



LUDWIG-
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UNIVERSITÄT
MÜNCHEN

INFORMED CHOICES, INCLUSIVE VOICES: EPISTEMIC JOURNEYS IN DEMOCRATIC DECISION MAKING

BEYOND THE CONDORCET JURY THEOREM

BIASES AND SOCIAL LEARNING

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The Condorcet Jury Theorem shows that groups can be wise.

What happens if its assumptions are not satisfied?

For instance, what if agents have different competences?

THREE VOTERS WITH DIFFERENT COMPETENCES

Take three voters with competences p_1, p_2, p_3 .

The probability of a correct majority decision is:

$$\begin{aligned}\Pr [F_{maj}(v_1, v_2, v_3) = a] &= \Pr[\text{the profile is one of } aab, aba, baa, aaa] \\ &= p_1p_2(1 - p_3) + p_1(1 - p_2)p_3 + (1 - p_1)p_2p_3 + p_1p_2p_3 \\ &= p_1p_2 + p_2p_3 + p_1p_3 - 2p_1p_2p_3.\end{aligned}$$

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With different competences things unravel a bit.

THINGS UNRAVEL A BIT

The group is no longer better than its members:

$$\Pr [F_{maj}(0.9, 0.6, 0.55) = a] = 0.77.$$

Enlarging the group does not necessarily make it better:

$$\Pr [F_{maj}(0.9, 0.6, 0.55, 0.55, 0.55) = a] = 0.76.$$

And if the p_i 's approach $1/2$ very fast then:

$$\lim_{n \rightarrow \infty} \Pr [F_{maj}(v_1, \dots, v_n) = a] \neq 1.$$

JACOB PAROUSH

Group competence still goes asymptotically to 1 as n goes to infinity, if the competence of each agent is above $0.5 + \varepsilon$, for some $\varepsilon > 0$.



Paroush, J. (1997). Stay away from fair coins: A Condorcet jury theorem. *Social Choice and Welfare*, 15(1), 15–20.



BERNARD GROFMAN

Or if the average competence is a fixed number above 0.5.

Grofman, B., Owen, G., & Feld, S. L. (1983). Thirteen theorems in search of the truth. *Theory and Decision*, 15(3), 261–278.

What about something more general?

For instance, if the competences are drawn from some probability distribution...



We've assumed competence is above $\frac{1}{2}$.

What would be a reason for it to be below $\frac{1}{2}$?

Pop quiz time!

A large steel truss bridge, likely the Manhattan Bridge, spans across a body of water. The bridge features a complex network of steel beams and supports, with two prominent towers. In the background, a city skyline with various buildings is visible under a clear blue sky. The water in the foreground is calm with some ripples.

This bridge connects Manhattan to
what other New York borough?

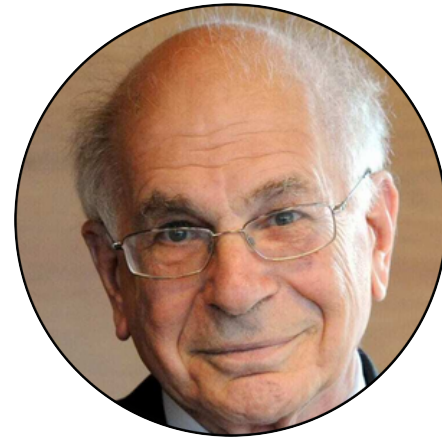
- Brooklyn
- Queens

The image shows a large steel truss bridge, the Ed Koch Queensboro Bridge, spanning a wide river. The bridge has two prominent stone towers with arches at the base. The sky is clear and blue. In the background, there are several high-rise apartment buildings and some greenery along the riverbank.

This bridge connects Manhattan to
what other New York borough?

- Brooklyn
- Queens

THE ED KOCH QUEENSBORO BRIDGE



DANIEL KAHNEMAN

You thought it was Brooklyn, didn't you?

Humans have biases!

Kahneman, D. (2013). *Thinking, Fast and Slow*. Farrar, Straus and Giroux.

BRYAN CAPLAN
Most people can't be relied on take good
decisions.



Caplan, B. (2011). *The Myth of the Rational Voter: Why Democracies Choose Bad Policies*. Princeton University Press.



