LMU LUDWIG-MAXIMILIANS-UNIVERSITÄT MÜNCHEN

NOVEMBER 19, 2024

MODEL AGENTS: SOCIAL BEHAVIOR THROUGH THE FORMAL LENS

# THE WISDOM OF CROWDS

Adrian Haret
a.haret@lmu.de

Sometimes groups can be smart.

FRANCIS GALTON

If I may be allowed!

I can personally attest to the surprising accuracy of group judgment.

I mean, of course...

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the ox! 🐂

# COUNTRY MATTERS

A weight-judging competition was carried on at the annual show of the West of England Fat Stock and Poultry Exhibition, in Plymouth, in 1907.

Competitors would try to guess the weight of an ox that had been slaughtered and laid out in the market.

Guesses would be submitted on paper tickets.

Closest guesses would receive prizes.

James Holland. *Country Fair.*

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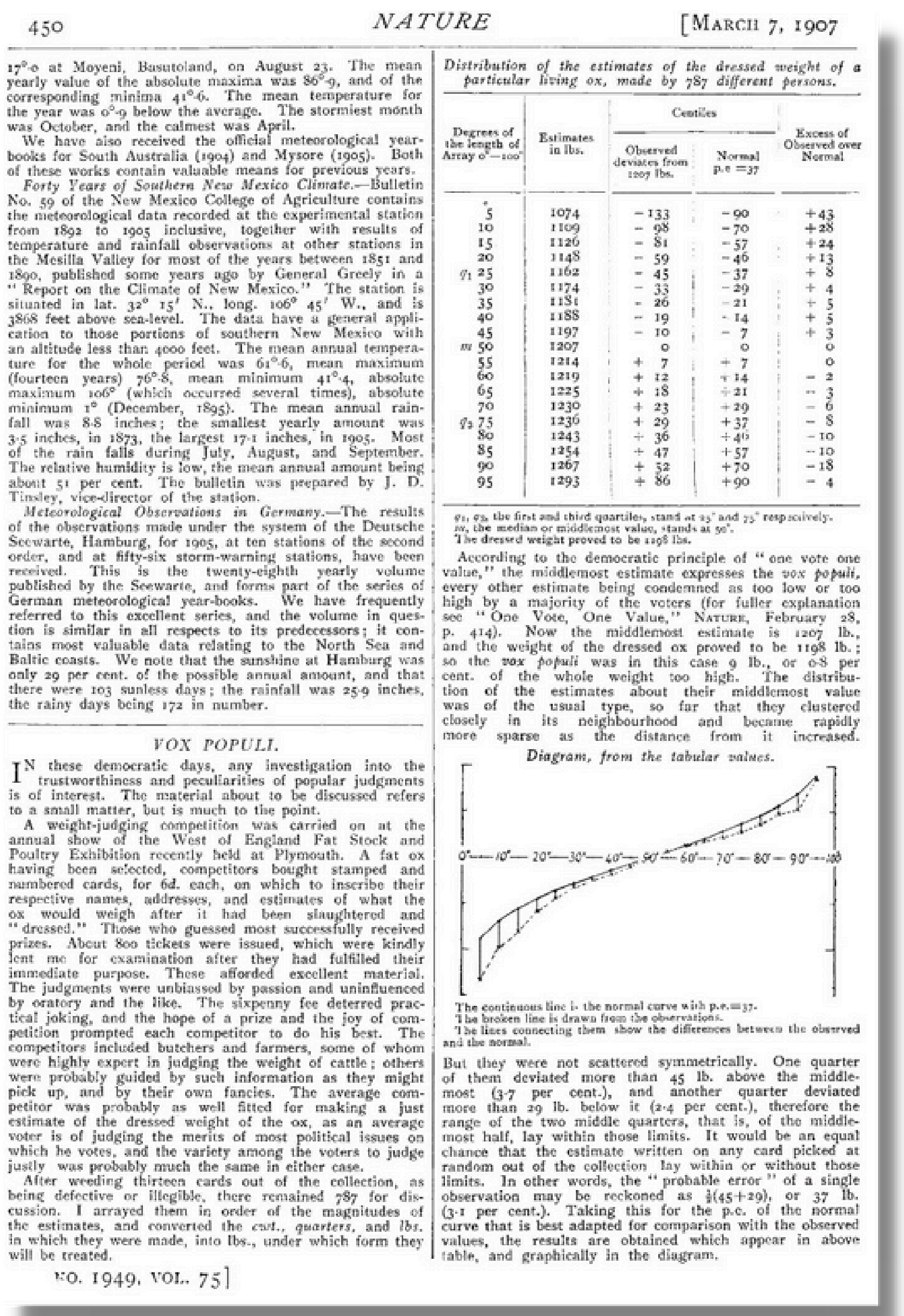

450 *NATURE* [MARCH 7, 1907

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*Distribution of the estimates of the dressed weight of a particular living ox, made by 787 different persons.*

| Degrees of the length of Array 0°—100° | Estimates in lbs. | Centiles: Observed deviates from 1207 lbs. | Centiles: Normal p.e =37 | Excess of Observed over Normal |
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*Diagram, from the tabular values.*

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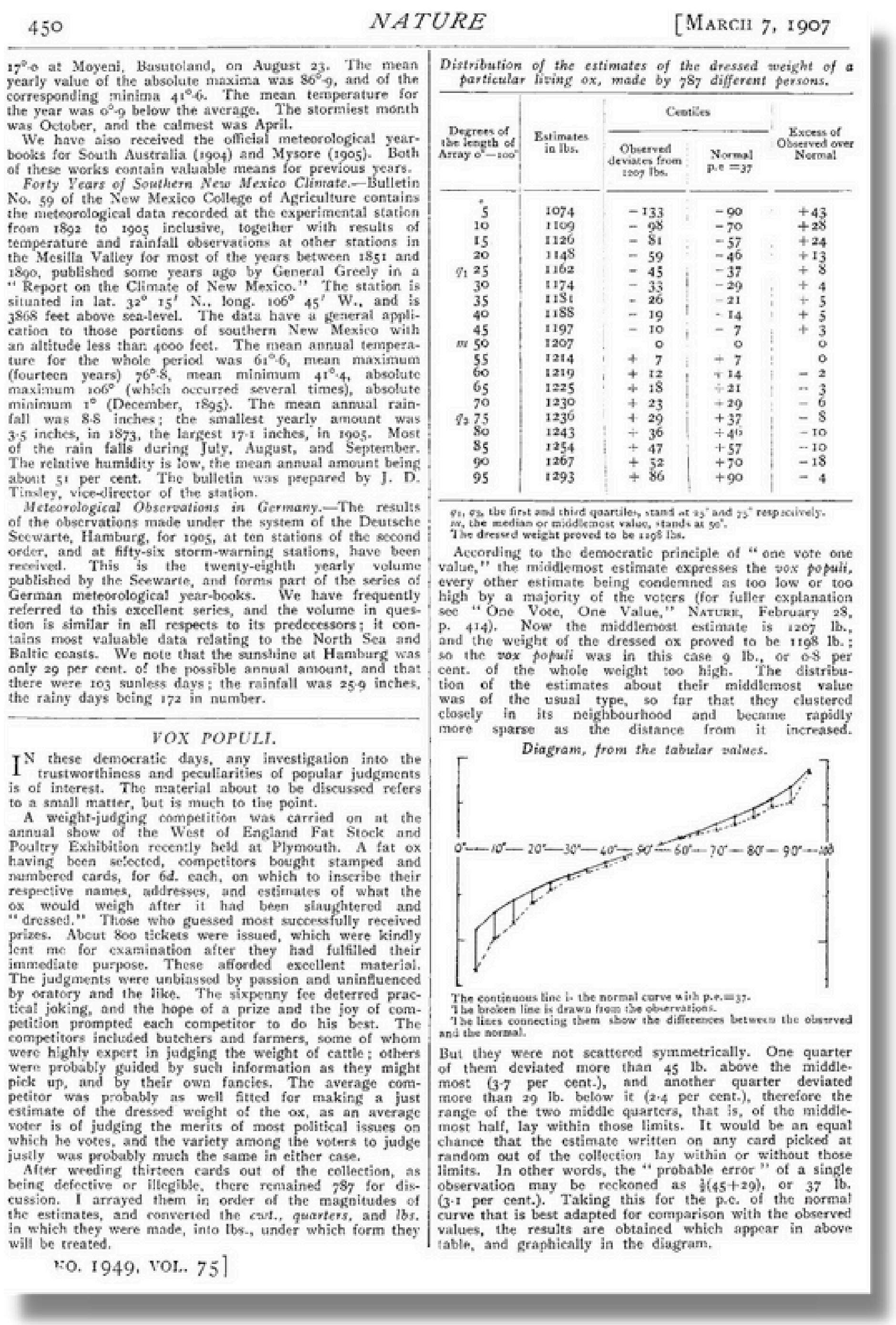



