STRATEGIC MINDS: THE GAME THEORY OF COOPERATION, COORDINATION AND COLLABORATION

DO COOPERATORS SURVIVE?

MIXED STRATEGIES AND EVOLUTIONARILY STABLE STRATEGIES

Adrian Haret
a.haret@lmu.de

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Pure Nash equilibria always exist.

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Except when they don't.

Matching Pennies

Two players have a penny each.

They decide on a face and reveal it at the same time.

If the faces match, player 1 wins $1, player 2 loses $1.

If the faces do not match, player 2 wins $1, player 1 loses $1.

payoffs

	Heads	Tails
Heads	1, -1	-1, 1
Tails	-1, 1	1, -1

Pareto optimal strategies

all

pure Nash equilibria

none

There is, however, a different way to play this game.

JOHN NASH

Sometimes the best thing to do is to flip a coin.

MIXED STRATEGIES

DEFINITION

A *mixed strategy* for player i is a probability ditribution over actions, written $s_i = (p_1, \ldots, p_j, \ldots)$, where p_i is the probability with which player i plays action j.

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Note that the pure strategies we've been dealing with so far are special cases of mixed strategies, in which one action is played with probability 1.

With mixed strategies, how are players supposed to play?

They aim to maximize *expected utility*.

EXPECTED UTILITY

An agent has to decide between different actions.

	state 1 (p_1)	...	state ℓ (p_ℓ)
action 1	$u(\text{action 1}, \text{state 1})$	$\ldots$	$u(\text{action 1}, \text{state } \ell)$
...	$\ldots$	$\ldots$	
action k	$u(\text{action } k, \text{state 1})$	$\ldots$	$u(\text{action } k, \text{state } \ell)$

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An agent goes with the action that maximizes this.

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Back to the Matching Pennies game.

FINDING MIXED EQUILIBRIA

Assume Player 1 has strategy $s_1 = (0.9, 0.1)$. What should Player 2 do?

payoffs

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Pareto optimal strategies

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Assume Player 1 has strategy $s_1 = (0.9, 0.1)$. What should Player 2 do?

$$\begin{aligned}\mathbb{E}[\text{Heads}] &= (-1) \cdot 0.9 + 1 \cdot 0.1 \\ &= -0.8.\end{aligned}$$

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Would it make sense for Player 2 to mix between Heads and Tails, say with $s_2' = (0.3, 0.7)$?

$$\begin{aligned} \mathbb{E}[s_2'] &= (-0.8) \cdot 0.3 + 0.8 \cdot 0.7 \\ &= 0.32 \\ &< 0.8. \end{aligned}$$

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No mixing gives better results than always going for Tails, so Player 2 wants to play $s_2 = (0, 1)$.

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Is $s = (s_1, s_2)$ a Nash equilibrium?

No! If Player 2 plays $s_2 = (0, 1)$, Player 1 wants to switch to $s_1' = (0, 1)$.

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FINDING MIXED EQUILIBRIA

In general, if Player 1 plays a mixed strategy that makes Player 2 prefer one action over another, Player 2 will just start playing that action all the time.

Player 2 sees the opportunity and goes for it!

payoffs

	Heads (q)	Tails ($1 - q$)
Heads (p)	1, -1	-1, 1
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So this cannot be a Nash equilibrium.

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The only way to avoid this is for Player 1 to play a strategy $s_1^* = (p, 1-p)$ that makes Player 2 indifferent between their actions, which means that:

$$\mathbb{E}[\text{Heads}] = \mathbb{E}[\text{Tails}] \text{ iff } (-1)\cdot p + 1\cdot(1-p) = 1\cdot p + (-1)\cdot(1-p)$$
$$\text{iff } p = {}^1\!/_2.$$

So Player 1 wants to play $s_1^* = ({}^1\!/_2, {}^1\!/_2)$.

payoffs

	Heads (q)	Tails (1 - q)
Heads (0.5)	1, -1	-1, 1
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$$\text{iff } p = 1/2.$$

So Player 1 wants to play $s_1^* = (1/2, 1/2)$.

Similarly, Player 2 wants to play $s_2^* = (1/2, 1/2)$.

This is the mixed Nash equilibrium.

payoffs

	Heads (0.5)	Tails (0.5)
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$$\boldsymbol{s}^* = \big((1/2, 1/2), (1/2, 1/2)\big)$$

JOHN NASH

This works for finding mixed Nash equilibria in general.

NASH'S THEOREM

THEOREM (NASH, 1951)

Any game with a finite number of players and finite actions has a Nash equilibrium in mixed strategies.

Nash, J. (1951). Non-Cooperative Games. *Annals of Mathematics*, 54(2), 286–295.

JOHN NASH

I got the Nobel prize for this result!

Fun fact: humans are not that good at randomizing.

ARIEL RUBINSTEIN

In experiments, they keep trying to detect patterns, are susceptible to stories and framing effects.

Mookherjee, D., & Sopher, B. (1994). Learning Behavior in an Experimental Matching Pennies Game. *Games and Economic Behavior*, 7(1), 62–91.

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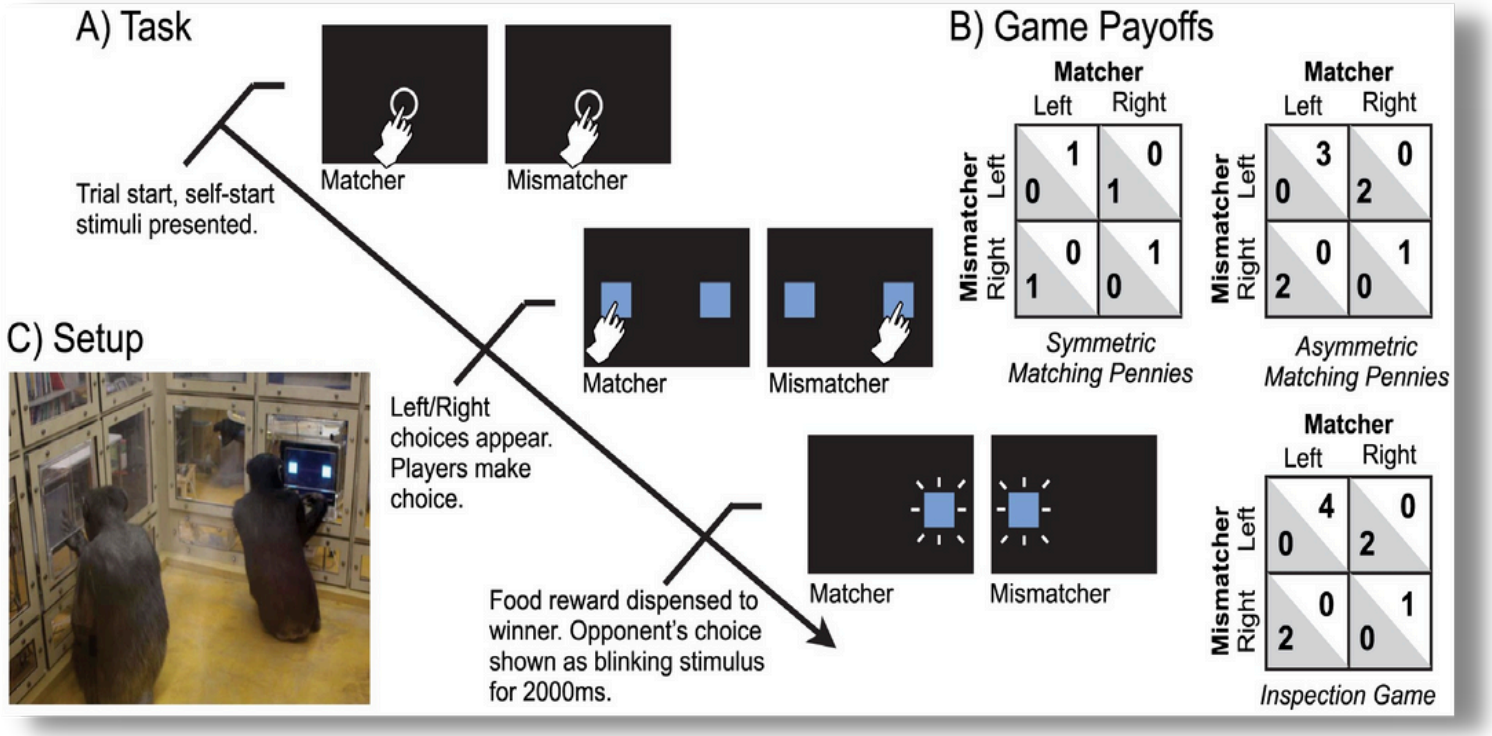
COLIN CAMERER

Interestingly, chimps seem to be pretty good at it.

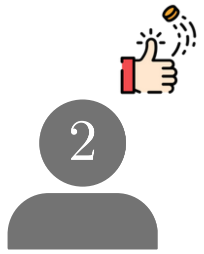


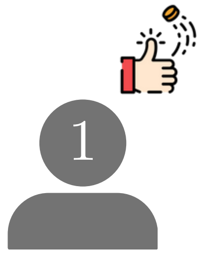
Martin, C. F., Bhui, R., Bossaerts, P., Matsuzawa, T., & Camerer, C. (2014). Chimpanzee choice rates in competitive games match equilibrium game theory predictions. *Nature: Scientific Reports*, 4, 5182.

What do the probabilities in a mixed strategy mean?

WHAT IS A MIXED STRATEGY ABOUT?

It can describe a randomization process between pure strategies inside a player’s head.