JASON BRENNAN
Especially when it comes to political issues.

Brennan, J. (2017). *Against Democracy*. Princeton University Press.



HÉLÈNE LANDEMORE
Yeah let's not exaggerate.



Landemore, H. (2013). *Democratic Reason: Politics, Collective Intelligence, and the Rule of the Many*. Princeton University Press.

What does this p even mean, anyway?

Does it make sense to rate people's accuracies? Especially if predicting rare, or unique, events?



GLENN BRIER

Sure! Check out the Brier score.

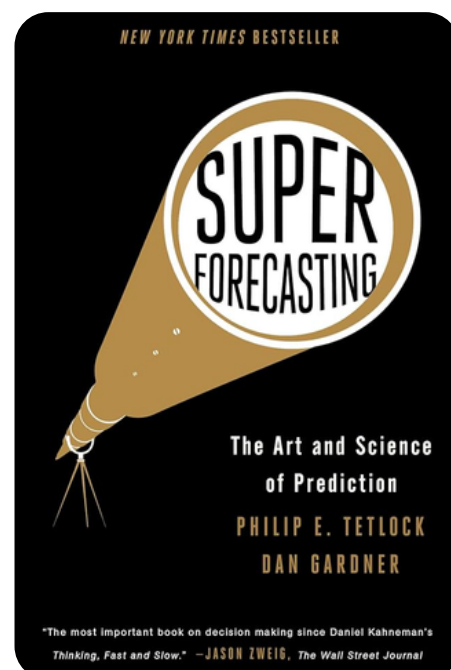
CONDORCET

Even so: is it realistic to assume that $p > 0.5$?



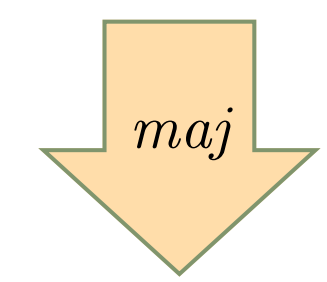
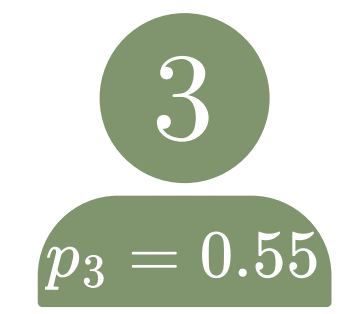
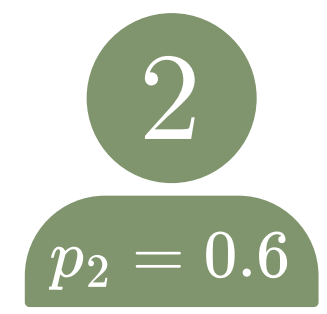
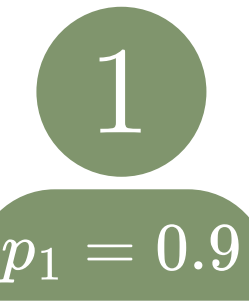
PHILIP E. TETLOCK

Some people seem to manage it: superforecasters. 🔍



Tetlock, P. E., & Gardner, D. (2016). *Superforecasting: The Art and Science of Prediction*. Random House.