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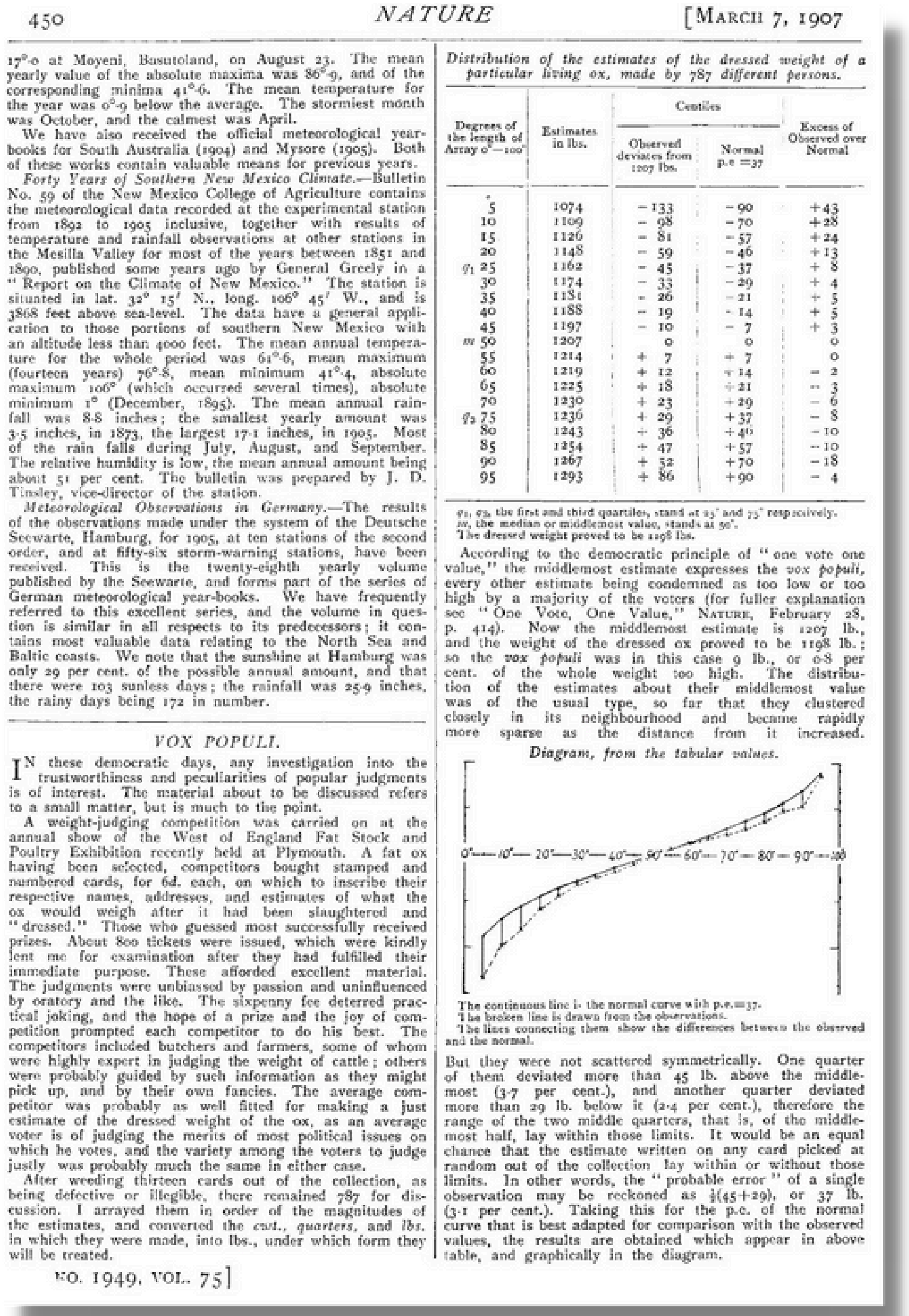

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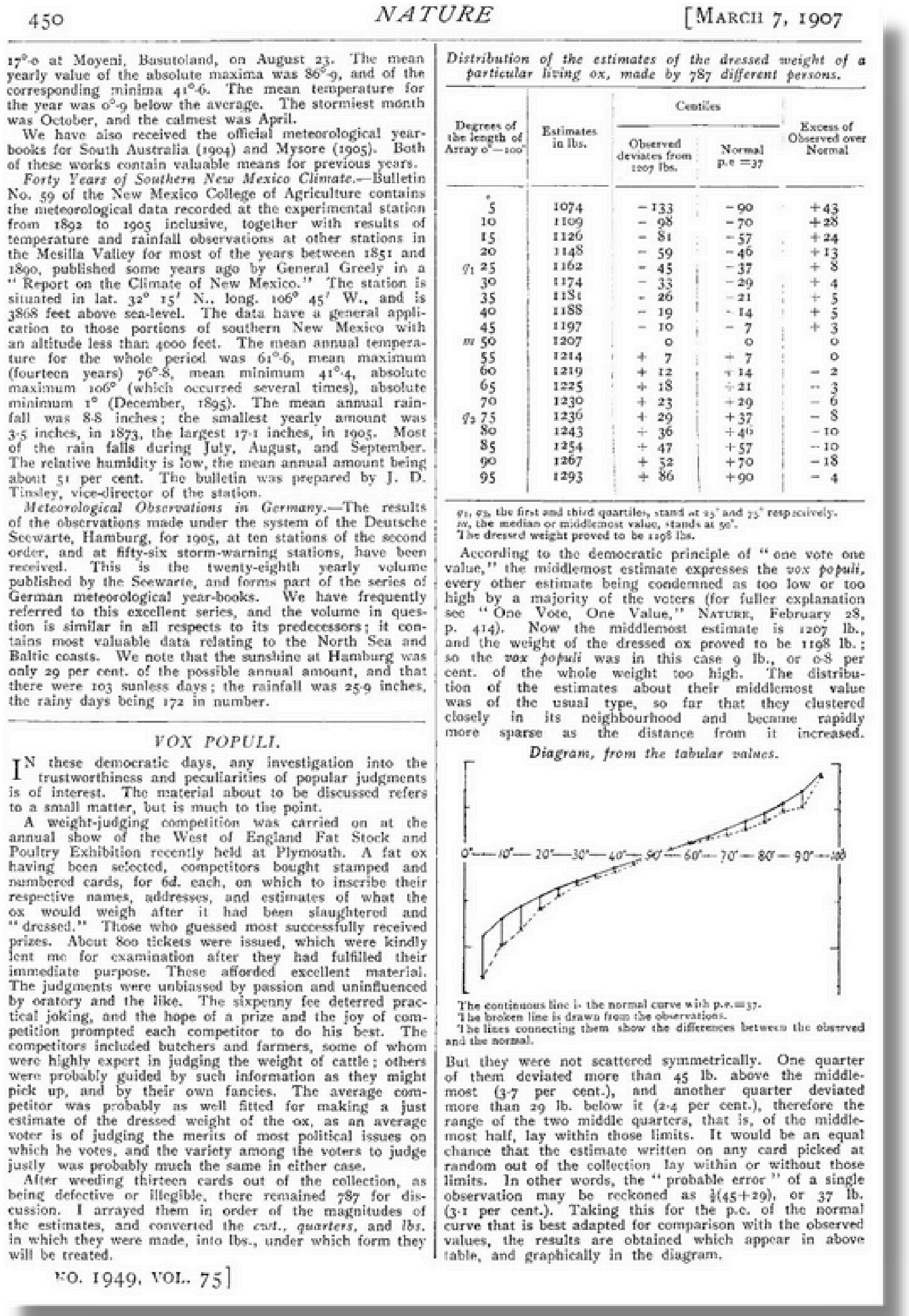


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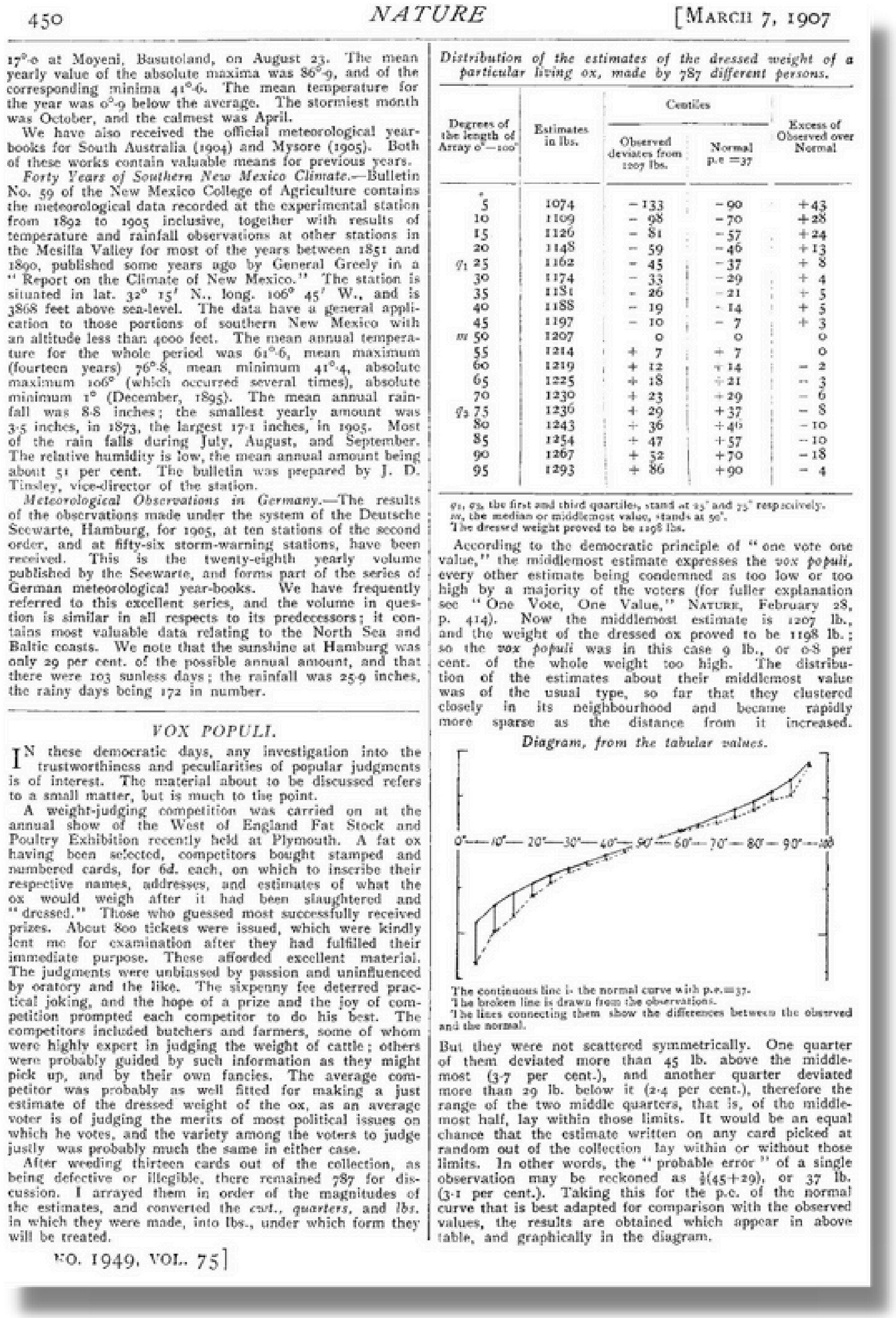

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| 55 | 1214 | + 7 | + 7 | 0 |
| 60 | 1219 | + 12 | +14 | − 2 |
| 65 | 1225 | + 18 | +21 | − 3 |
| 70 | 1230 | + 23 | +29 | − 6 |
| $q_3$ 75 | 1236 | + 29 | +37 | − 8 |
| 80 | 1243 | + 36 | +46 | −10 |
| 85 | 1254 | + 47 | +57 | −10 |
| 90 | 1267 | + 52 | +70 | −18 |
| 95 | 1293 | + 86 | +90 | − 4 |

$q_1$, $q_3$, the first and third quartiles, stand at 25° and 75° respectively.
*m*, the median or middlemost value, stands at 50°.
The dressed weight proved to be 1198 lbs.