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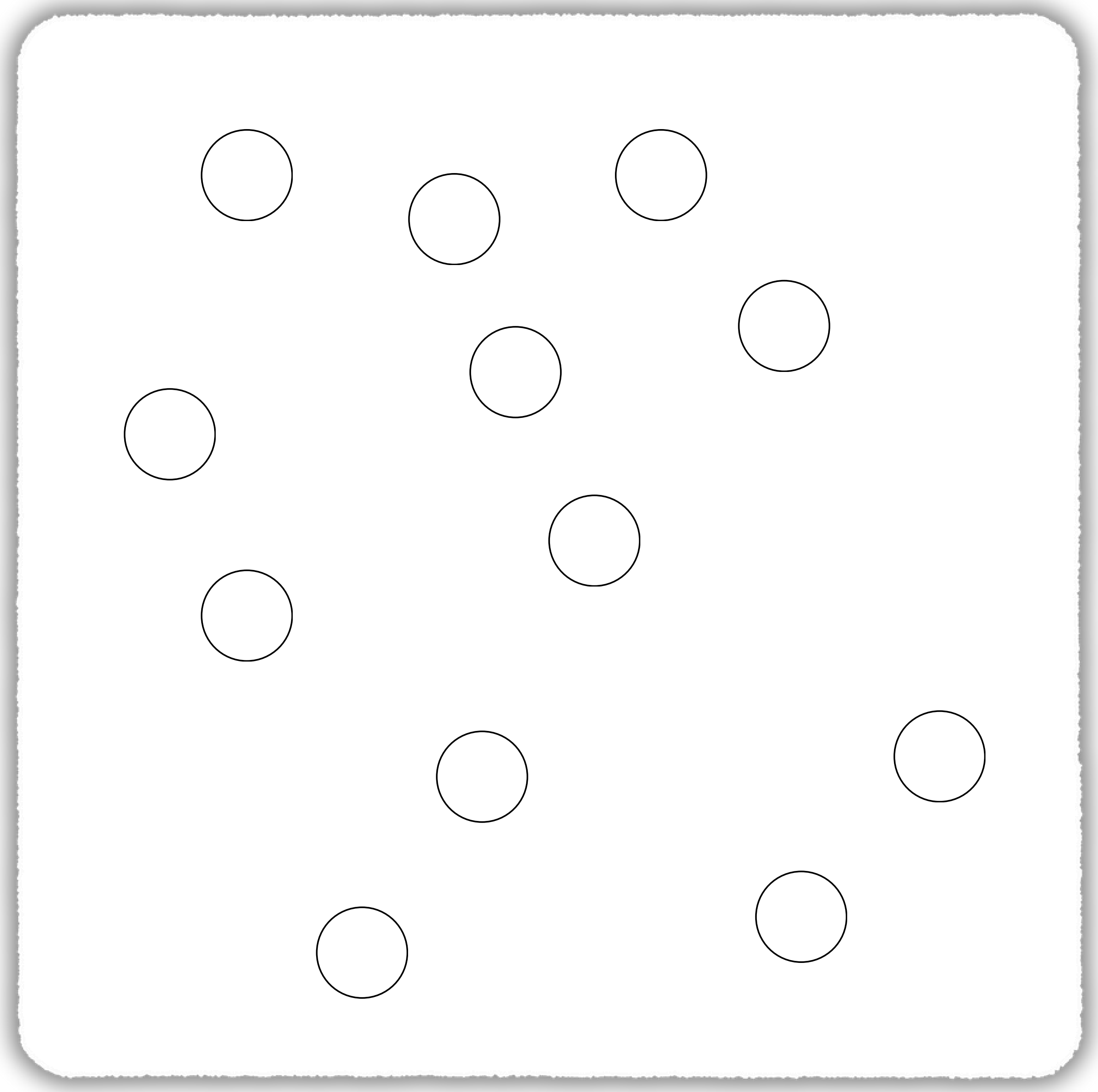
It can also describe a (large) population of players playing pure strategies, in proportions described by the probabilities.



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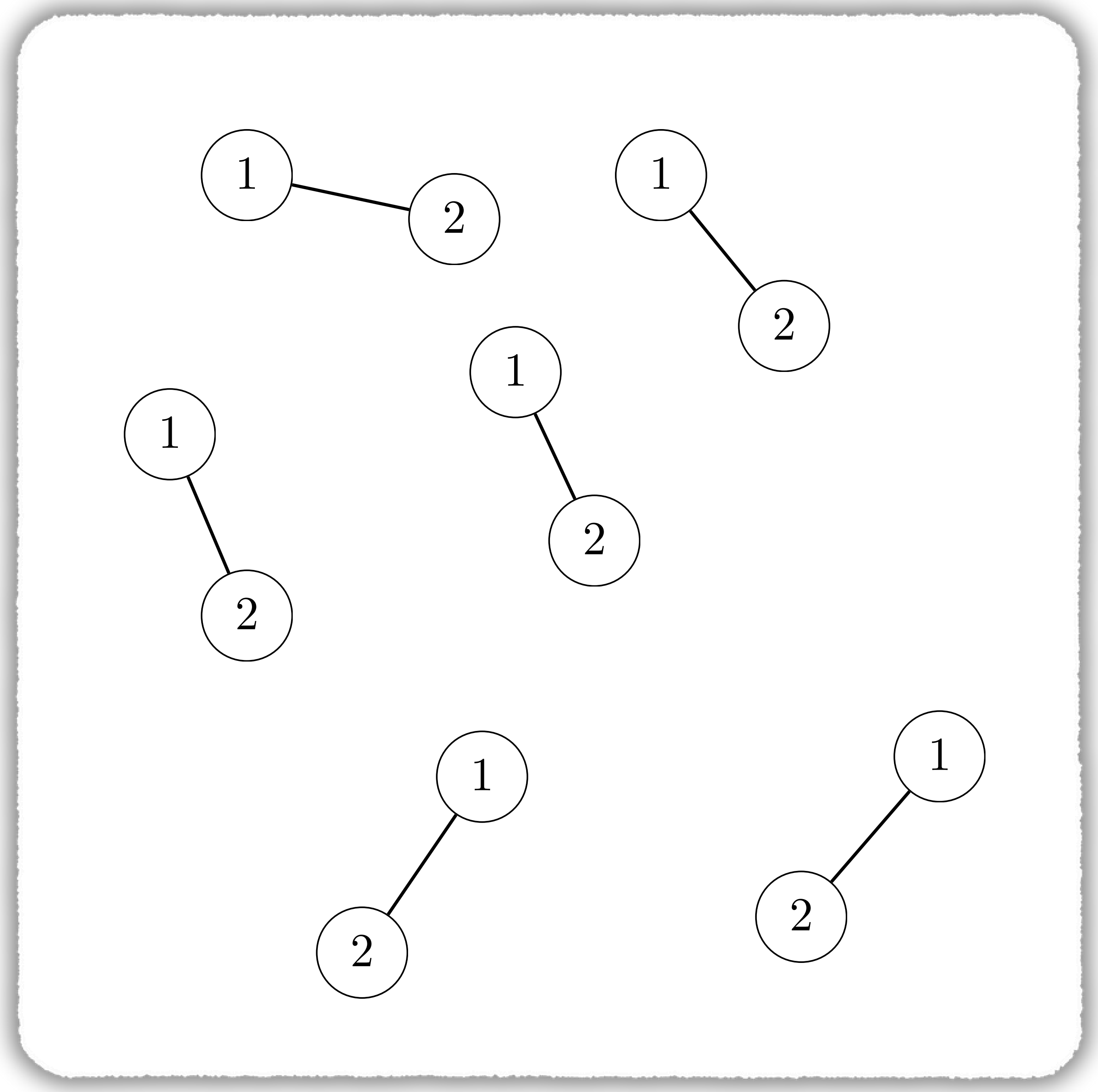

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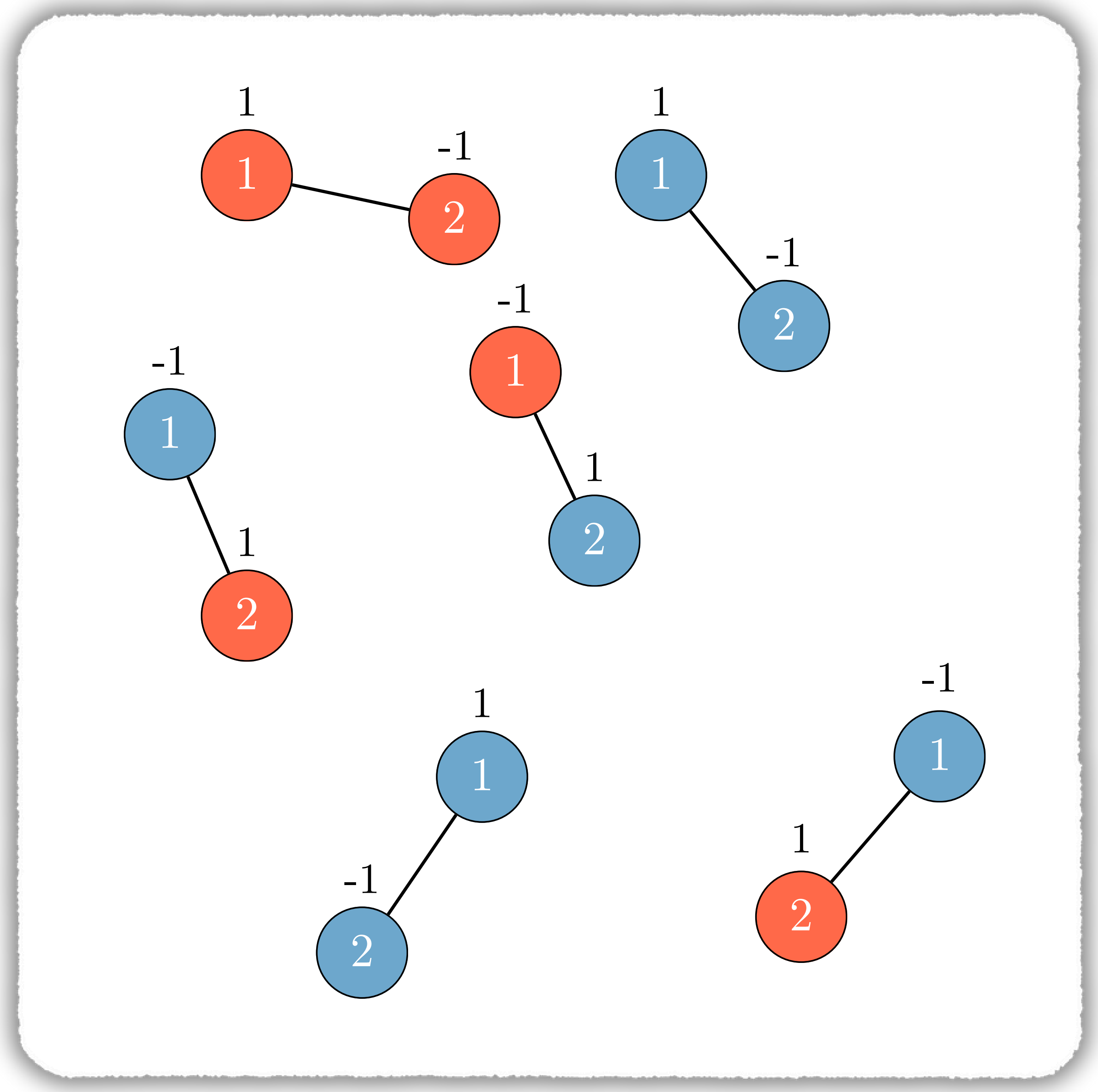

Some strategies (aka, players that play those strategies) are successful, others are not.

Like life.

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Like life.

And, like in life, we can assume successful strategies thrive.

And the others… well, they go extinct.

JOHN MAYNARD SMITH

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Secondly, and more importantly, in seeking the solution of a game, the concept of human rationality is replaced by that of evolutionary stability.