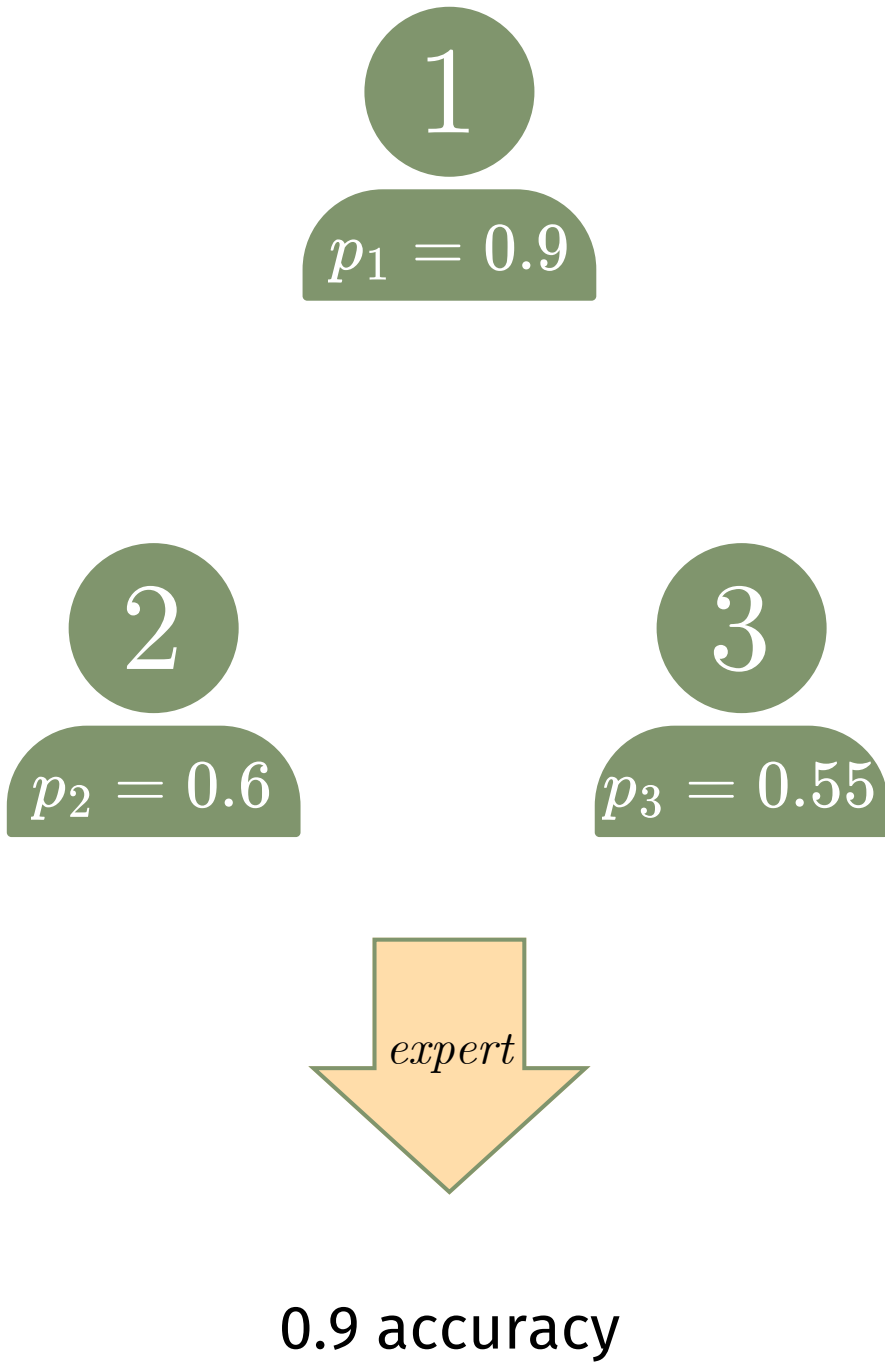
Is majority the best decision rule to use?



0.77 accuracy

See? We'd do better by going with the expert.

PLATO



See? We'd do better by going with the expert.

PLATO



LLOYD SHAPLEY

If we want to maximize group accuracy, then the best decision rule is a *weighted* voting rule.

Where the weight of voter i is proportional to:

$$\log \left(\frac{p_i}{1 - p_i} \right)$$

These weights might be negative! If the competences is bad.

Shapley, L., & Grofman, B. (1984). Optimizing group judgmental accuracy in the presence of interdependencies. *Public Choice*, 43(3), 329–343.