According to the democratic principle of "one vote one value," the middlemost estimate expresses the *vox populi*, every other estimate being condemned as too low or too high by a majority of the voters (for fuller explanation see "One Vote, One Value," NATURE, February 28, p. 414). Now the middlemost estimate is 1207 lb., and the weight of the dressed ox proved to be 1198 lb.; so the *vox populi* was in this case 9 lb., or 0·8 per cent. of the whole weight too high. The distribution of the estimates about their middlemost value was of the usual type, so far that they clustered closely in its neighbourhood and became rapidly more sparse as the distance from it increased.

*Diagram, from the tabular values.*



The continuous line is the normal curve with p.e.=37.
The broken line is drawn from the observations.
The lines connecting them show the differences between the observed and the normal.

But they were not scattered symmetrically. One quarter of them deviated more than 45 lb. above the middlemost (3·7 per cent.), and another quarter deviated more than 29 lb. below it (2·4 per cent.), therefore the range of the two middle quarters, that is, of the middlemost half, lay within those limits. It would be an equal chance that the estimate written on any card picked at random out of the collection lay within or without those limits. In other words, the "probable error" of a single observation may be reckoned as ½(45+29), or 37 lb. (3·1 per cent.). Taking this for the p.e. of the normal curve that is best adapted for comparison with the observed values, the results are obtained which appear in above table, and graphically in the diagram.

The mechanism at play here is captured in a simple model.

To get a feeling for it, let’s play a game.

# GUESS-THE-LOGO GAME

Guess the correct version of the logo.

Keep track of your score!

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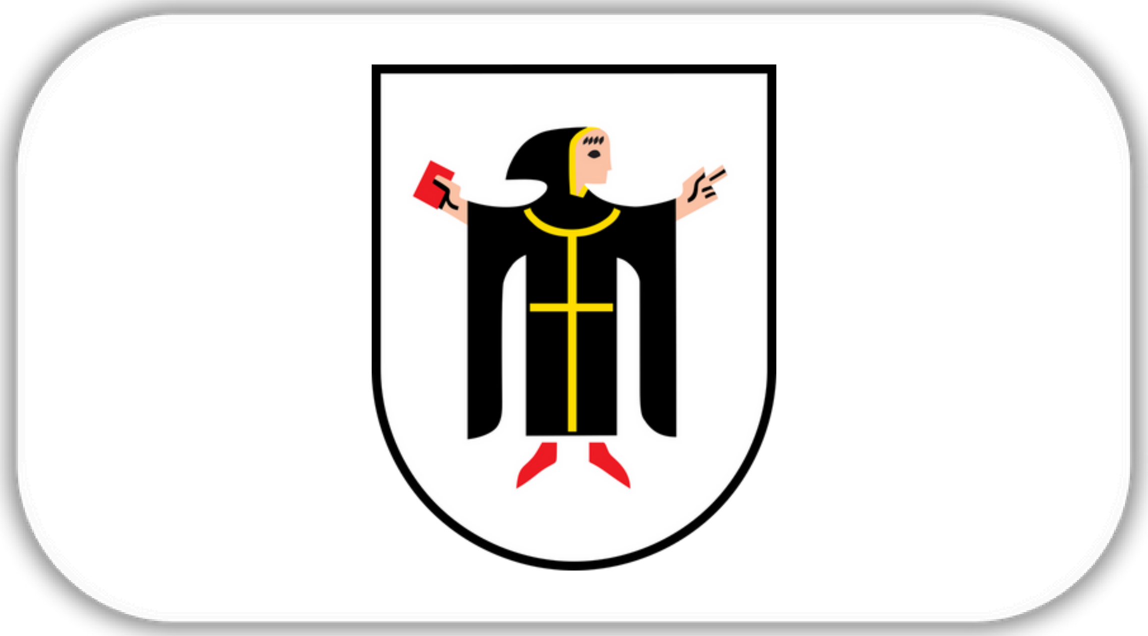

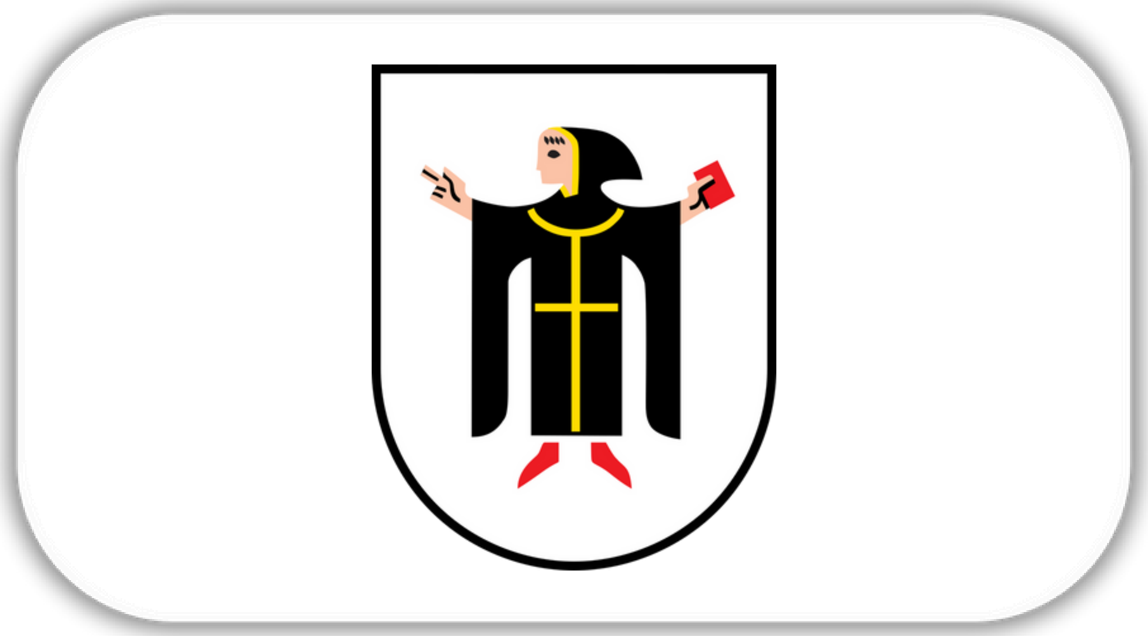

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How did you do?

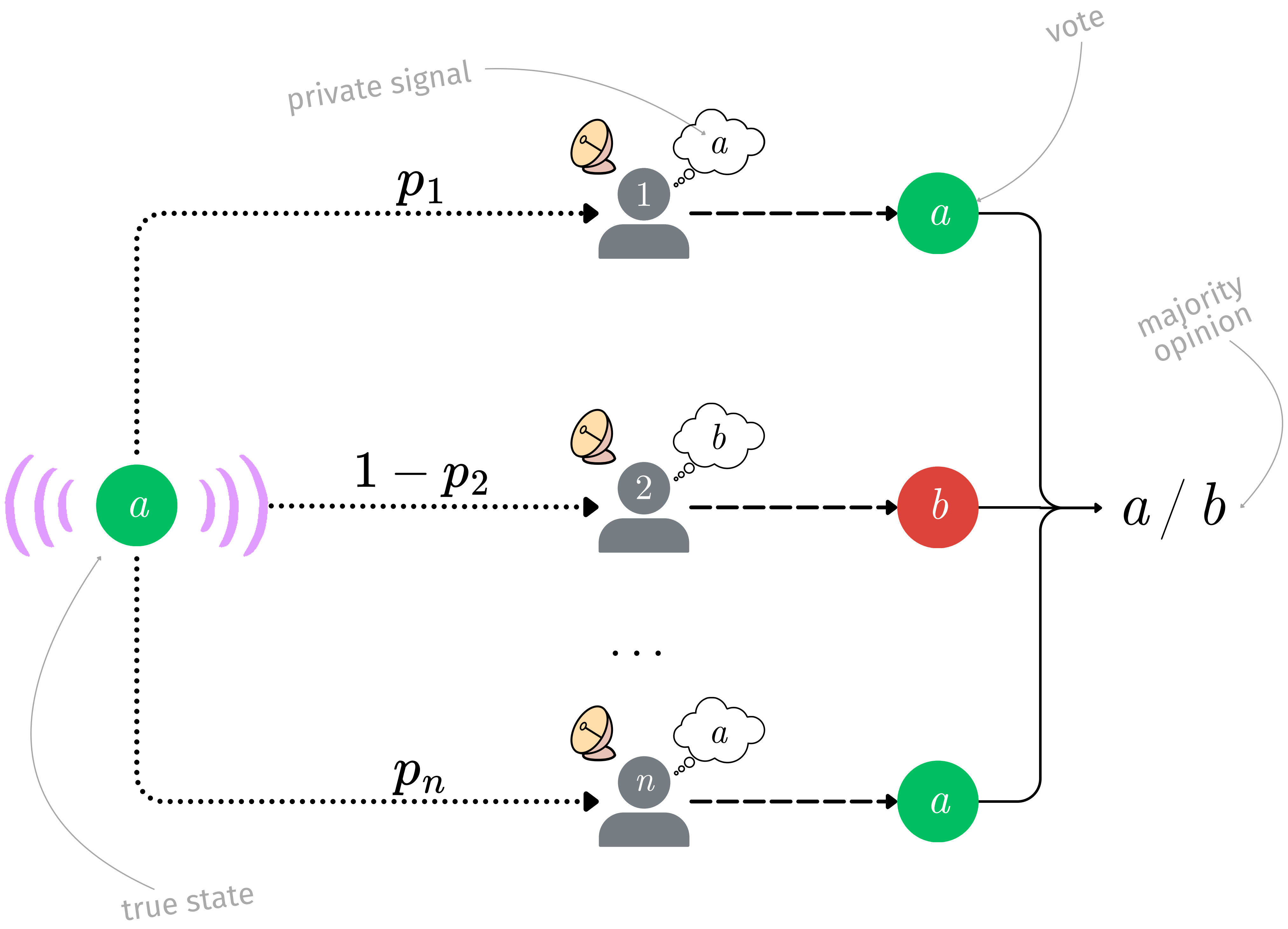
And how did the group do?