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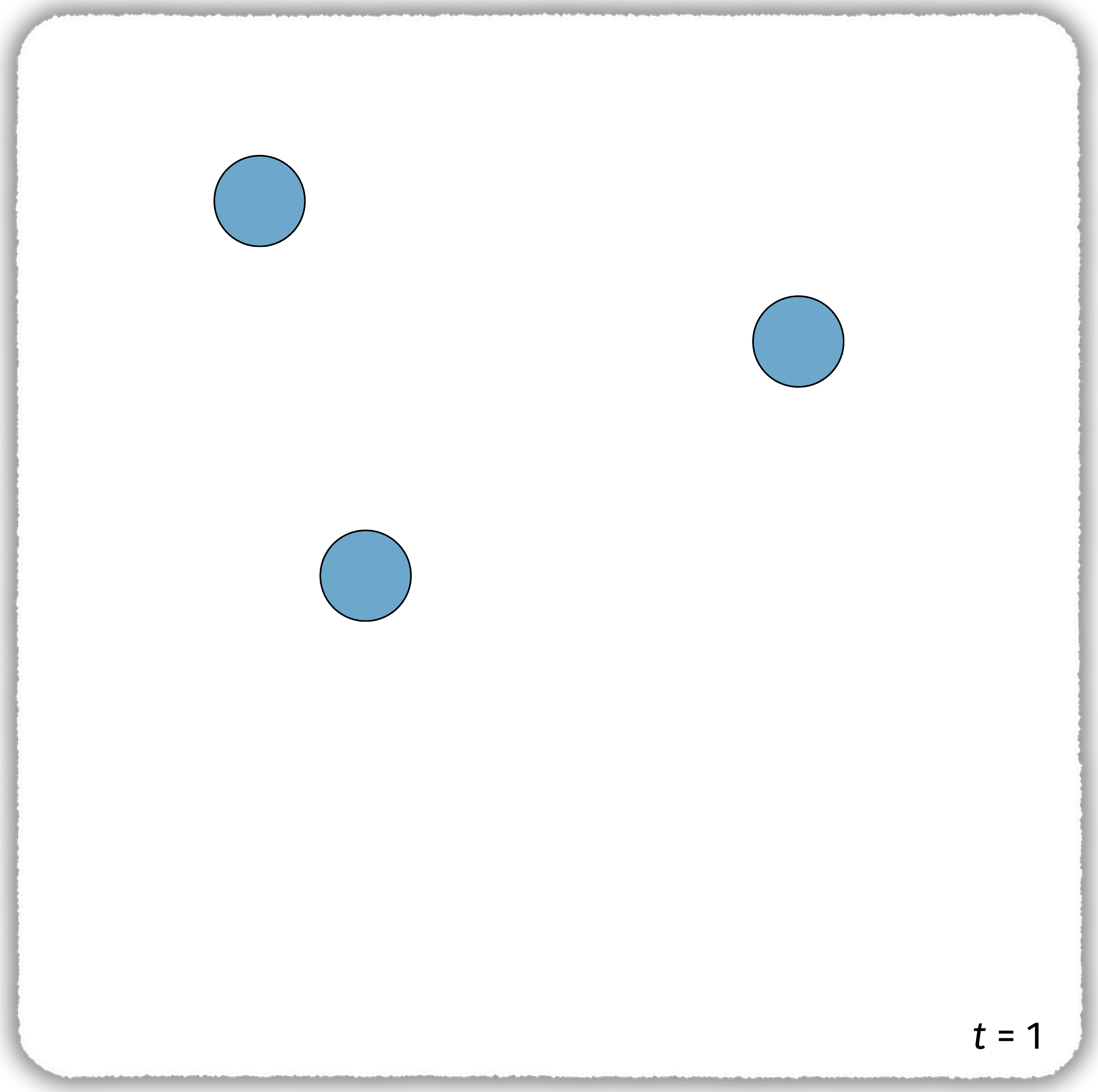
Consider a world of cooperators...



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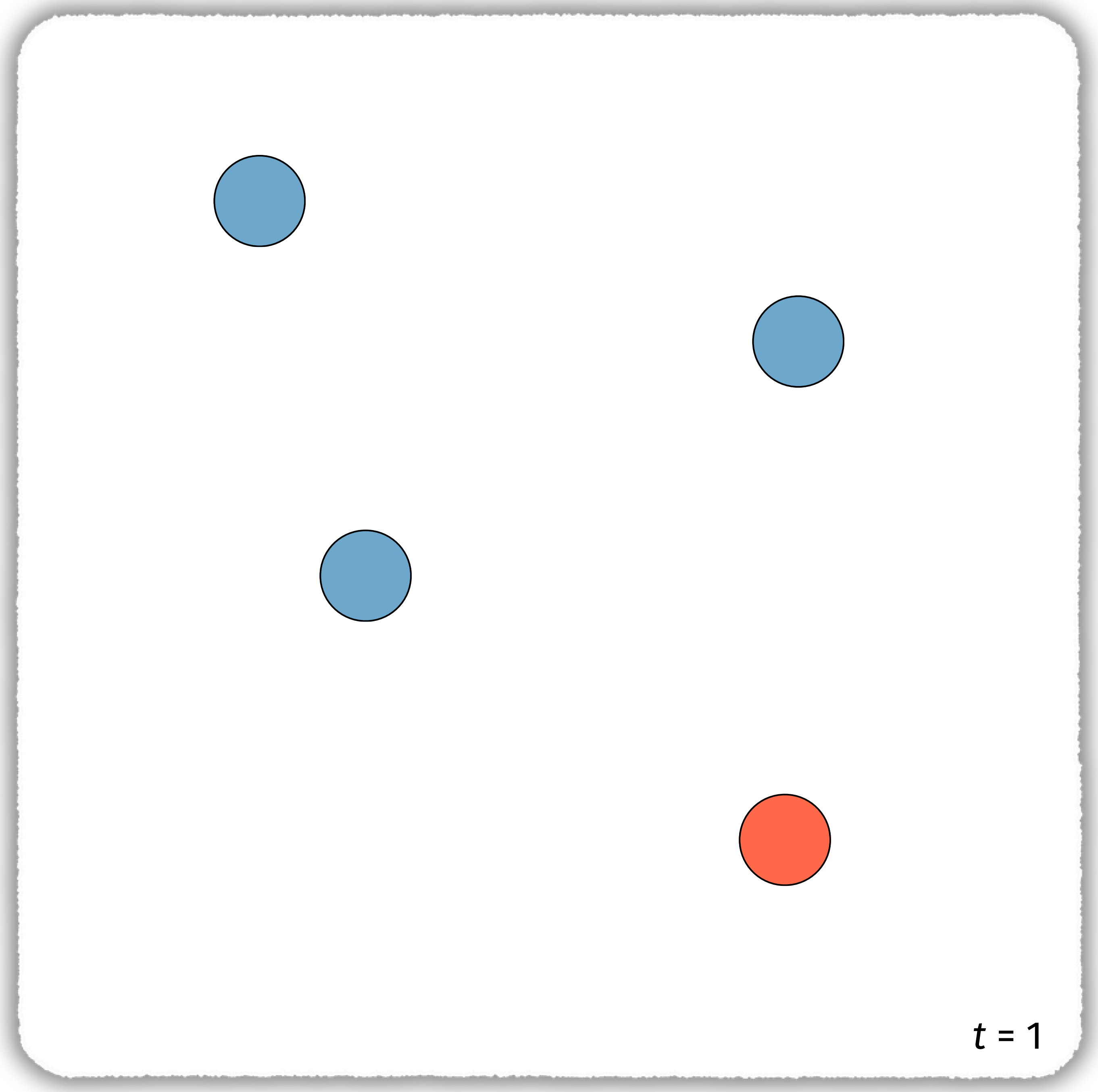
...in which one wild defector shows up.



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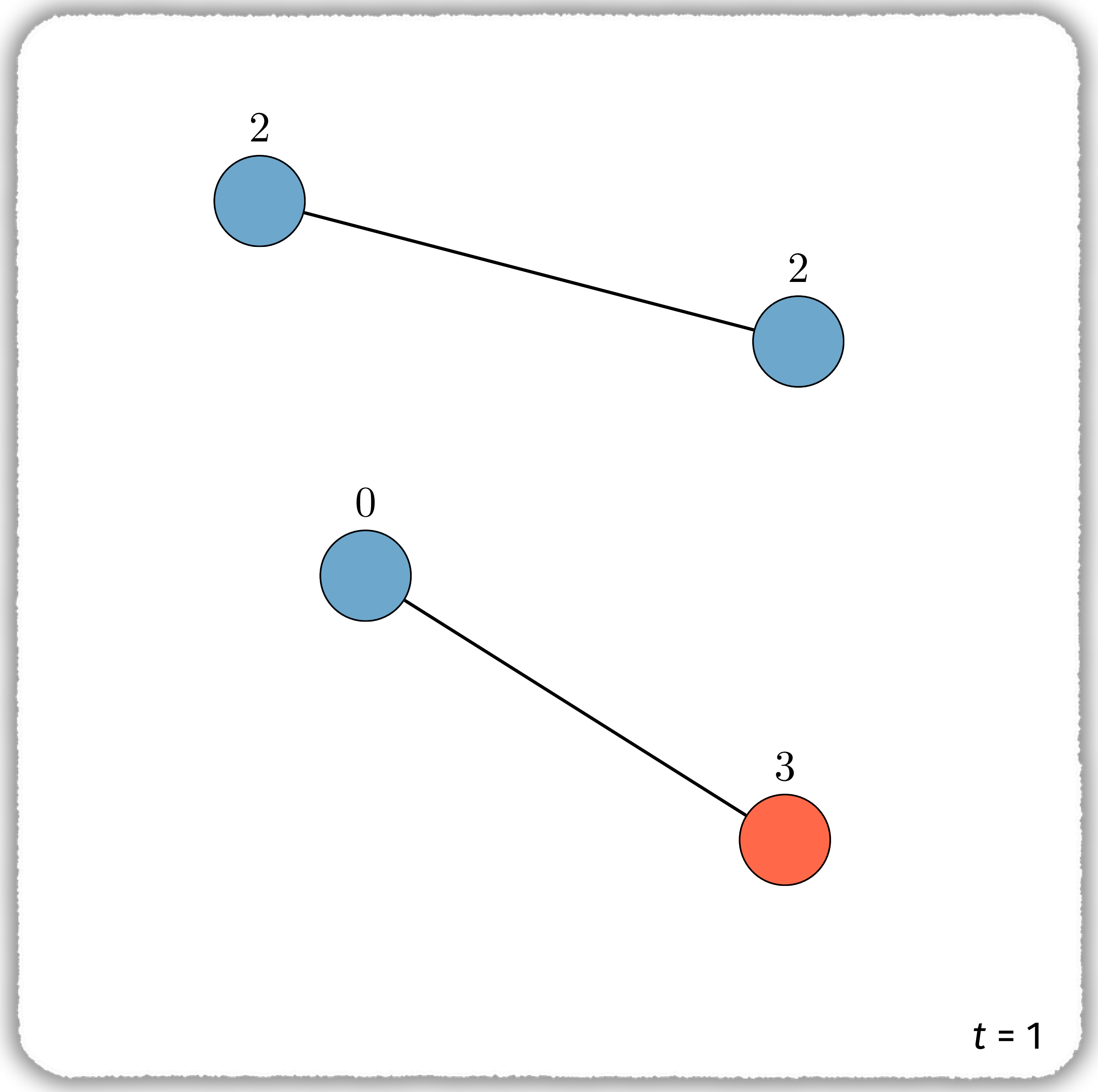

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1 \ 2	Cooperate	Defect
Cooperate	2, 2	0, 3
Defect	3, 0	1, 1