$$w_1 = 0.95$$

1

$$p_1 = 0.9$$

$$w_2 = 0.18$$

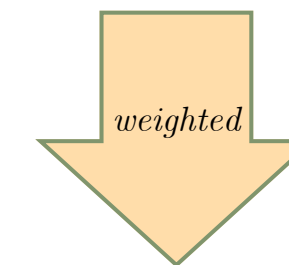
2

$$p_2 = 0.6$$

$$w_3 = 0.09$$

3

$$p_3 = 0.55$$



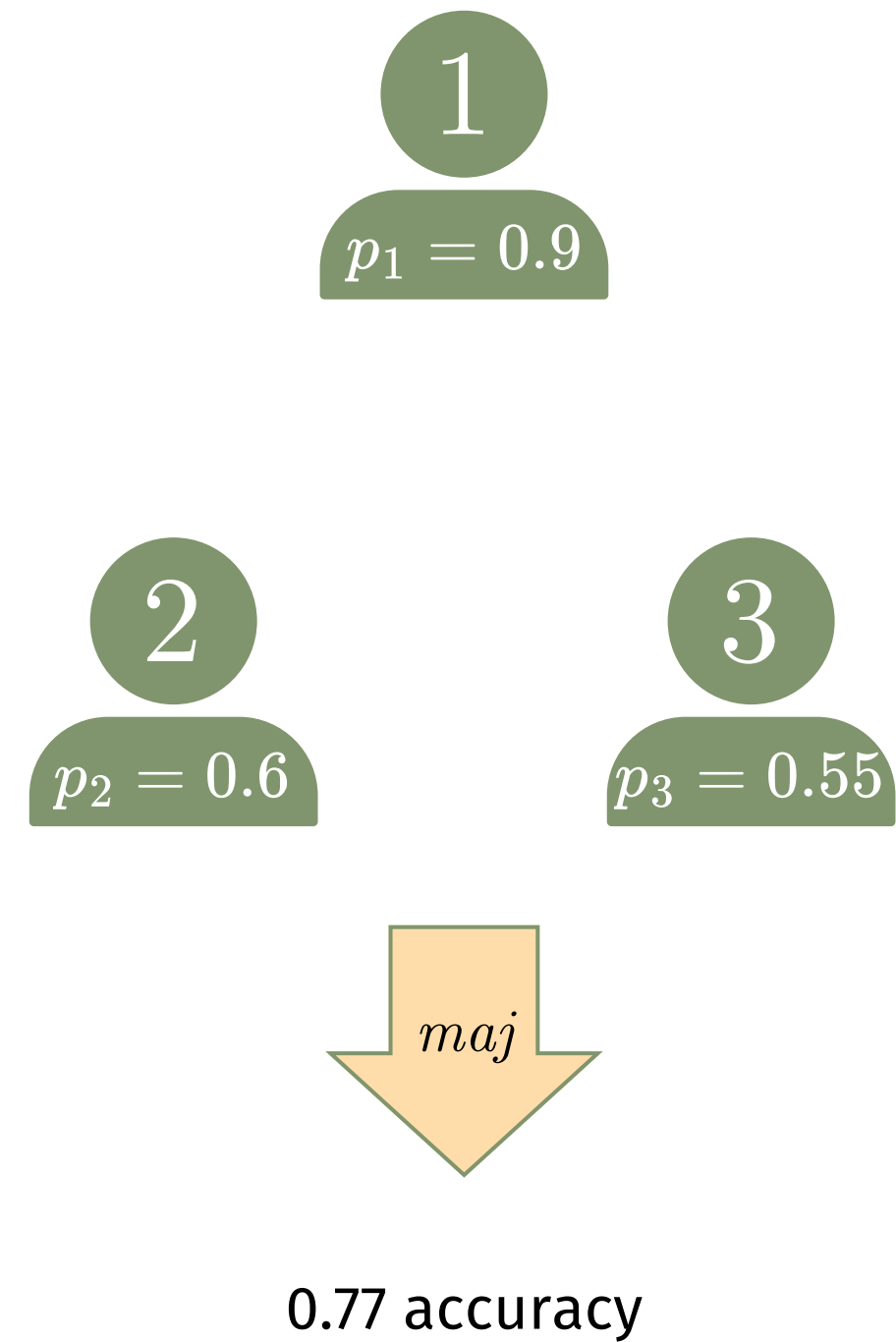
0.9 accuracy

Assigning voters *negative* weight might not go well with democratic ideals...

Chasing accuracy might conflict with fairness.