Here’s the model we’re working with.

# AGENTS AS NOISY ESTIMATORS OF THE TRUTH

A number of *agents* vote on two *alternatives*, one of which is *correct*.

Each agent has a specific *competence*, i.e., the probability of voting for the correct alternative.



It’s possible that everyone ends up voting for the wrong thing, e.g., if they get the wrong signal.

But how likely is this?…

# NOTATION

we assume $n$ is odd so as not to worry about ties

| | |
|---|---|
| voters | $N = \{1, \ldots, n\}$ |
| alternatives | $A = \{a, b\}$ |
| correct alternative | $a$ |
| voter $i$'s vote | $v_i \in A$ |
| profile of votes | $\boldsymbol{v} = (v_1, \ldots, v_n)$ |
| voter $i$'s competence | $\Pr[v_i = a] = p_i$, with $p_i \in [0, 1]$ |
| majority vote | $F_{maj}(\boldsymbol{v}) = x$, such that $v_i = x$ for a (strict) majority of voters |

$i$'s guess of the right answer

we write profiles as words: ($a$, $a$, $b$, $a$, ...) ⇾ $aaba$...

probability that $i$ gets the right answer

THE MARQUIS DE CONDORCET

I want to make some assumptions.

# ASSUMPTIONS

**(Competence)** Agents are better than random at being correct:

$$p_i > \frac{1}{2}, \text{ for any voter } i \in N.$$

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$$p_i = p_j = p, \text{ for all voters } i, j \in N.$$

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**(Equal Competence)** All agents have the same competence:

$$p_i = p_j = p, \text{ for all voters } i, j \in N.$$

**(Independence)** Voters vote independently of each other:

$$\Pr[v_i = x, v_j = y] = \Pr[v_i = x] \cdot \Pr[v_j = y], \text{ for all voters } i, j \in N.$$

THE MARQUIS DE CONDORCET

I claim that under these conditions, the majority tends to get it right!

We want to understand the probability that the majority opinion is correct:

$$\Pr\left[F_{maj}(v_1, \ldots, v_n) = a\right]$$

Computing the accuracy of the group gets more and more involved as the number of agents grows.

But let's start simple.

# ONE VOTER

The profile is $v = (v_1)$.

The probability of a correct decision is:

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$$\begin{aligned} \Pr\left[F_{maj}(v_1) = a\right] &= \Pr\left[v_1 = a\right] \\ &= p \\ &> 1/2. \end{aligned}$$

by the Equal Competence assumption

by the Competence assumption

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The probability of a correct decision is:

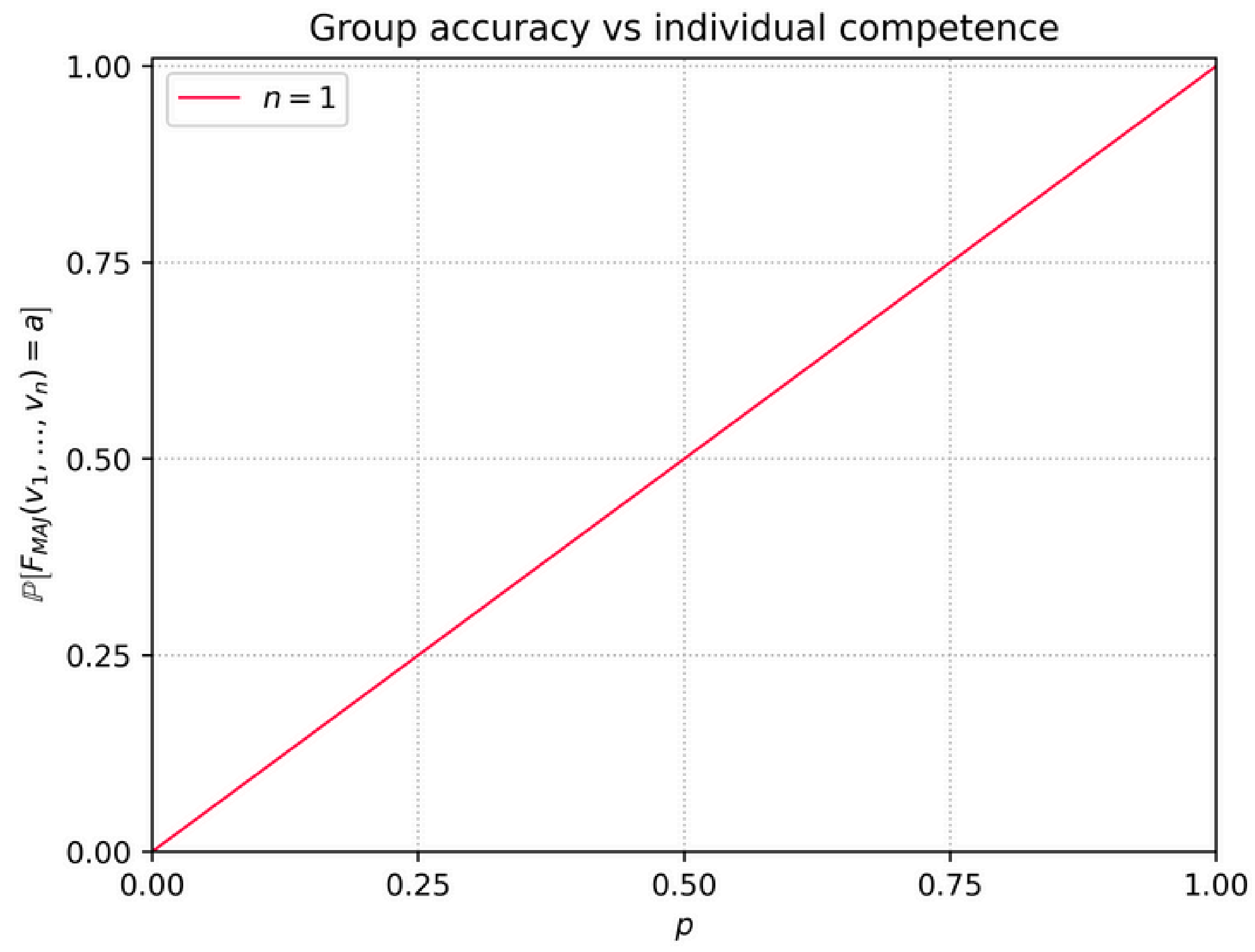
$$\Pr\left[F_{maj}(v_1) = a\right] = \Pr\left[v_1 = a\right]$$
$$= p$$
$$> 1/2.$$

by the Equal Competence assumption

by the Competence assumption

Note that as $p$ grows, so does group accuracy.

in this case, trivially



# TWO VOTERS

The profile is $\boldsymbol{v} = (v_1, v_2)$.

# TWO VOTERS

The profile is $\boldsymbol{v} = (v_1, v_2)$.

Oh wait, we're not looking at this case.

# THREE VOTERS

The profile is $\boldsymbol{v} = (v_1, v_2, v_3)$.

The probability of a correct majority decision is:

$$\Pr\left[F_{maj}(\boldsymbol{v}) = a\right] = \Pr\left[\text{a majority of voters in } \boldsymbol{v} \text{ are correct}\right]$$

# THREE VOTERS

The profile is $\boldsymbol{v} = (v_1, v_2, v_3)$.

The probability of a correct majority decision is:

$$\begin{aligned}\Pr\left[F_{maj}(\boldsymbol{v}) = a\right] &= \Pr\left[\text{a majority of voters in } \boldsymbol{v} \text{ are correct}\right] \\ &= \Pr\left[\boldsymbol{v} \text{ is either } aab, aba, baa, \text{ or } aaa\right] \\ &= \Pr\left[\boldsymbol{v} = aab\right] + \Pr\left[\boldsymbol{v} = aba\right] + \Pr\left[\boldsymbol{v} = baa\right] + \Pr\left[\boldsymbol{v} = aaa\right]\end{aligned}$$

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$$
\begin{aligned}
\Pr\left[F_{maj}(\boldsymbol{v}) = a\right] &= \Pr\left[\text{a majority of voters in } \boldsymbol{v} \text{ are correct}\right] \\
&= \Pr\left[\boldsymbol{v} \text{ is either } aab, aba, baa, \text{ or } aaa\right] \\
&= \Pr\left[\boldsymbol{v} = aab\right] + \Pr\left[\boldsymbol{v} = aba\right] + \Pr\left[\boldsymbol{v} = baa\right] + \Pr\left[\boldsymbol{v} = aaa\right] \\
&= \Pr\left[v_1 = a\right] \cdot \Pr\left[v_2 = a\right] \cdot \Pr\left[v_3 = b\right] + \\
&\quad \Pr\left[v_1 = a\right] \cdot \Pr\left[v_2 = b\right] \cdot \Pr\left[v_3 = a\right] + \\
&\quad \Pr\left[v_1 = b\right] \cdot \Pr\left[v_2 = a\right] \cdot \Pr\left[v_3 = a\right] + \\
&\quad \Pr\left[v_1 = a\right] \cdot \Pr\left[v_2 = a\right] \cdot \Pr\left[v_3 = a\right] \\
&= p \cdot p \cdot (1-p) + p \cdot (1-p) \cdot p + (1-p) \cdot p \cdot p + p \cdot p \cdot p \\
&= 3p^2(1-p) + p^3 \\
&> p.
\end{aligned}
$$

by the Independence assumption

by the Equal Competence assumption

by the Competence assumption

# THREE VOTERS

The profile is $\boldsymbol{v} = (v_1, v_2, v_3)$.