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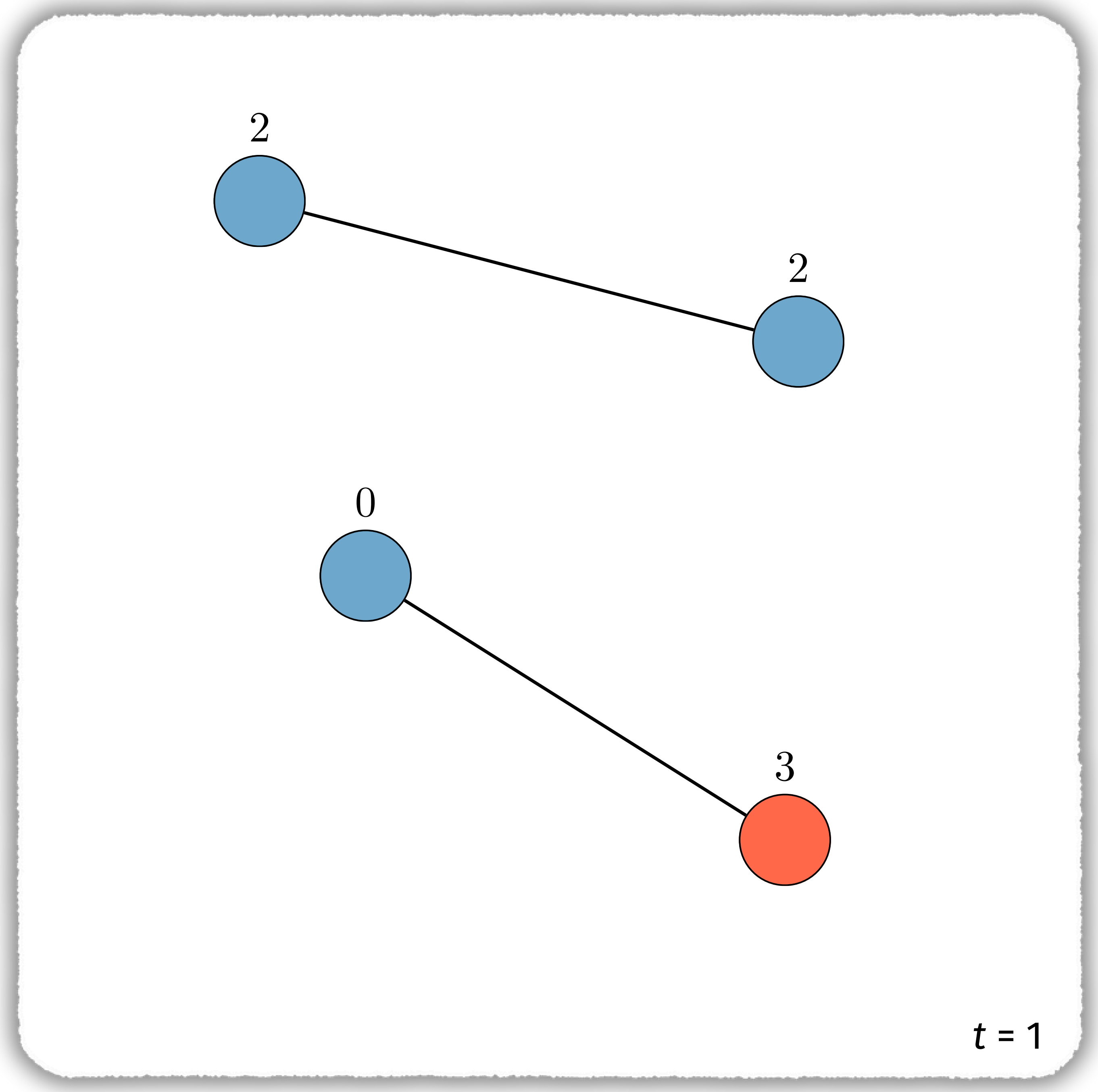

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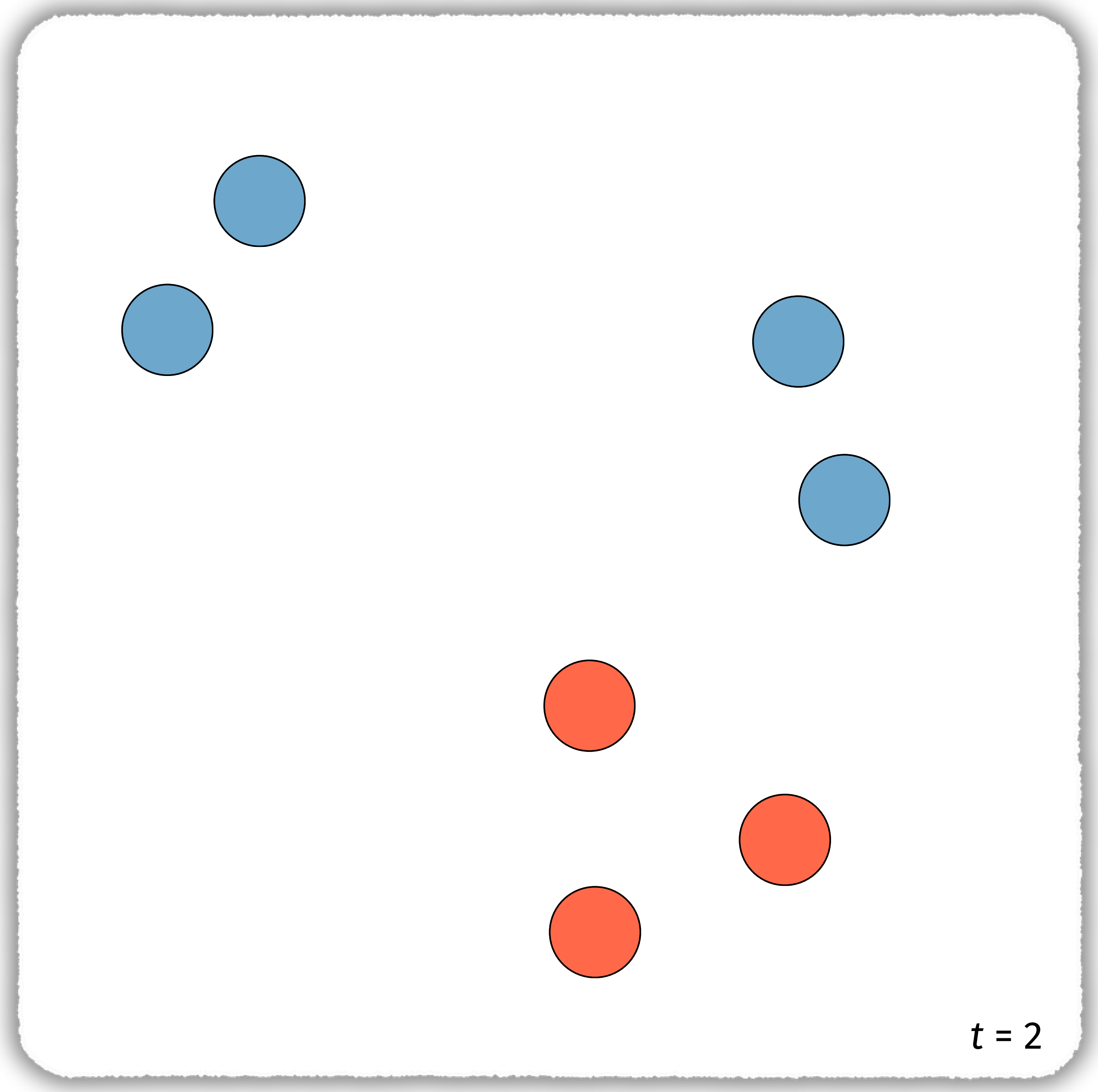

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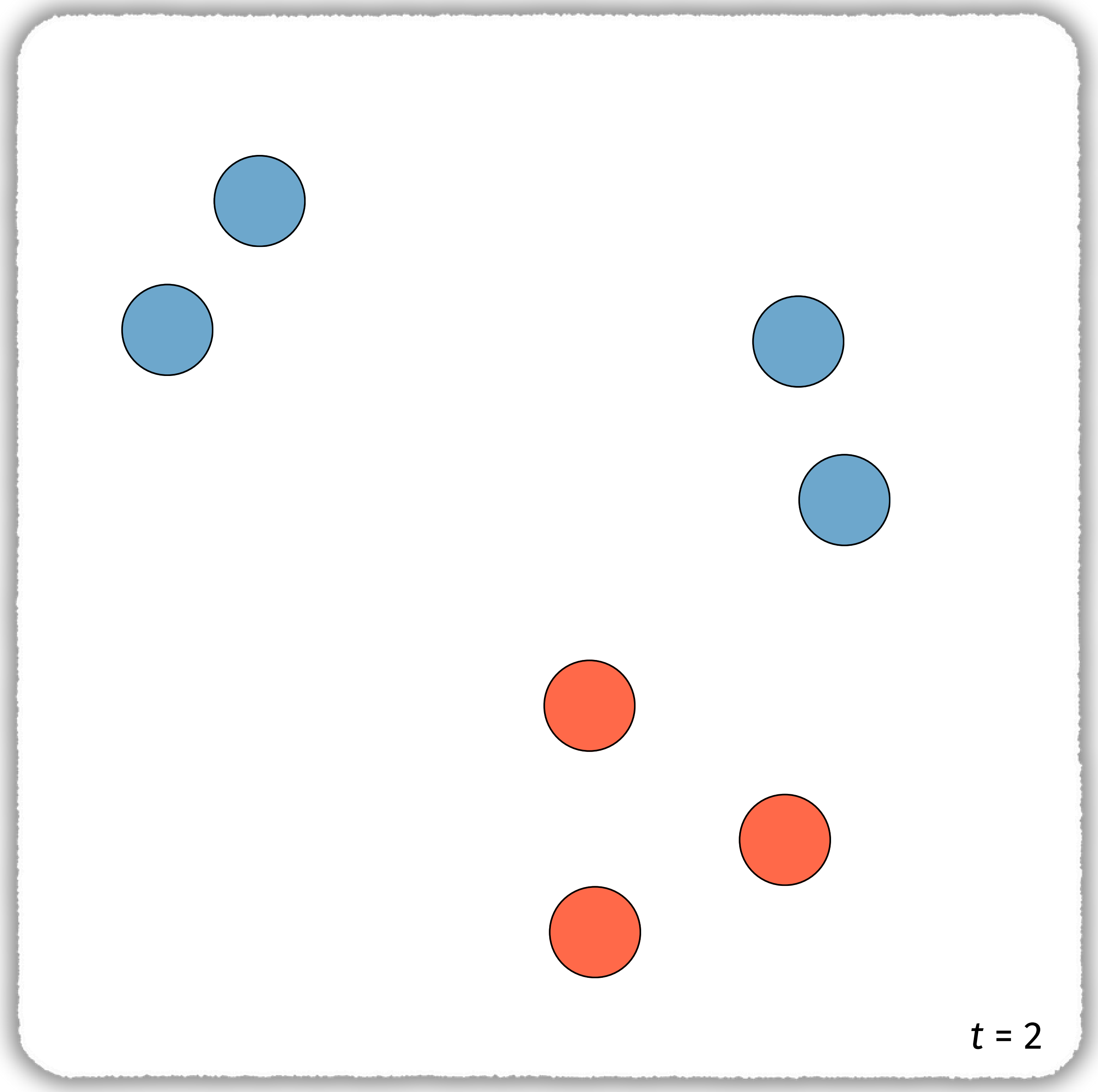