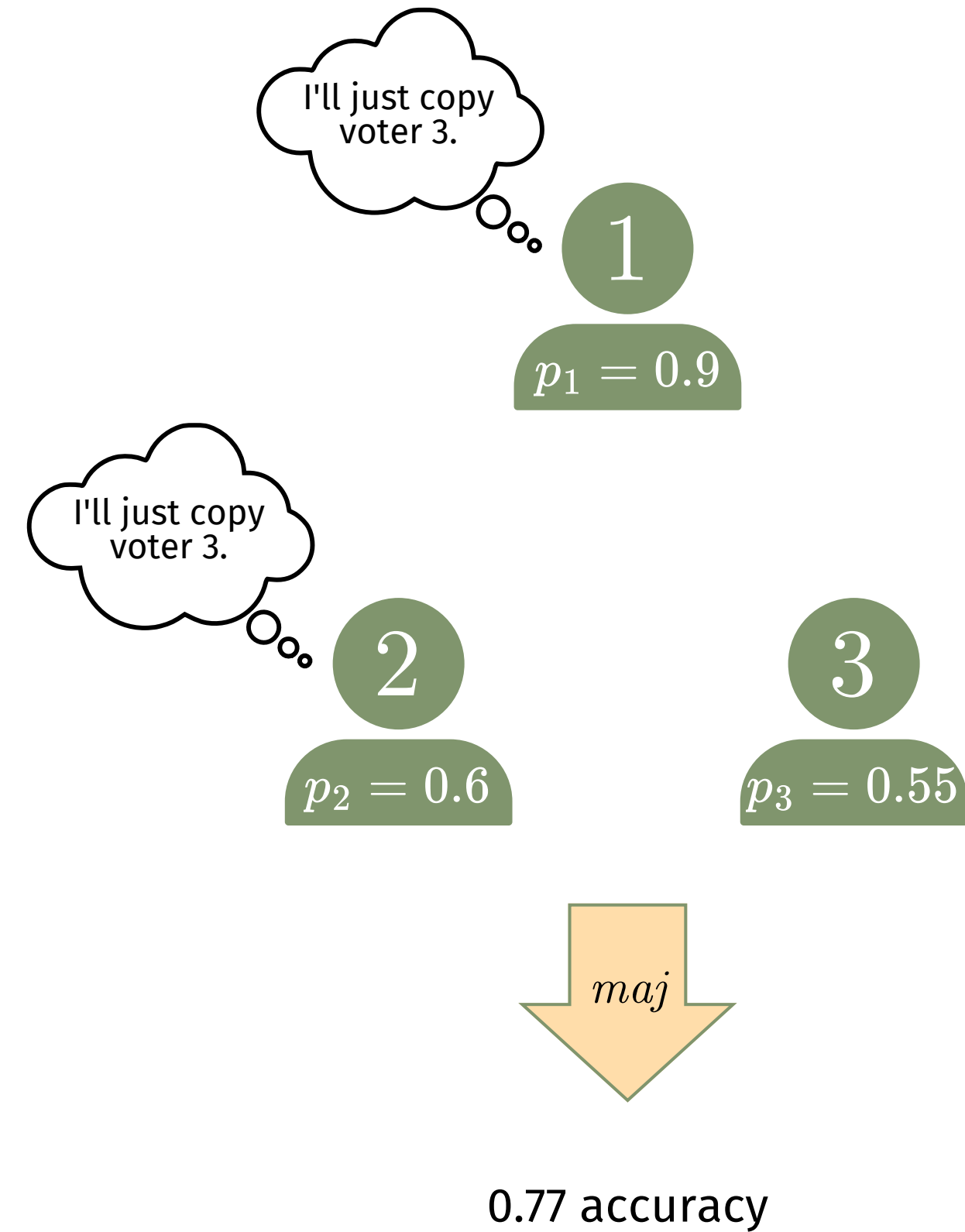
What about independent voters?

Majority opinion with independent voters achieves accuracy of 0.77.



