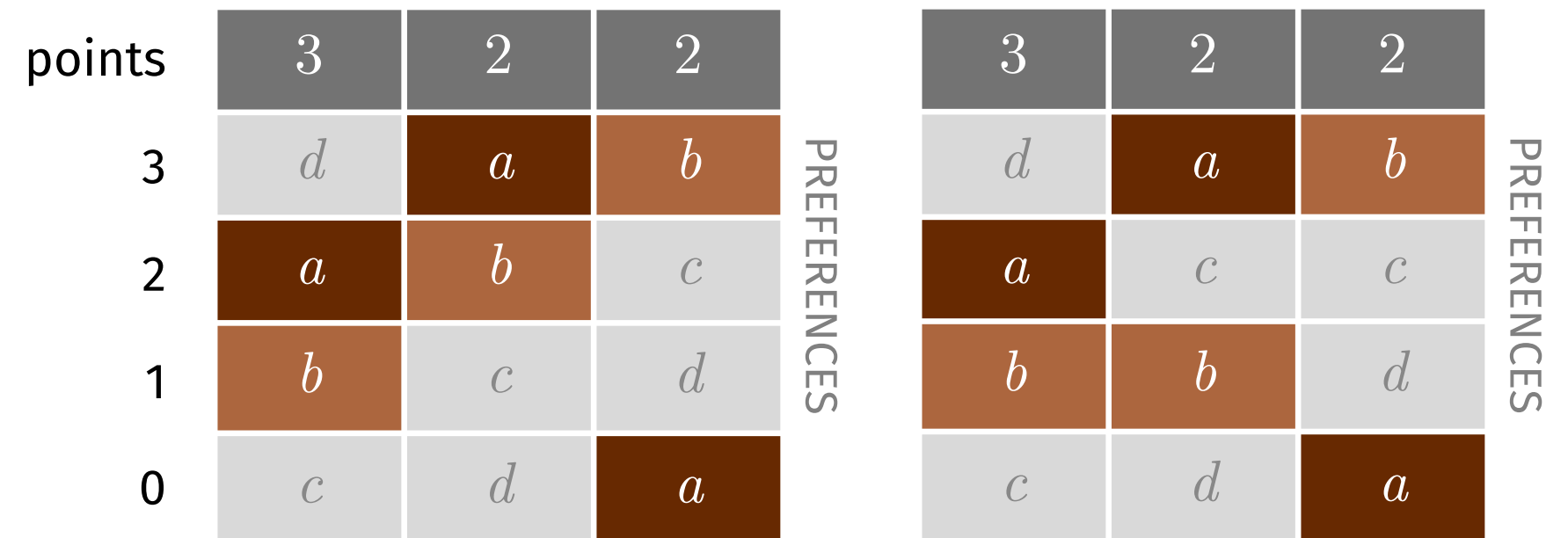
A voting rule satisfies *Independence of Irrelevant Alternatives (IIA)* if the final ranking on any two alternatives x and y depends on how voters rank x and y , and *nothing else*.

BORDA FAILS IIA

By raising c in their rankings, the middle voters change the final ranking between a and b .

Even though the middle winners to not change their rankings for a and b .




The final ranking between a and b depends on how voters rank c , and thus fails IIA.



Something along these lines
happened at the 1995 World Figure
Skating Championship.

1999 WFS CHAMPIONSHIP

Towards the end of the competition, Nicole Bobek is ranked second and Surya Bonaly third.

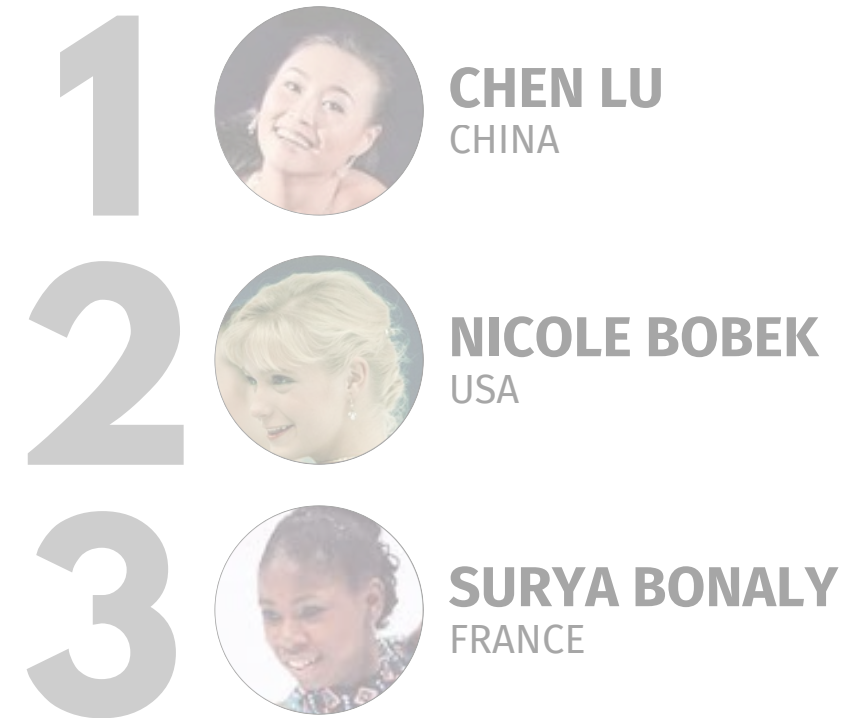
- 1  **CHEN LU**
CHINA
- 2  **NICOLE BOBEK**
USA
- 3  **SURYA BONALY**
FRANCE

1999 WFS CHAMPIONSHIP

Towards the end of the competition, Nicole Bobek is ranked second and Surya Bonaly third.

After Michelle Kwan's performance, they swap places.

Even though Kwan came in fourth!



To avoid this type of situation, the International Skating Union changed its voting rule.

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Several times...