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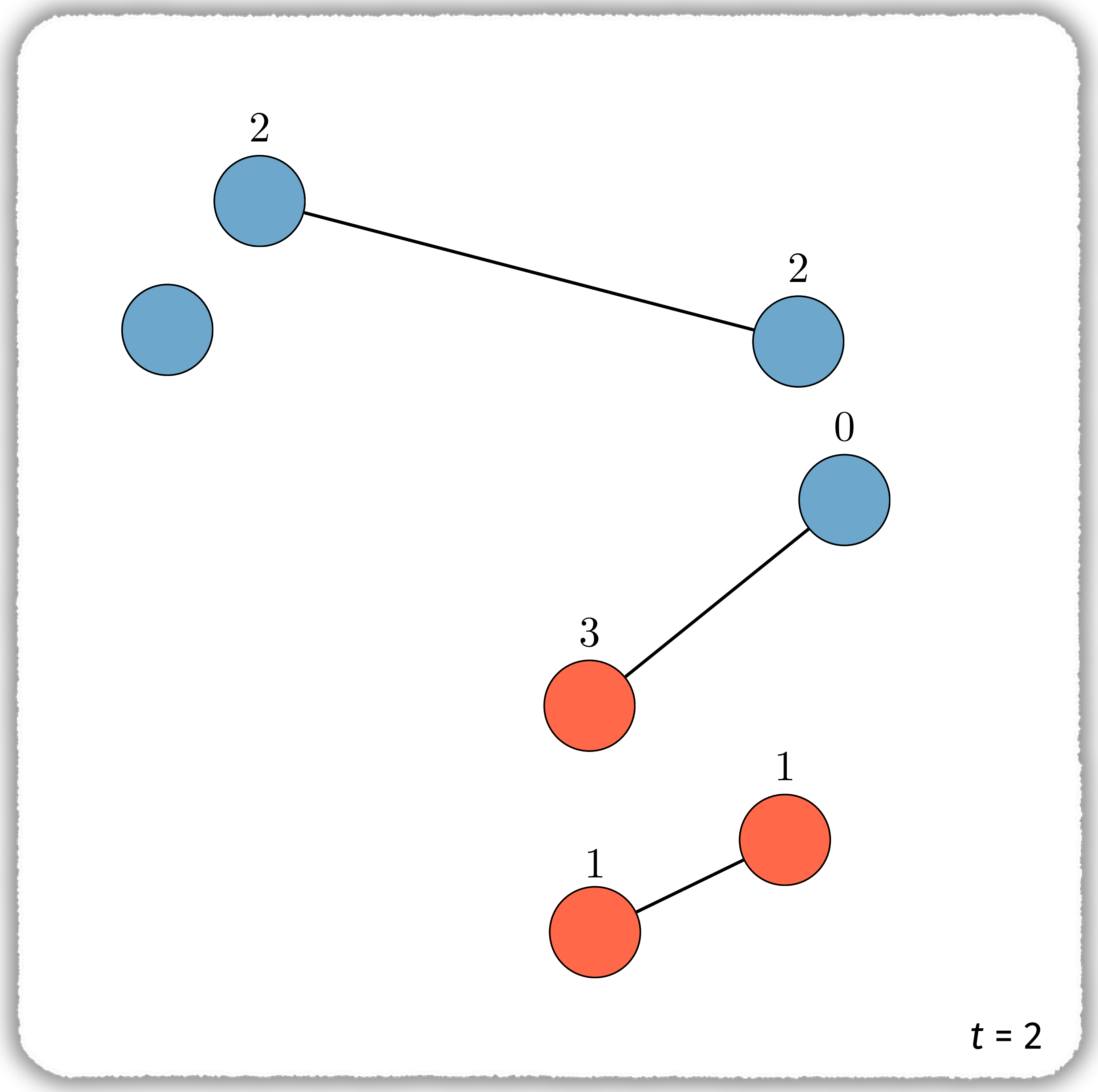

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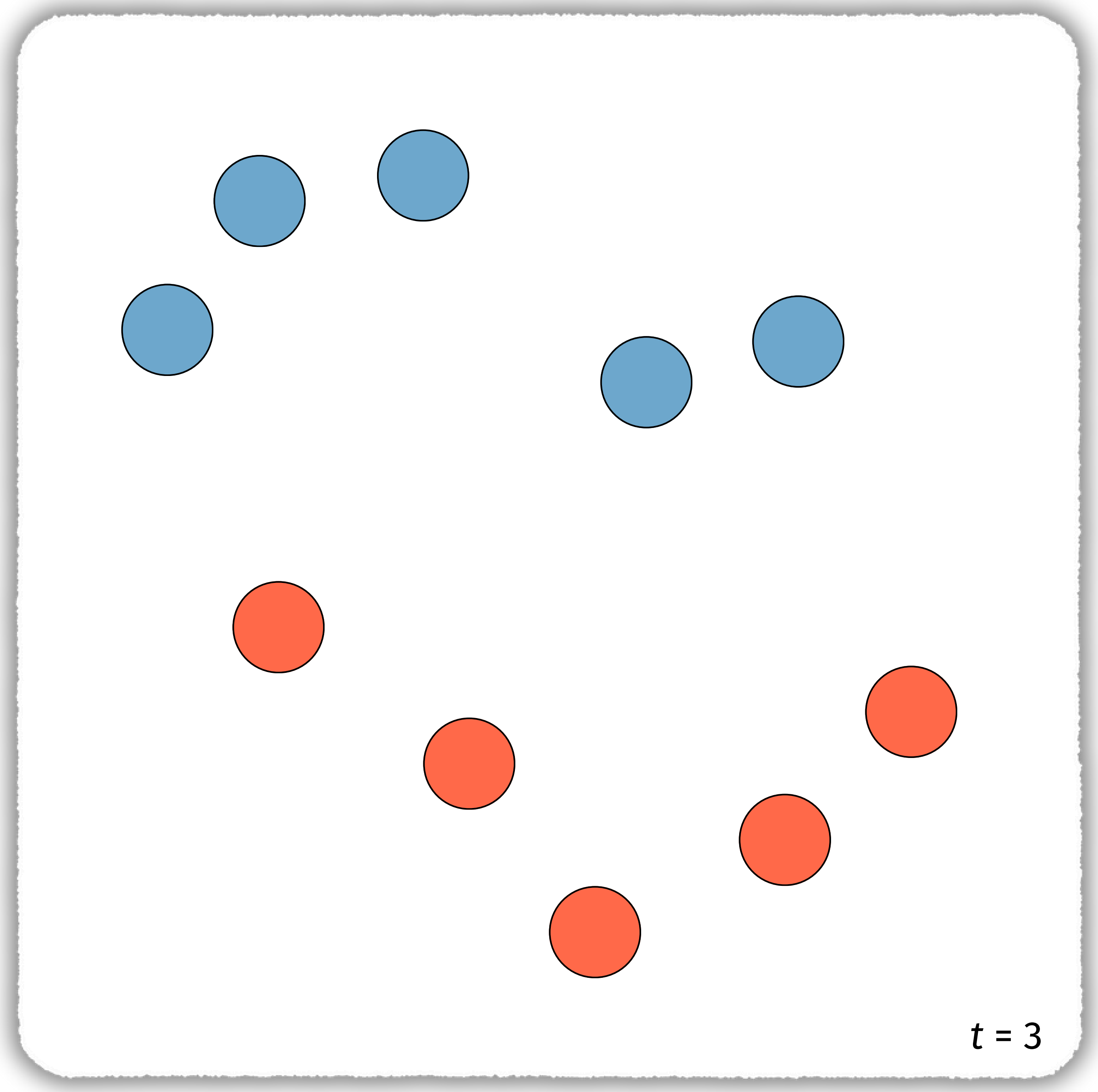

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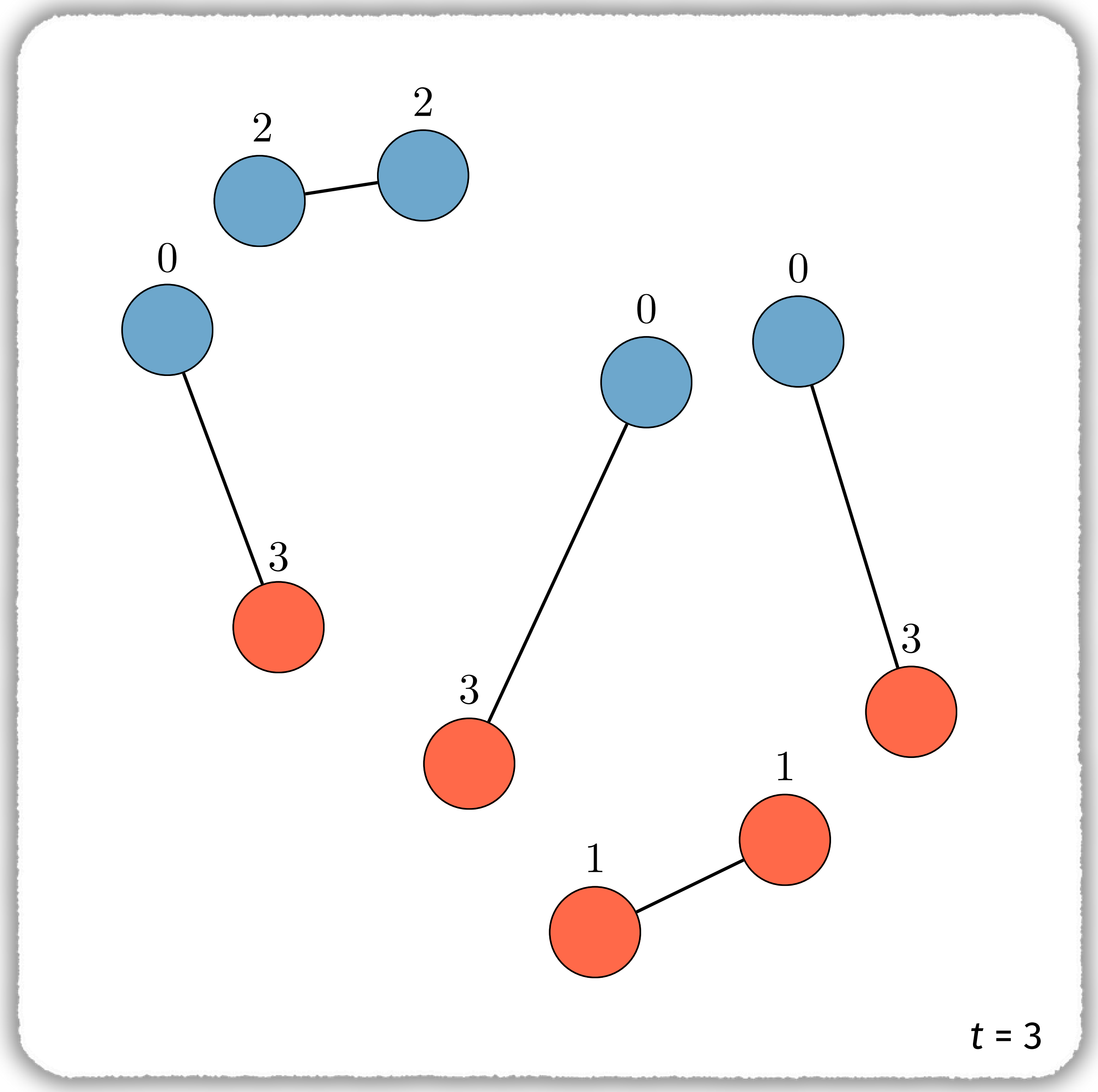