The probability of a correct majority decision is:

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&= \Pr\left[v_1 = a\right] \cdot \Pr\left[v_2 = a\right] \cdot \Pr\left[v_3 = b\right] + \\
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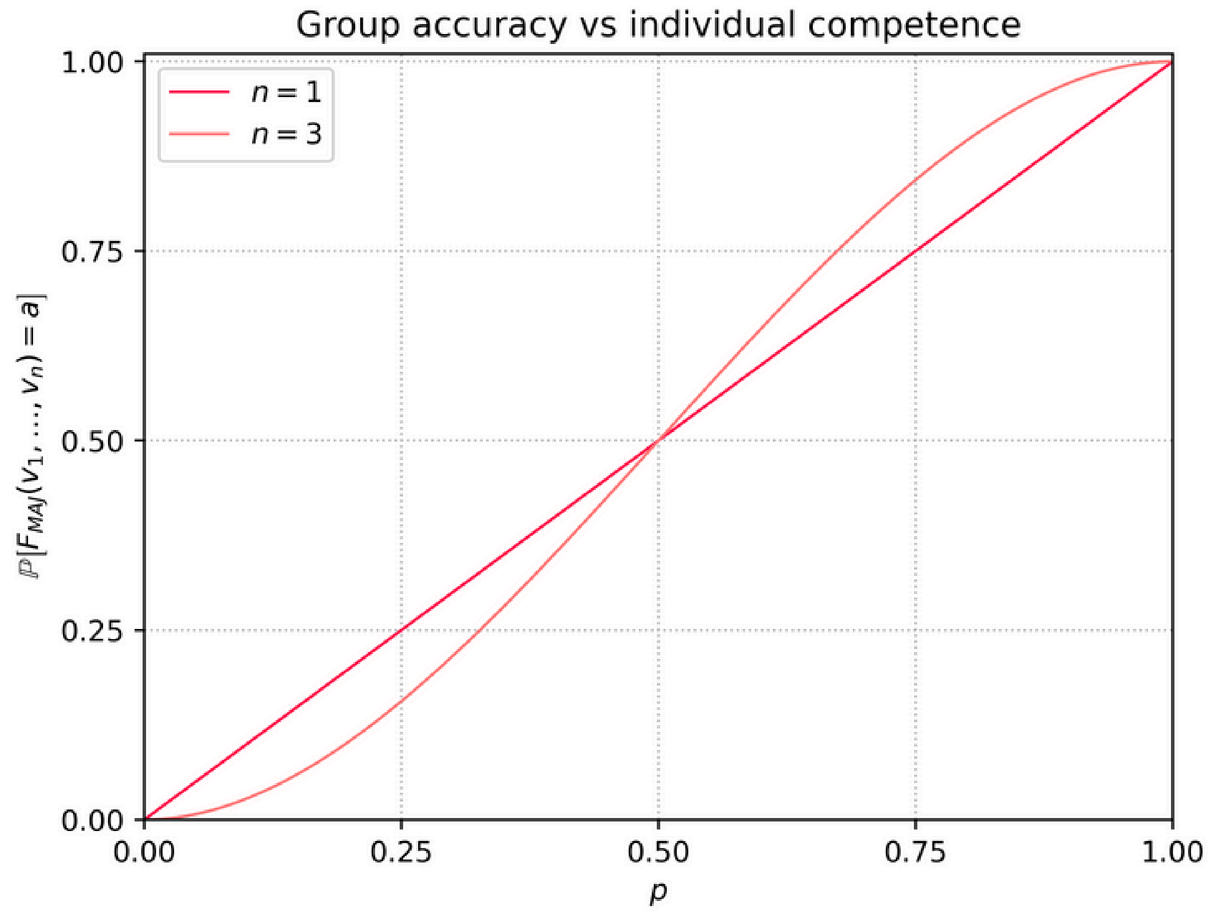
&\quad\; \Pr\left[v_1 = b\right] \cdot \Pr\left[v_2 = a\right] \cdot \Pr\left[v_3 = a\right] + \\
&\quad\; \Pr\left[v_1 = a\right] \cdot \Pr\left[v_2 = a\right] \cdot \Pr\left[v_3 = a\right] \\
&= p \cdot p \cdot (1-p) + p \cdot (1-p) \cdot p + (1-p) \cdot p \cdot p + p \cdot p \cdot p \\
&= 3p^2(1-p) + p^3 \\
&> p.
\end{aligned}$$

by the Independence assumption

by the Equal Competence assumption

by the Competence assumption




Note that as $p$ grows, so does group accuracy.

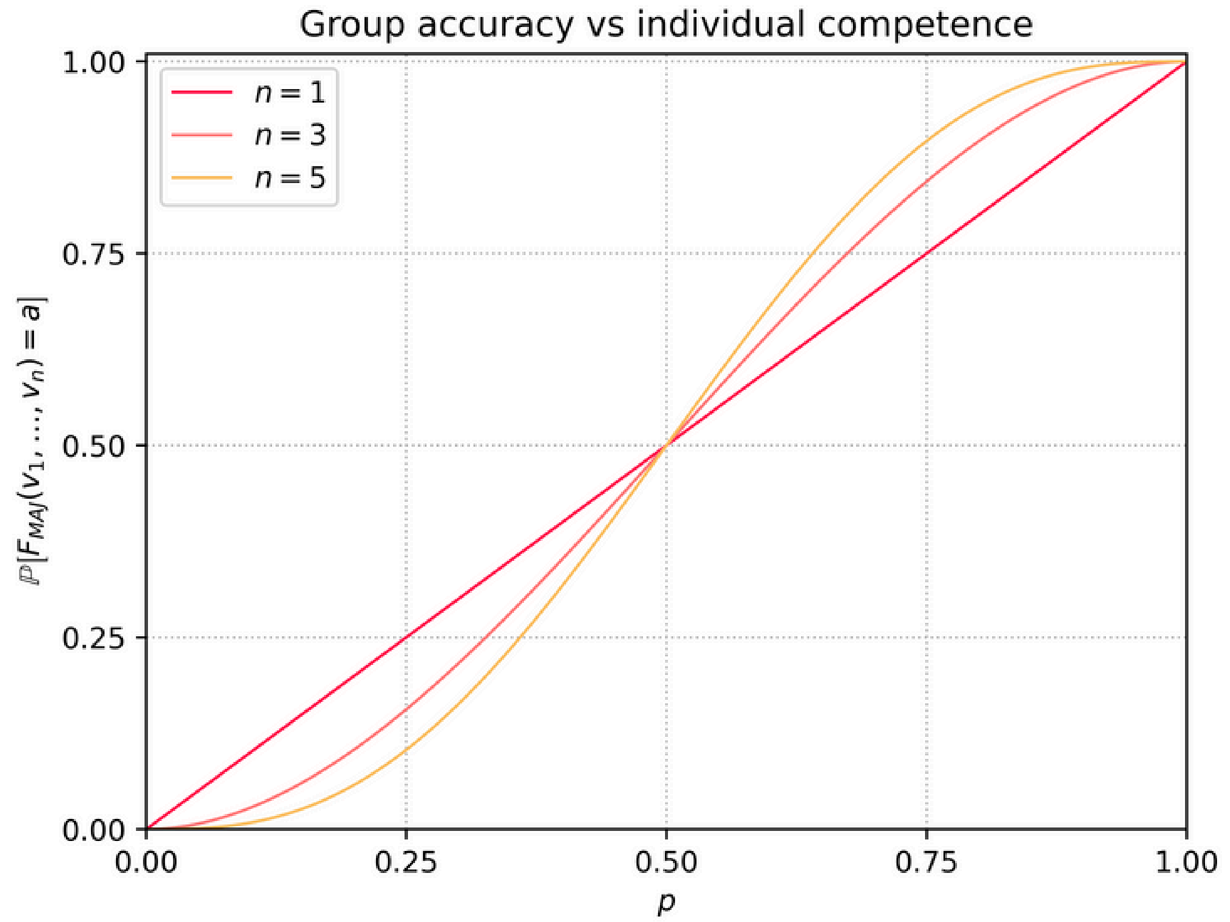
And a group of size 3 is more likely to be correct than a group of size 1.

# FIVE VOTERS

The profile is $\boldsymbol{v} = (v_1, v_2, v_3, v_4, v_5)$.

The probability of a correct majority decision is:

$$
\begin{aligned}
\Pr\left[F_{maj}(\boldsymbol{v}) = a\right] &= \Pr\left[\text{a majority of voters in } \boldsymbol{v} \text{ are correct}\right] \\
&= \Pr\left[\text{either exactly } 3, 4 \text{ or } 5 \text{ voters in } \boldsymbol{v} \text{ are correct}\right] \\
&= \Pr\Big[\boldsymbol{v} \text{ is either } aaabb, aabab, abaab, baaab, aabba, ababa, baaba \\
&\qquad\qquad aaaab, aaaba, aabaa, abaaa, baaaa, \\
&\qquad\qquad \text{or } aaaaa\Big] \\
\ldots \\
&= 10 \cdot p^3(1-p)^2 + 5 \cdot p^4(1-p) + p^5 \\
&= \binom{5}{3} \cdot p^3(1-p)^2 + \binom{5}{4} \cdot p^4(1-p)^1 + \binom{5}{5} \cdot p^5
\end{aligned}
$$




Note, again, that as $p$ grows, so does group accuracy.

And a group of size $5$ is more likely to be correct than a group of size $3$.