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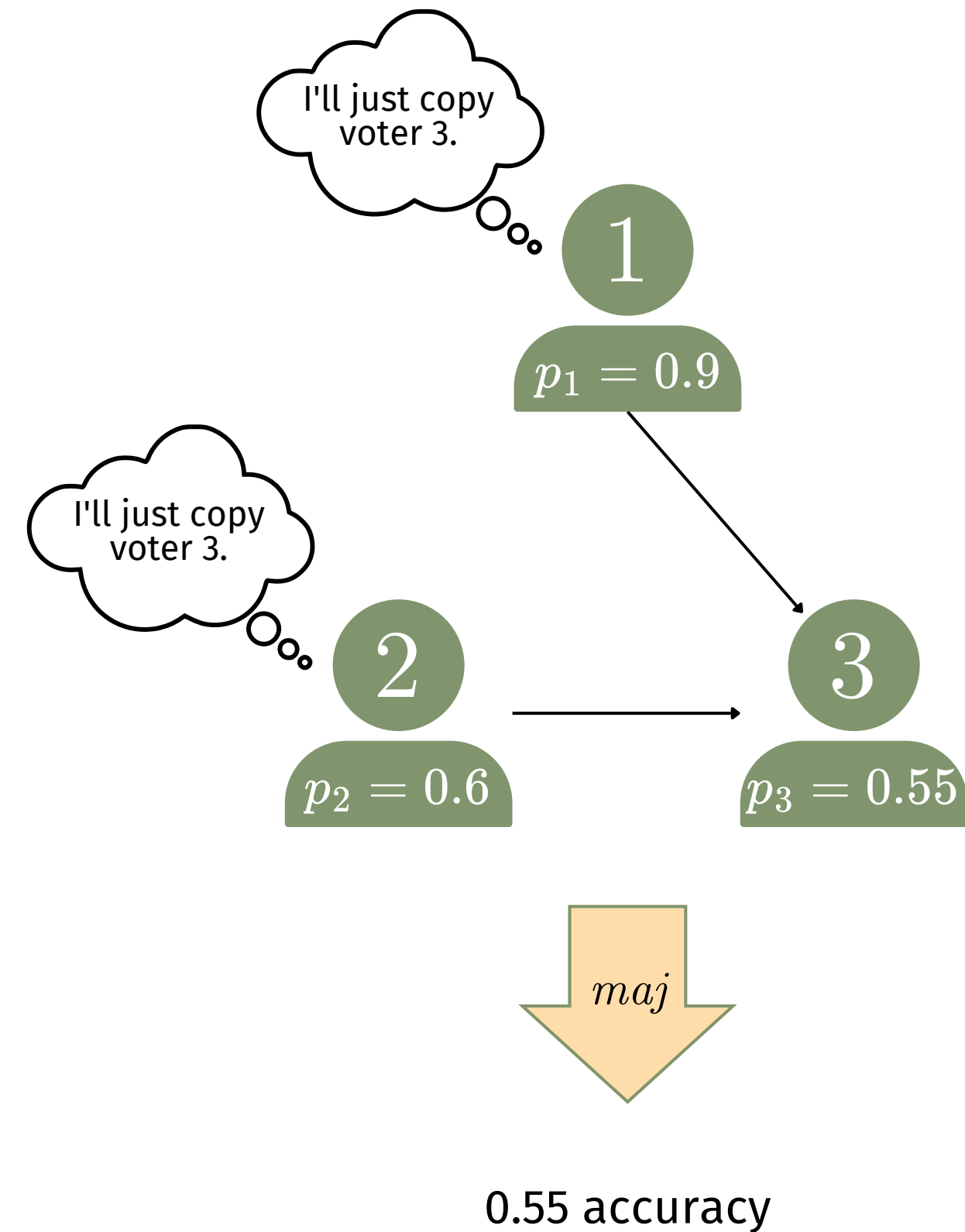
Suppose voters copy voter 3.



Majority opinion with independent voters achieves accuracy of 0.77.

Suppose voters copy voter 3.

The group is only as good as voter 3!



Introducing correlations between voters makes the optimistic conclusions of the Condorcet Jury Theorem go away.

But people are likely to become correlated: they talk to each other, observe each other, are exposed to similar sources of information.

In social networks.

MATTHEW O. JACKSON

**Social networks have an outsized influence
on people's beliefs and behaviors.**



Jackson, M. O. (2019). *The Human Network: How Your Social Position Determines Your Power, Beliefs, and Behaviors*. Knopf Doubleday.



NICHOLAS CHRISTAKIS

**Best predictor of smoking is whether your
friends smoke.**

Christakis, N. A., & Fowler, J. H. (2008). The collective dynamics of smoking in a large social network. *The New England Journal of Medicine*, 358(21), 2249–2258.