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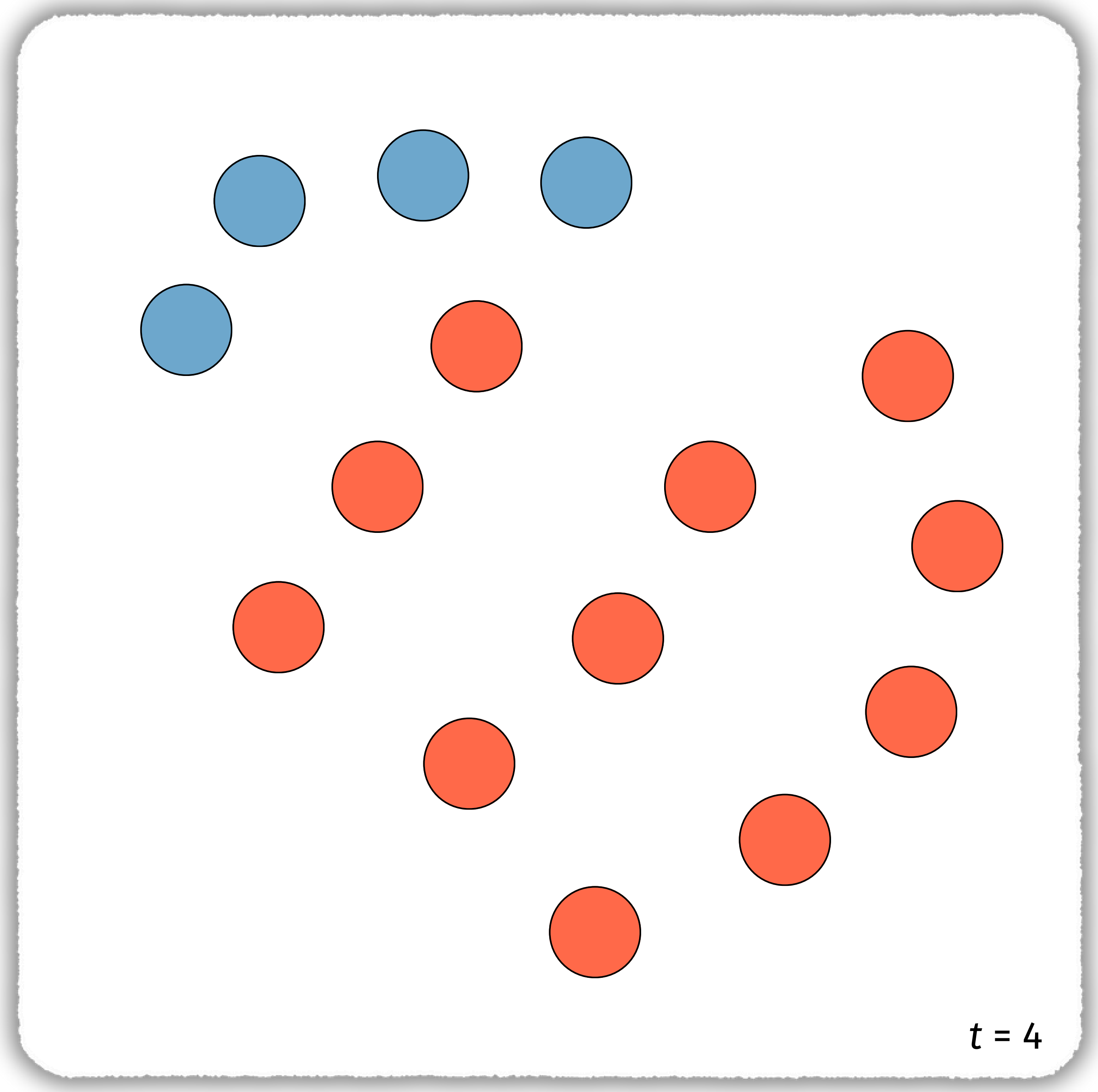

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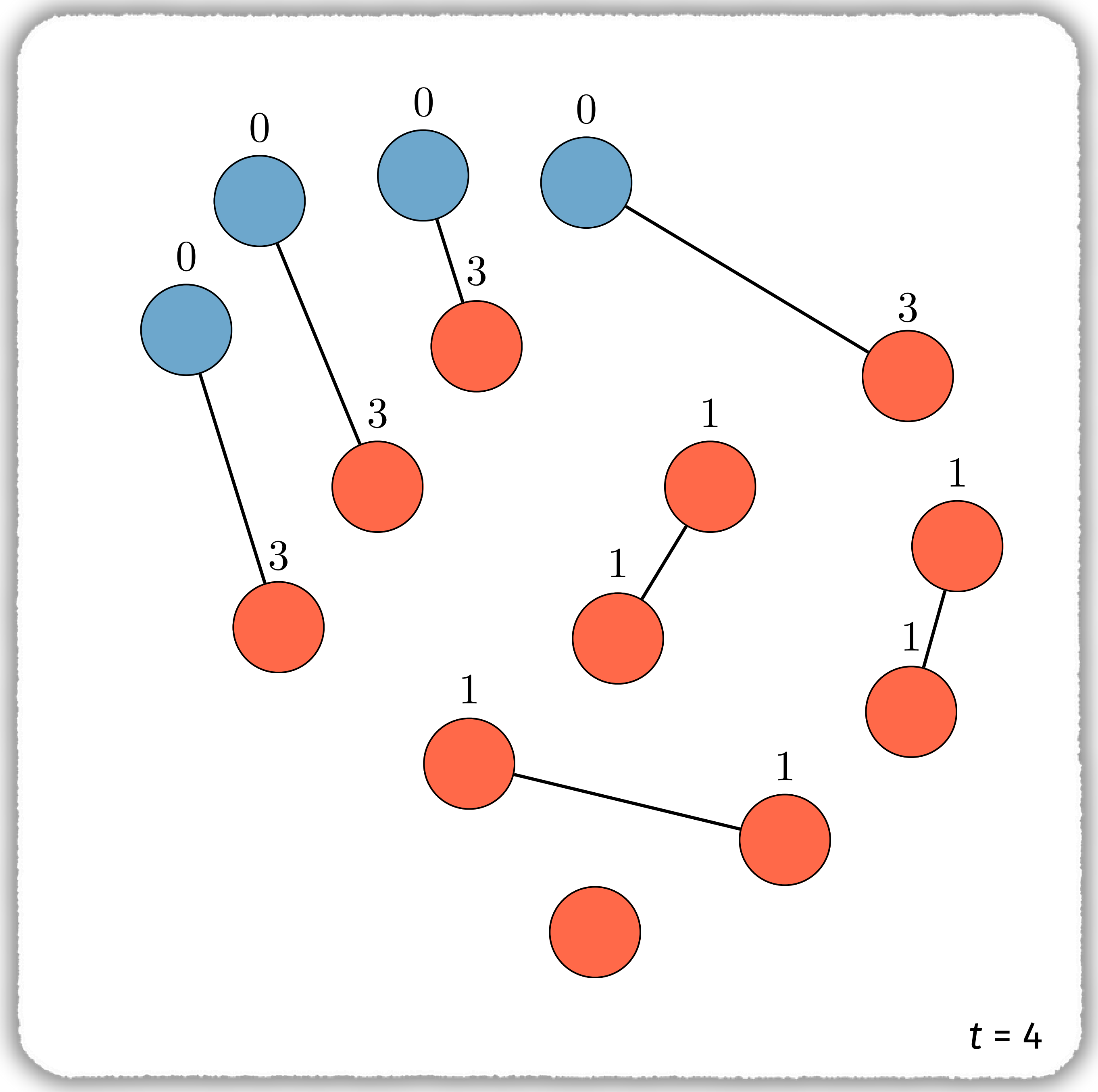

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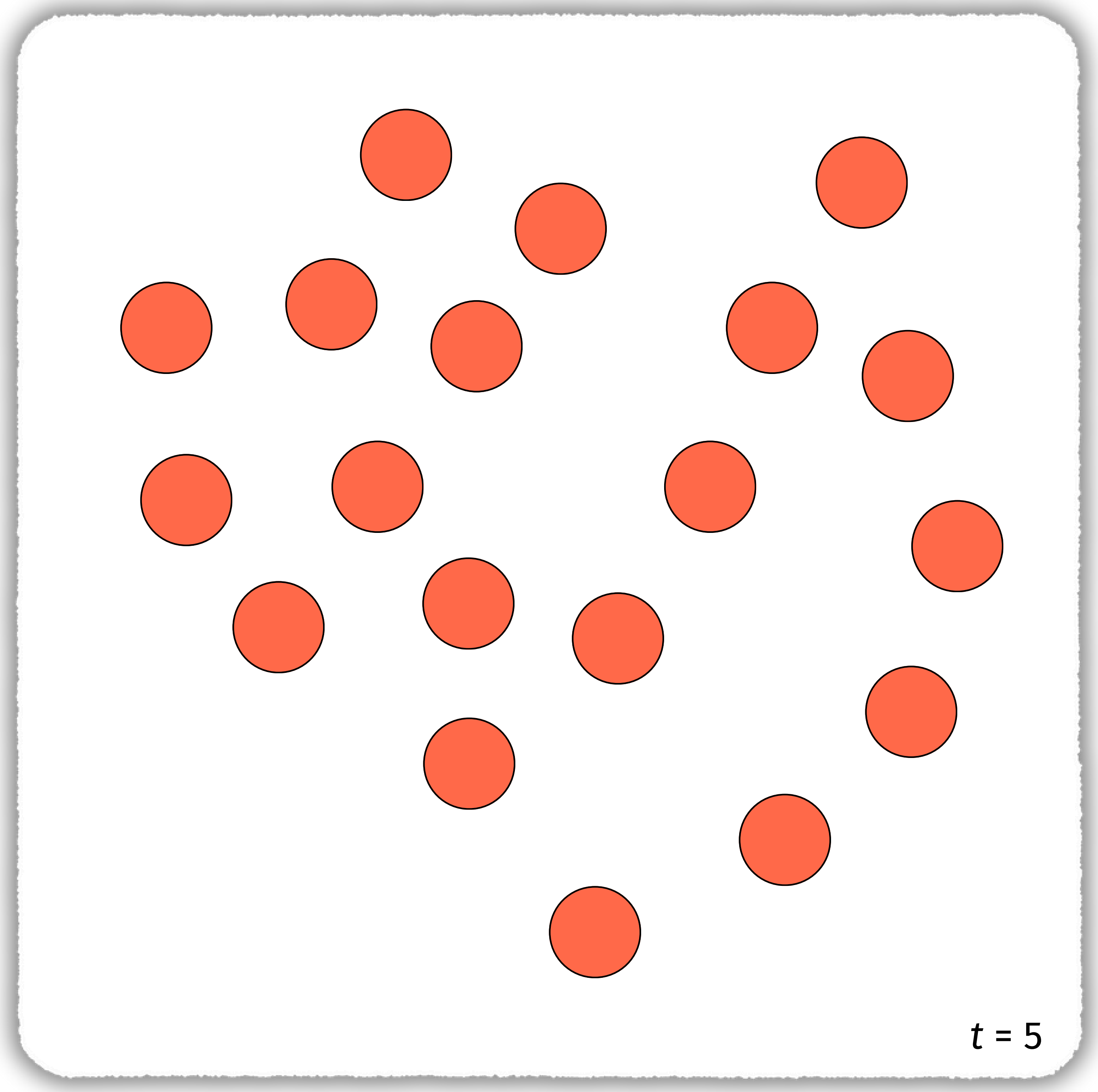