# ANY ODD NUMBER OF VOTERS

The profile is $\boldsymbol{v} = (v_1, \ldots, v_n)$.

The probability of a correct majority decision is:

$$\begin{aligned}
\Pr\left[F_{maj}(\boldsymbol{v}) = a\right] &= \Pr\left[\text{a majority of voters in } \boldsymbol{v} \text{ are correct}\right] \\
&= \Pr\left[\text{either exactly } \lfloor n/2 \rfloor + 1, \ldots, n-1, \text{ or } n \text{ voters in } \boldsymbol{v} \text{ are correct}\right] \\
&= \binom{n}{\lfloor n/2 \rfloor + 1} \cdot p^{\lfloor n/2 \rfloor + 1}(1-p)^{n-(\lfloor n/2 \rfloor + 1)} + + \binom{n}{n-1} p^{n-1}(1-p)^1 + \binom{n}{n} p^n \\
&= \sum_{i=\lfloor n/2 \rfloor + 1}^{n} \binom{n}{i} p^i (1-p)^{n-i}.
\end{aligned}$$

THE MARQUIS DE CONDORCET

By the croissants of my ancestors, I claim that groups improve with size!

# THE CONDORCET JURY THEOREM (CJT)

**THEOREM**

For $n$ voters, with $n$ odd, all of whom have accuracy $p > 1/2$ and vote independently of each other, it holds that:

**(1)** The accuracy of the group improves as the size of the group grows:

$$\Pr\left[F_{maj}(v_1, \ldots, v_{n+2}) = a\right] > \Pr\left[F_{maj}(v_1, \ldots, v_n) = a\right].$$

**(2)** The accuracy of the group is at least as good as the accuracy of the inidividual members:

$$\Pr\left[F_{maj}(v_1, \ldots, v_n) = a\right] \geq p.$$

**(3)** As $n$ goes to infinity, the accuracy of the group approachs $1$ asymptotically:

$$\lim_{n \to \infty} \Pr\left[F_{maj}(v_1, \ldots, v_n) = a\right] = 1.$$

# How to prove this?

# PROOF OF CLAIM 1: LARGER GROUPS HAVE BETTER ACCURACY

This is shown by analyzing the expression for the accuracy of a group of *n* voters.

And deriving a recurrence relation for it, in terms of the accuracy of a group of *n* - 2 voters.

$$\sum_{i=\lfloor n/2 \rfloor+1}^{n} \binom{n}{i} p^i (1-p)^{n-i}$$

# PROOF OF CLAIM 2: THE GROUP IS BETTER THAN ITS MEMBERS

This follows from Claim 1, as single voters are just groups of size 1.

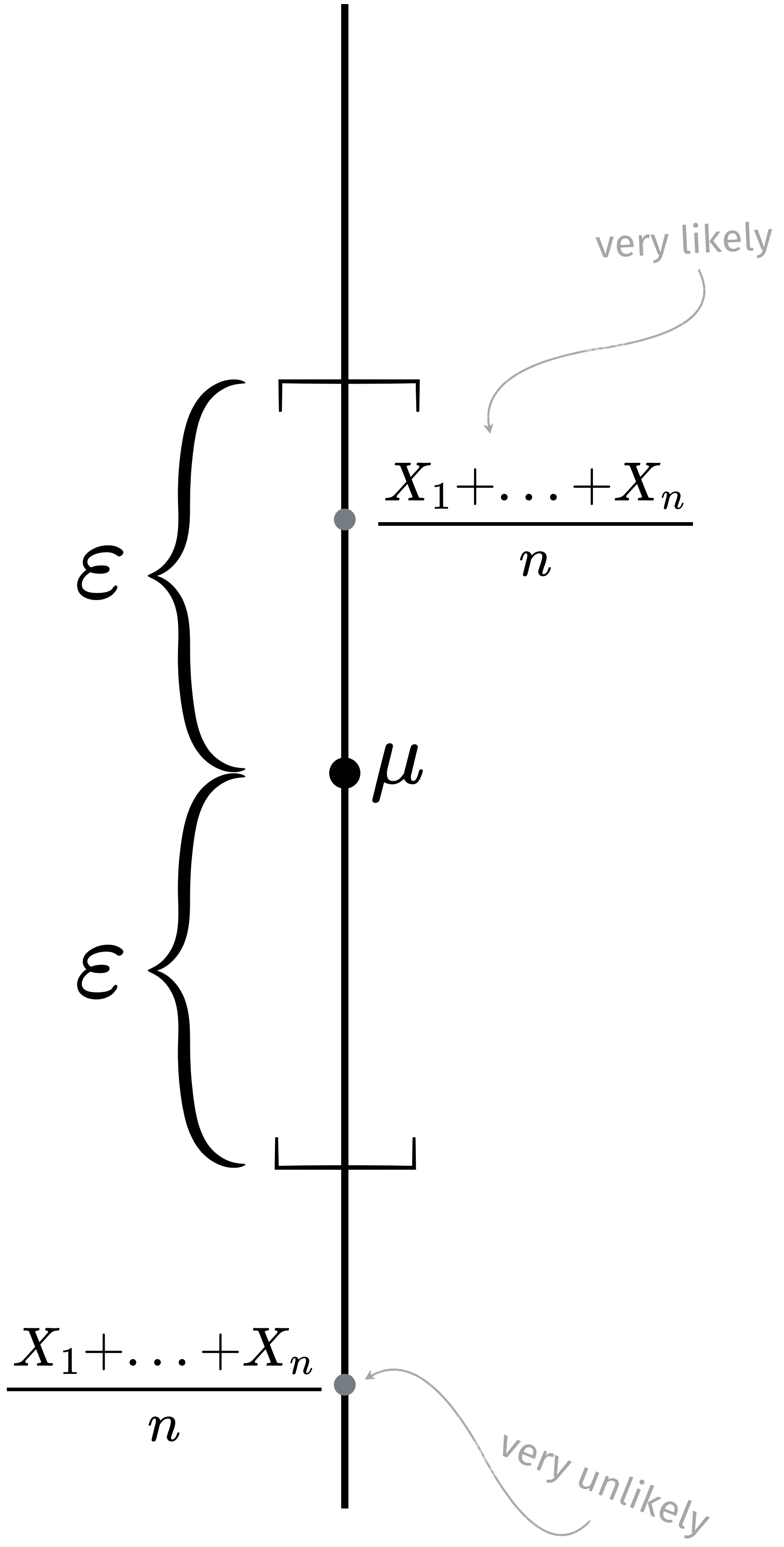
# PROOF OF CLAIM 3: IN THE LIMIT, ACCURACY IS PERFECT

Use the *Law of Large Numbers.*

# THE (WEAK) LAW OF LARGE NUMBERS

**THEOREM**
If $X_1, \ldots, X_n$ are independent and identically distributed (i.i.d.) random variables such that $\mathbb{E}[X_i] = \mu$, then, for any $\varepsilon > 0$, it holds that:

$$\lim_{n \to \infty} \Pr\left[\left|\frac{X_1 + \cdots + X_n}{n} - \mu\right| < \varepsilon\right] = 1.$$



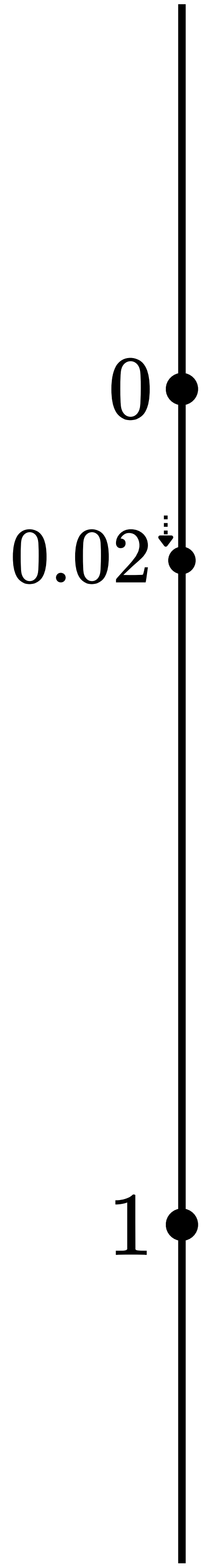
# THE LAW OF LARGE NUMBERS: INTUITION

Consider random variables that take value 1 with probability 0.02, and 0 with probability 0.98:

$$X_i = \begin{cases} 1, & \text{with probability } 0.02, \\ 0, & \text{with probability } 0.98, \end{cases}$$

The expected value of such a variable is:

$$\mathbb{E}[X_i] = 1 \cdot 0.02 + 0 \cdot 0.98 = 0.02.$$



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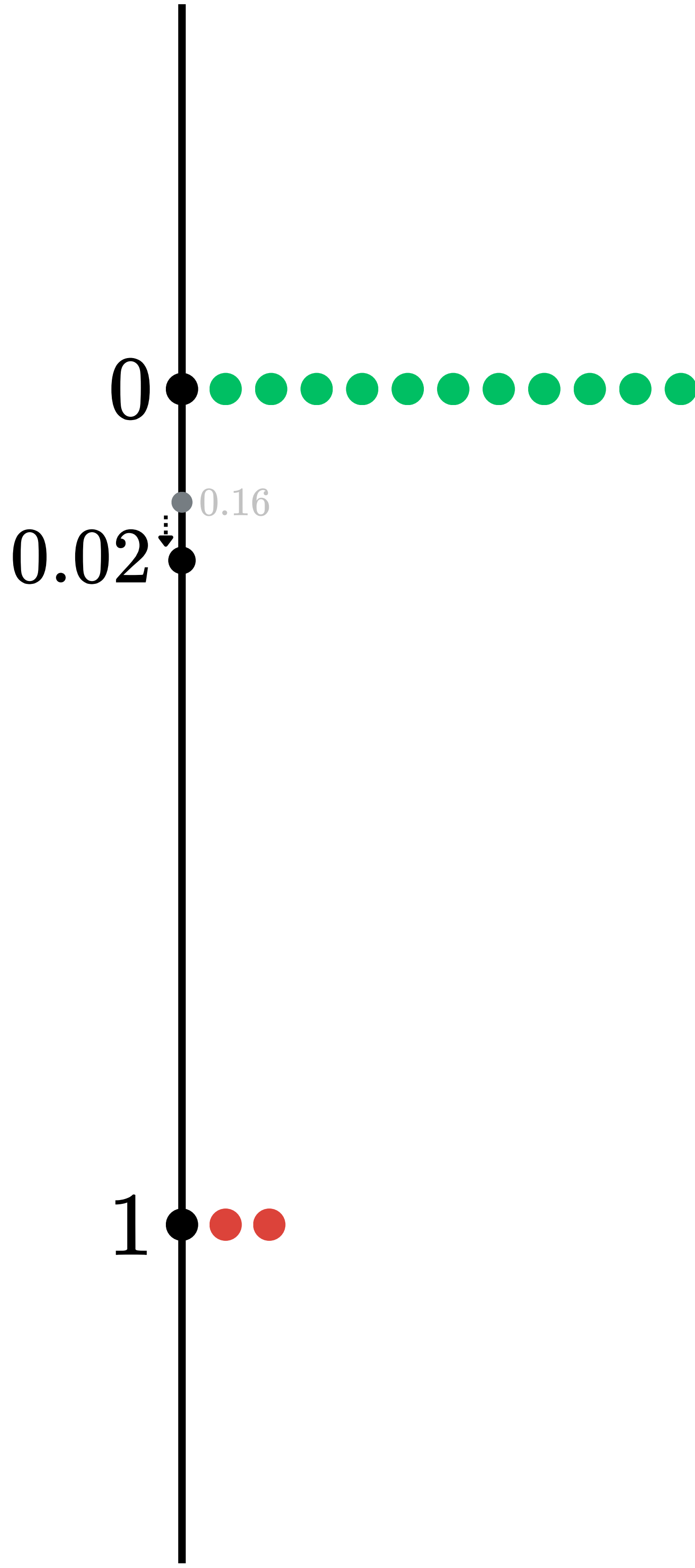
The expected value of such a variable is:

$$\mathbb{E}[X_i] = 1 \cdot 0.02 + 0 \cdot 0.98 = 0.02.$$

If we sample a large number of such variables (e.g., a million), we'd expect about 2% of them to take value 1.

Or the average to be very close to 0.02.



In our setting, random variables keep track of whether agents are correct or not.

# PROOF OF CLAIM 3: IN THE LIMIT, ACCURACY IS PERFECT

Random variables keep track of whether agents are correct:

$$X_i = \begin{cases} 1, & \text{if voter } i \text{ is correct, i.e., if } v_i = a, \\ 0, & \text{otherwise.} \end{cases}$$

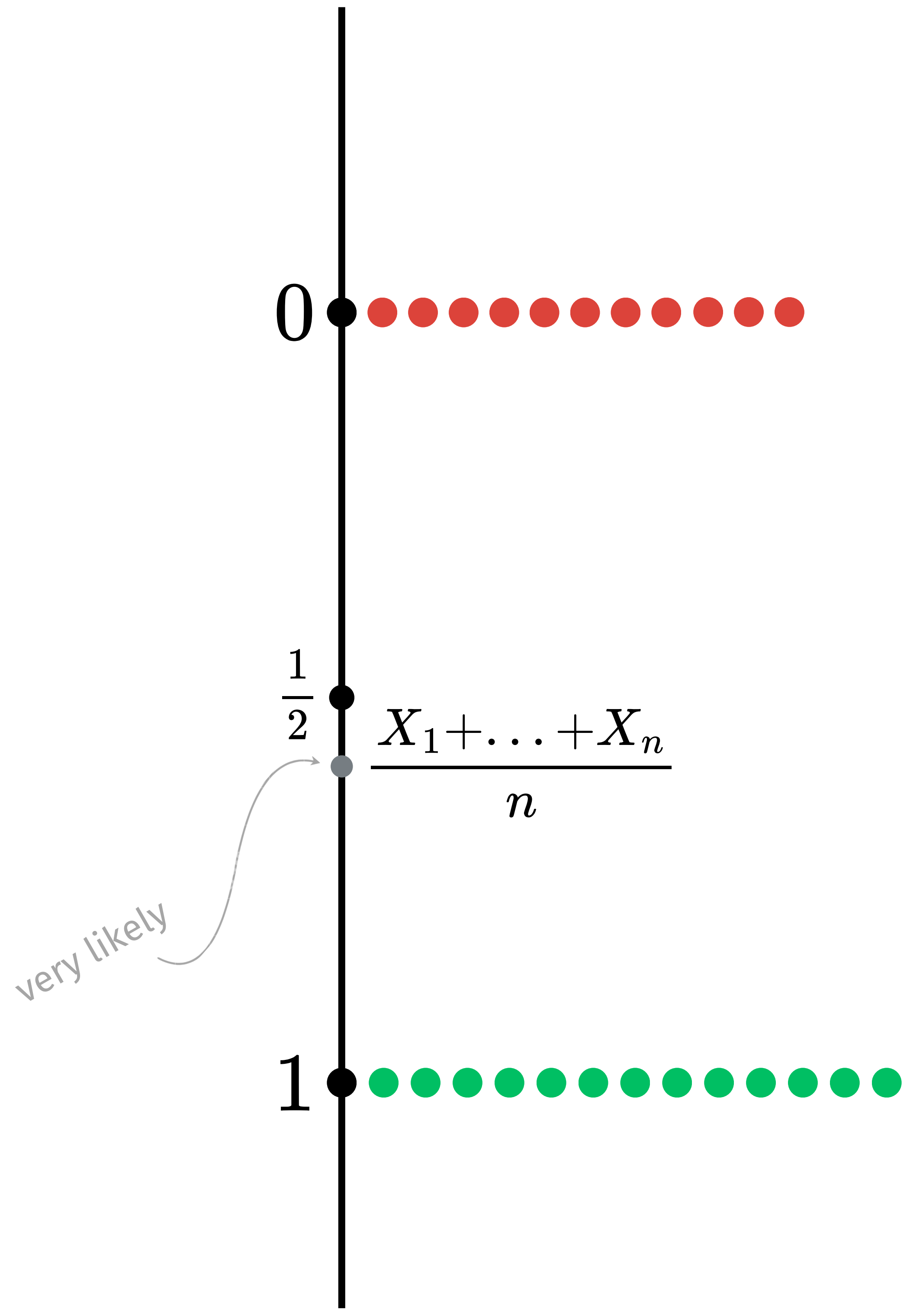
We want to show that, in the long run, more than half of these take value 1:

$$X_1 + \ldots + X_n > \frac{n}{2}$$

Or, equivalently:

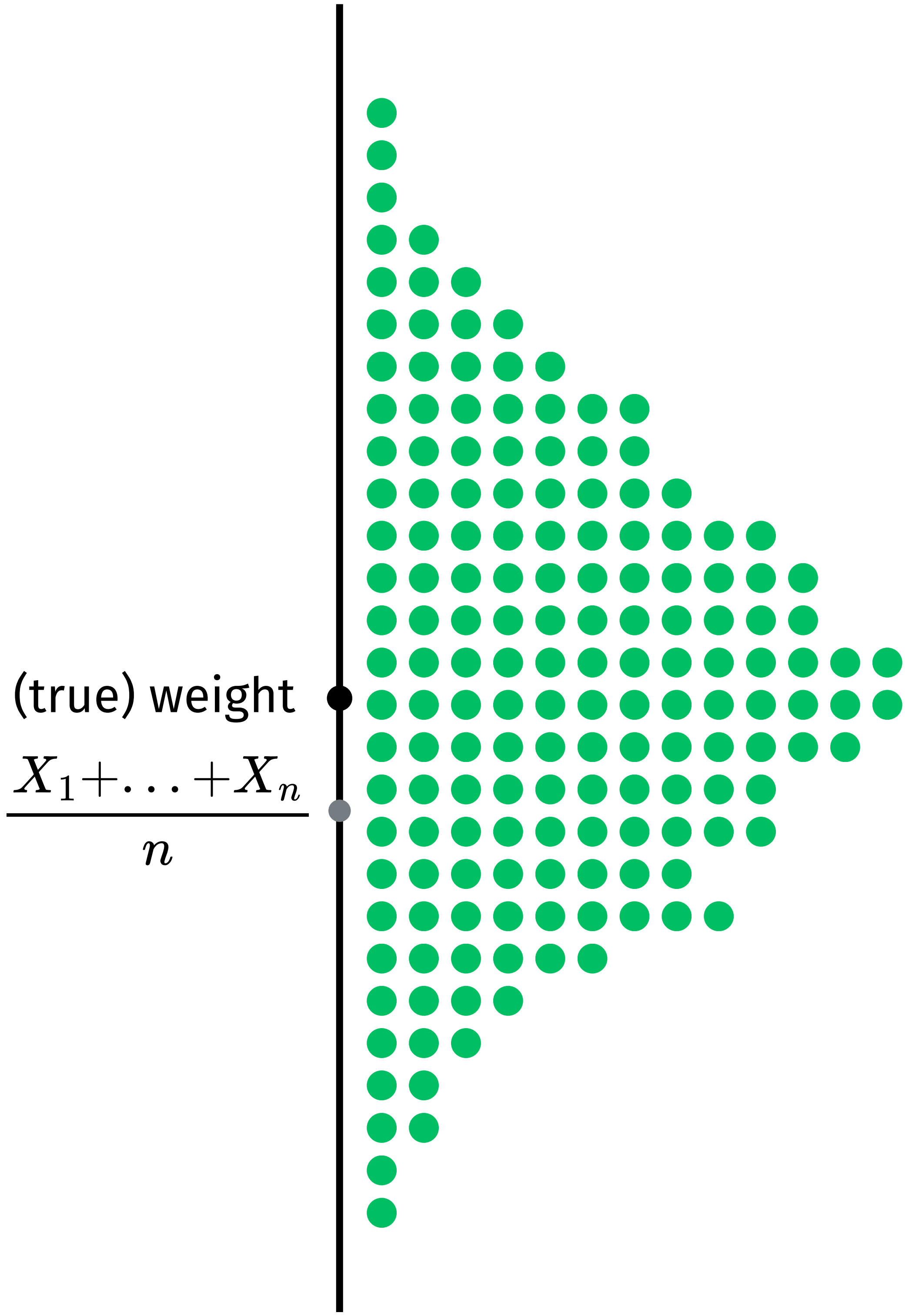
$$\frac{X_1 + \ldots + X_n}{n} > \frac{1}{2}$$

The bias in favor of the truth ensures that, on average, and in the long run, more people vote for it than not.