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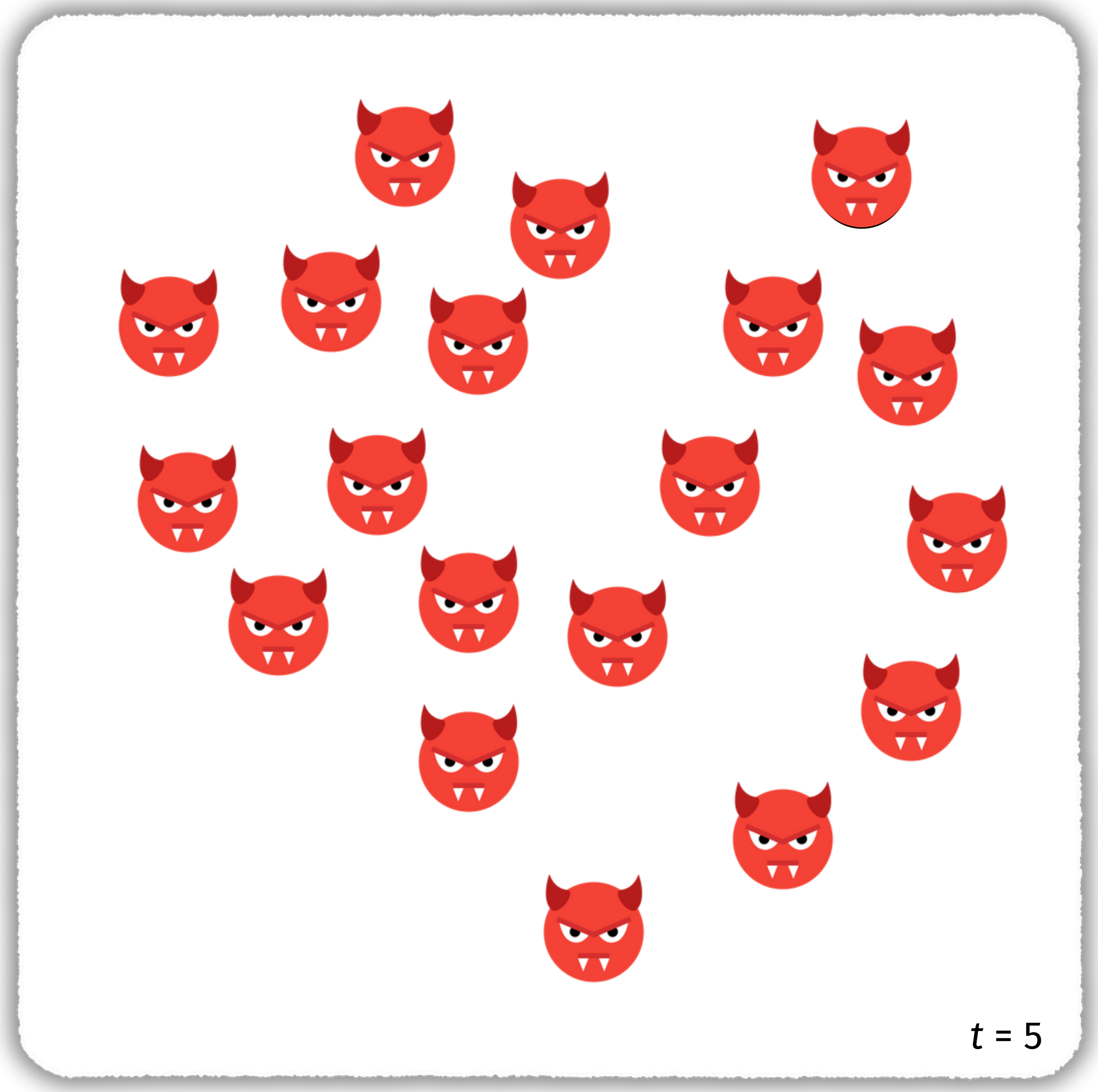
Individuals are paired randomly with each other and play a Prisoner's Dilemma.

The payoffs determine how many copies of these players make it to the next round.

What happens in the long run?

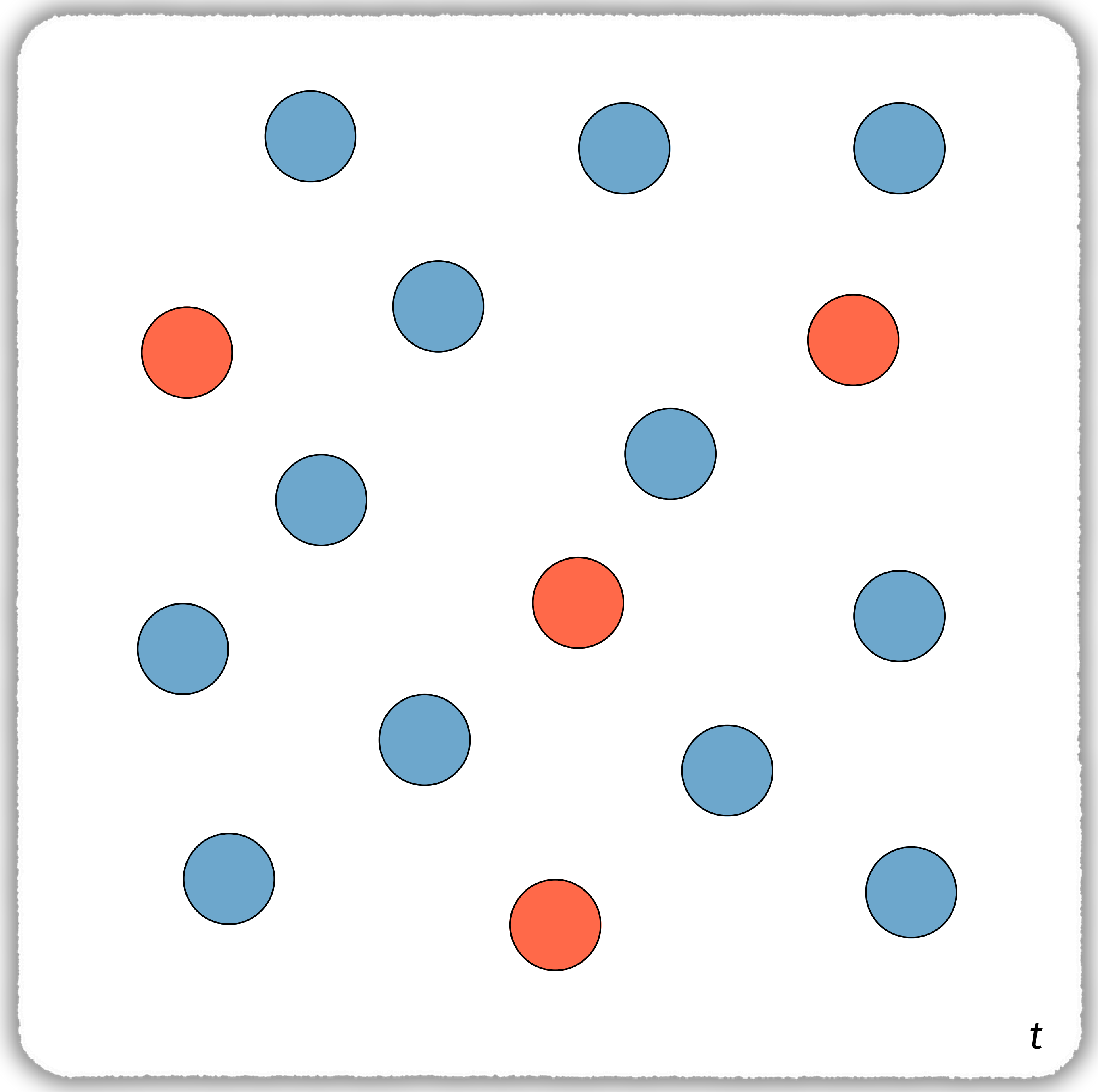
Defectors inherit the earth!



In a well-mixed population (i.e., equal probability of being paired with anyone else), defectors drive cooperators to extinction.

THE CALCULUS

Assume that at some time t there are c cooperators and d defectors.

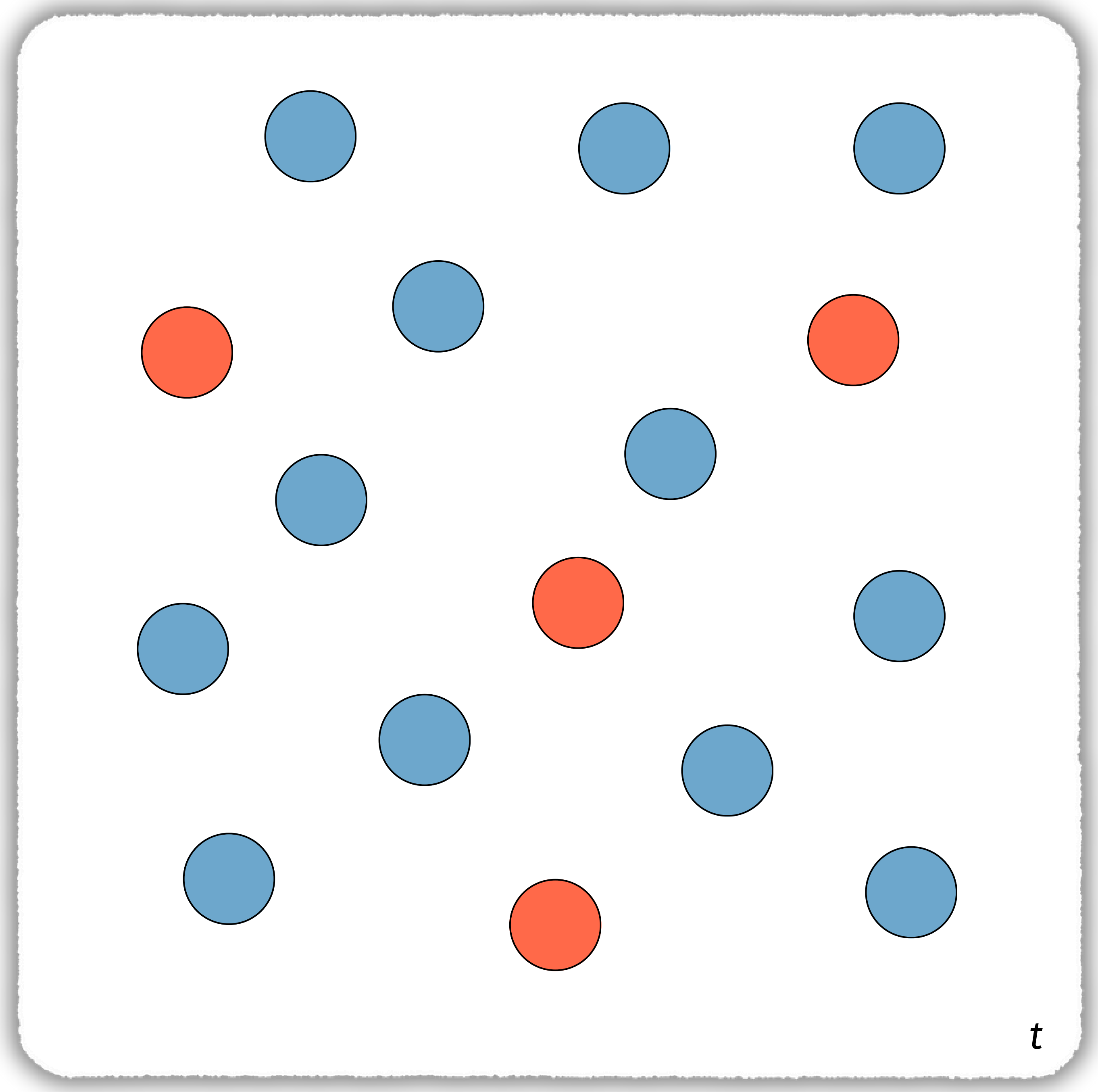


THE CALCULUS

Assume that at some time t there are c cooperators and d defectors.

The expected payoff of a cooperator is:

$$\begin{aligned}\mathbb{E}[\text{Cooperate}] &= u(\text{Cooperate}, \text{Cooperate}) \cdot \Pr[\text{match w/ cooperator}] + \\ &\quad u(\text{Cooperate}, \text{Defect}) \cdot \Pr[\text{match w/ defector}] \\ &= 2 \cdot \frac{c-1}{c+d} + 0 \cdot \frac{d}{c+d}.\end{aligned}$$



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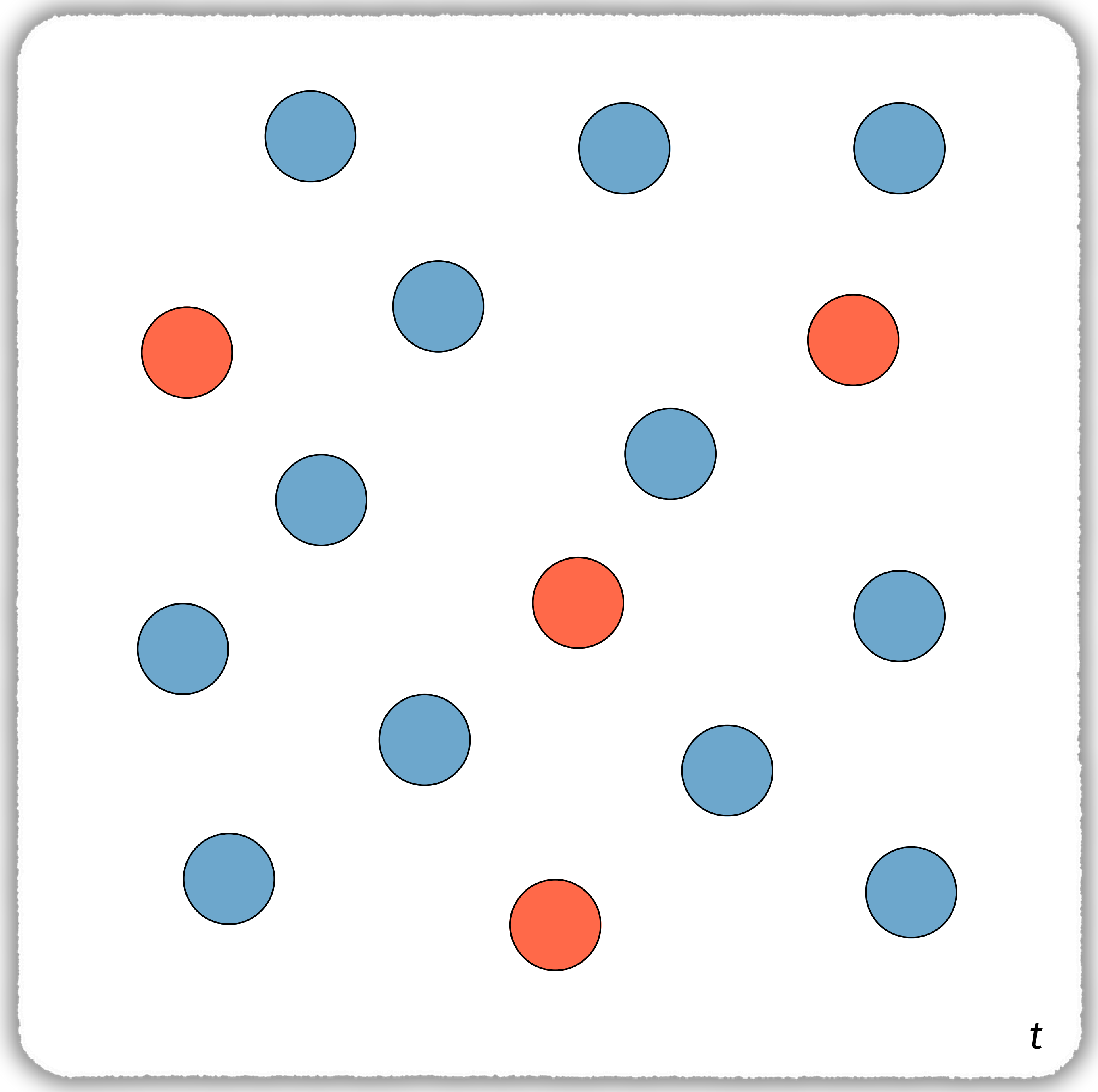
The expected payoff of a defector is:

$$\begin{aligned}\mathbb{E}[\text{Defect}] &= u(\text{Defect}, \text{Cooperate}) \cdot \Pr[\text{match w/ cooperator}] + \\ &\quad u(\text{Defect}, \text{Defect}) \cdot \Pr[\text{match w/ defector}] \\ &= 3 \cdot \frac{c}{c+d} + 1 \cdot \frac{d-1}{c+d}.\end{aligned}$$



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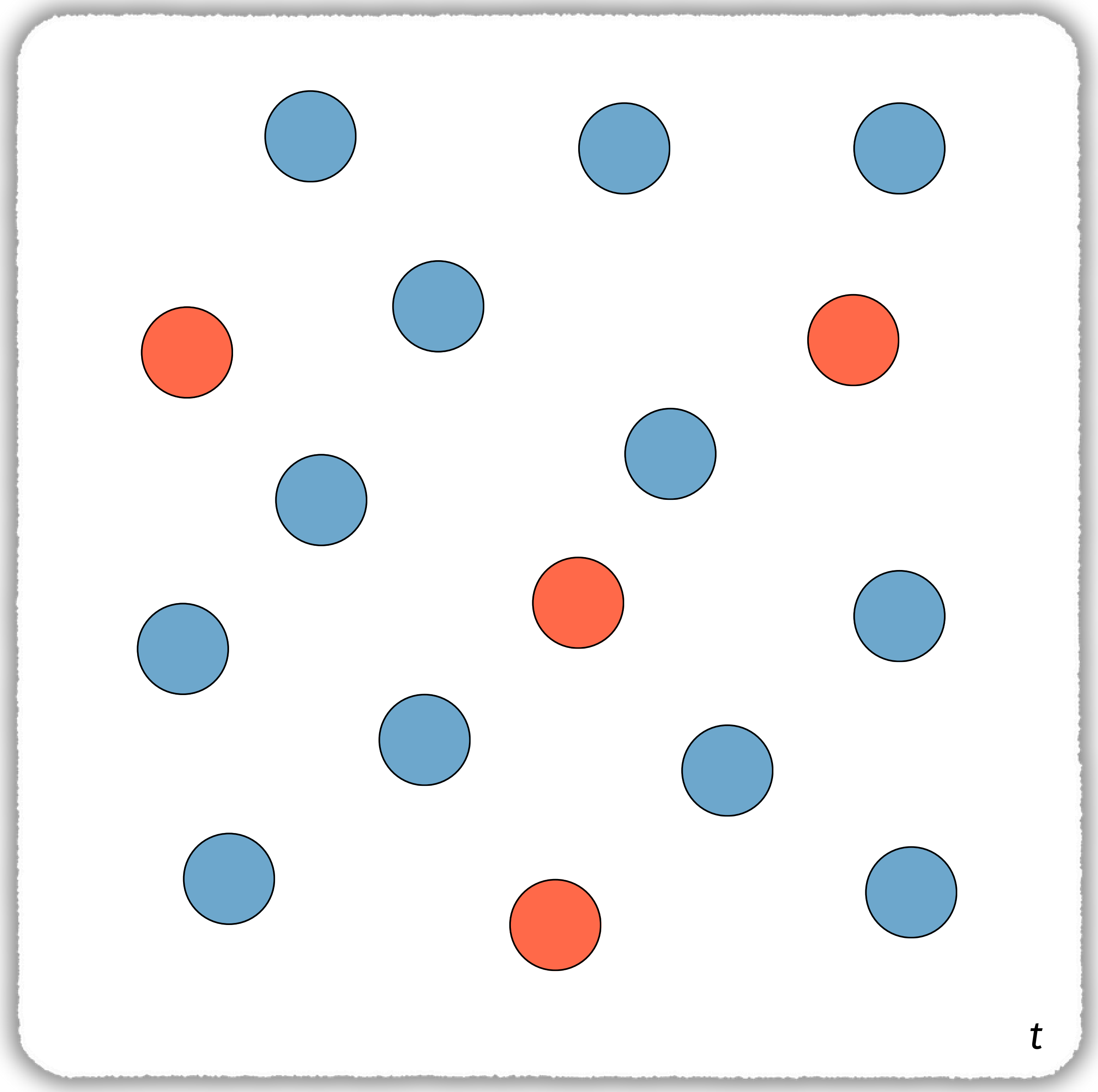
Note that $\mathbb{E}[\text{Defect}] > \mathbb{E}[\text{Cooperate}]$.

Defectors grow faster than cooperators.

Eventually, cooperators die out.



JOHN MAYNARD SMITH

Imagine a world of A players, and throw a B player in there.

The game between As and Bs is given by this game:

	A	B
A	$a,\ a$	$b,\ c$
B	$c,\ b$	$d,\ d$

What is the condition for selection to oppose the invasion of Bs?

GETTING TO AN EVOLUTIONARILY STABLE STRATEGY

Assume the proportions of Bs is ε, and the proportion of As is $1 - \varepsilon$.

	A (1 - ε)	B (ε)
A (1 - ε)	$a,\ a$	$b,\ c$
B (ε)	$c,\ b$	$d,\ d$

GETTING TO AN EVOLUTIONARILY STABLE STRATEGY

Assume the proportions of Bs is ε, and the proportion of As is $1 - \varepsilon$.

The expected payoff of A is greater than that of B if:

$$a(1-\varepsilon) + b\varepsilon > c(1-\varepsilon) + d\varepsilon,$$

	A (1 - ε)	B (ε)
A (1 - ε)	$a,\ a$	$b,\ c$
B (ε)	$c,\ b$	$d,\ d$

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which, if we ignore the ε terms, is equivalent to:

$$a > c.$$

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If, however, it happens that $a = c$, then we need:

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If, however, it happens that $a = c$, then we need:

$$b > d.$$

A strategy is evolutionarily stable if $a > c$, or $a = c$ and $b > d$.

	A (1 - ε)	B (ε)
A (1 - ε)	$a,\ a$	$b,\ c$
B (ε)	$c,\ b$	$d,\ d$

JOHN MAYNARD SMITH

Intuitively, an evolutionarily stable strategy is a genetically determined strategy that tends to persist once prevalent in the population.

EVOLUTIONARILY STABLE STRATEGY

DEFINITION

We write $u(s_i, s_j)$ for the payoff of strategy s_i against s_j.

Strategy s_i is an *evolutionarily stable strategy (ESS)* if:

(i) $u(s_i, s_i) > u(s_j, s_i)$, for all strategies $s_j \neq s_i$, or

(ii) $u(s_i, s_i) = u(s_j, s_i)$ and $u(s_i, s_j) > u(s_j, s_j)$, for all strategies $s_j \neq s_i$.

Note that in the Prisoner's Dilemma the ESS is defection.

	Cooperate	Defect
Cooperate	2, 2	0, 3
Defect	3, 0	1, 1

JOHN MAYNARD SMITH

This makes the problem of cooperation even more acute: how can cooperators survive when they can be so easily invaded by defectors?