FRANCIS GALTON

This probably also explains what happened at the country fair!



Let's sum up.

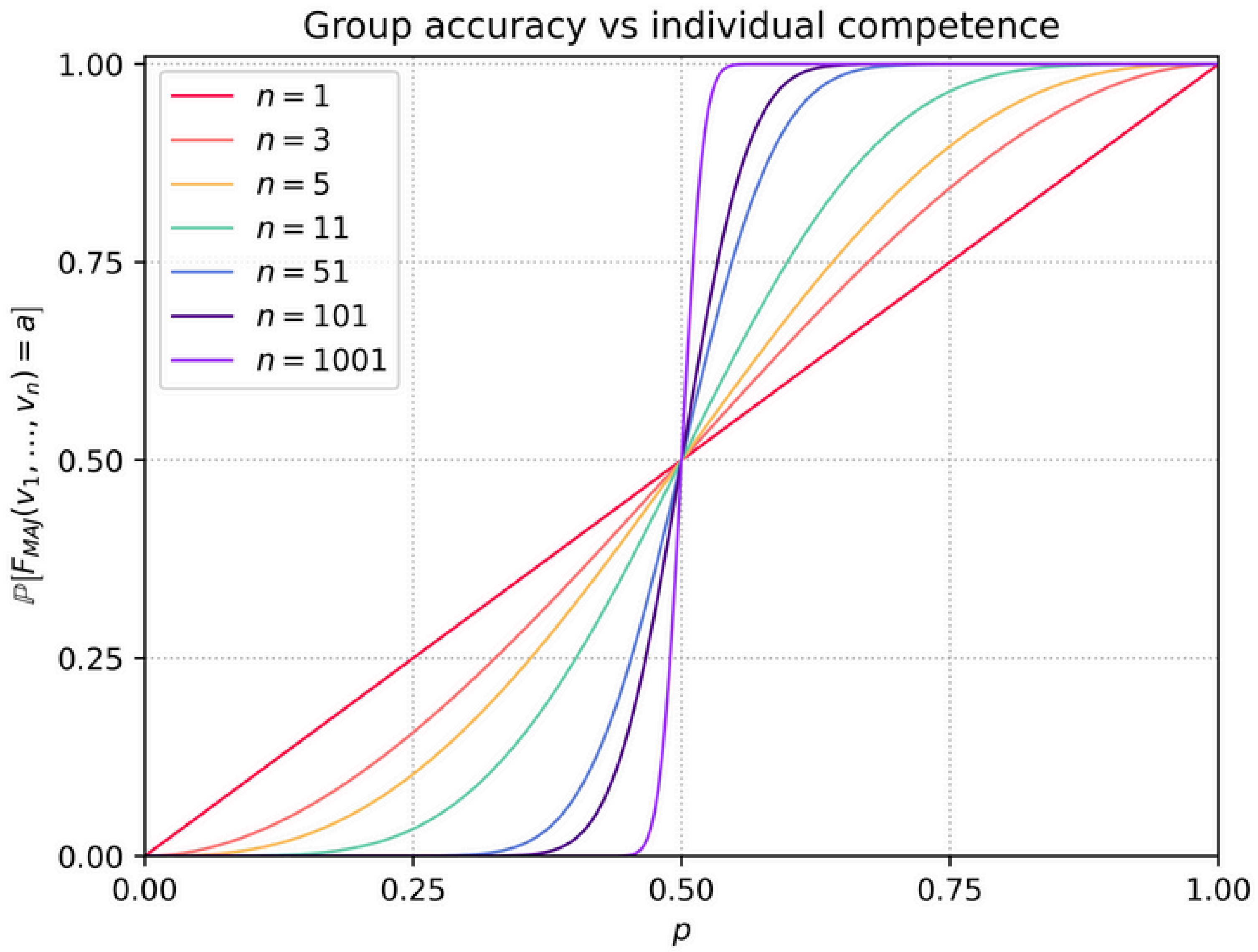
THE MARQUIS DE CONDORCET

Groups are better than their members.

The larger the group, the better.

In the limit, performance is perfect.

As long as people are better than random, and vote independently!



But people are not generally independent…

Can social structure interfere with group accuracy?

ODORIC OF PORDENONE

*In a province of the Grand Can there grow gourds, which, when they are ripe, open, and within them is found a little beast like unto a young lamb...*

Odoric of Pordenone [*trans.* Sir Henry Yule] (2002). *The Travels of Friar Odoric*. W.B. Eerdmans Publishing Company.

AD 1330

SIR JOHN MANDEVILLE

*In Tartary groweth a manner of fruit, as though it were gourds. And when they be ripe, men cut them a-two, and men find within a little beast, in flesh, in bone, and blood, as though it were a little lamb without wool. And men eat both the fruit and the beast. And that is a great marvel.*

*Of that fruit I have eaten... and found it wondirfulle.*

Mandeville, J. (1900). *The Travels of Sir John Mandeville. The Cotton Manuscript in modern spelling.* Macmillan and Co. Limited.

AD 1357 - 1371

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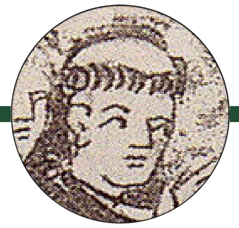

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AD 1357 - 1371

AD 1515 - 1553

BARON SIGISMUND VON HERBERSTEIN

*[...] a certain seed like that of a melon, but rather rounder and longer, from which, when it was set in the earth, grew a plant resembling a lamb, and attaining to a height of about two and a half feet...*

Sigmund Freiherr von Herberstein (1851). *Notes Upon Russia: Being a Translation of the Earliest Account of that Country, Entitled Rerum Moscoviticarum Commentarii.* Hakluyt Society.

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AD 1515 - 1553

## CLAUDE DURET

Duret, C. (1605). *Histoire Admirable des Plantes.*

AD 1605

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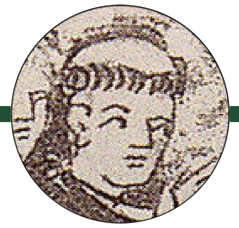
*In a province of the Grand Can there grow gourds, which, when they are ripe, open, and within them is found a little beast like unto a young lamb...*

Odoric of Pordenone [trans. Sir Henry Yule] (2002). *The Travels of Friar Odoric.* W.B. Eerdmans Publishing Company.



AD 1330

SIR JOHN MANDEVILLE

*In Tartary groweth a manner of fruit, as though it were gourds. And when they be ripe, men cut them a-two, and men find within a little beast, in flesh, in bone, and blood, as though it were a little lamb without wool. And men eat both the fruit and the beast. And that is a great marvel.*

*Of that fruit I have eaten... and found it wondirfulle.*

Mandeville, J. (1900). *The Travels of Sir John Mandeville. The Cotton Manuscript in modern spelling.* Macmillan and Co. Limited.

AD 1357 - 1371

AD 1515 - 1553

BARON SIGISMUND VON HERBERSTEIN

*[...] a certain seed like that of a melon, but rather rounder and longer, from which, when it was set in the earth, grew a plant resembling a lamb, and attaining to a height of about two and a half feet...*

Sigmund Freiherr von Herberstein (1851). *Notes Upon Russia: Being a Translation of the Earliest Account of that Country, Entitled Rerum Moscoviticarum Commentarii.* Hakluyt Society.

CLAUDE DURET

Duret, C. (1605). *Histoire Admirable des Plantes.*

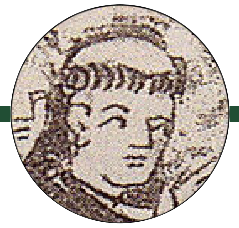
AD 1605

ATHANASIUS KIRCHER

*[...] we assert that it is a plant. Though its form be that of a quadruped, and the juice beneath its woolly covering be blood which flows if an incision be made in its flesh, these things will not move us. It will be found to be a plant.*

Kircher, A. (1641). *Magnes; sive de arte magneticâ opus tripartitum.*

AD 1641



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Kircher, A. (1641). *Magnes; sive de arte magneticâ opus tripartitum.*

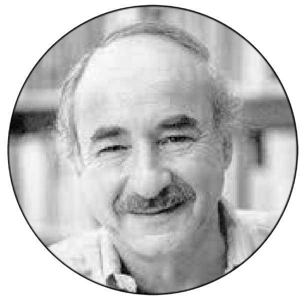
AD 1683

ENGELBERT KAEMPFER

*I have searched ad risum et nauseam for this zoophyte feeding on grass, but have found nothing.*

Kaempfer, E. (1712). *Amœnitatum Exoticarum politico-physico-medicarum fascicul.*

Let's model this.



MORRIS DEGROOT

Agents are represented by nodes in a social network, and update their opinions depending on the opinions of their peers.

# THE DEGROOT MODEL

| | |
|---|---|
| agents | $1, 2, \ldots, n$ |
| time | $t \in \{0, 1, 2, \ldots\}$ |
| belief of agent $i$ at $t$ | number between $0$ and $1$ |
| social network | directed graph with agents as vertices, and who-pays-attention-to-who as edges |
| agent $i$'s neighborhood | agents that $i$ pays attention to |
| weight on edge from $i$ to $j$ | number that indicates how much weight $i$ places on $j$'s opinion; we assume $i$ distributes a total weight of $1$ across $i$'s neighborhood |
| update rule | at time $t+1$ every agent updates their belief to a weighted average over the beliefs of neighbors |

## Agents

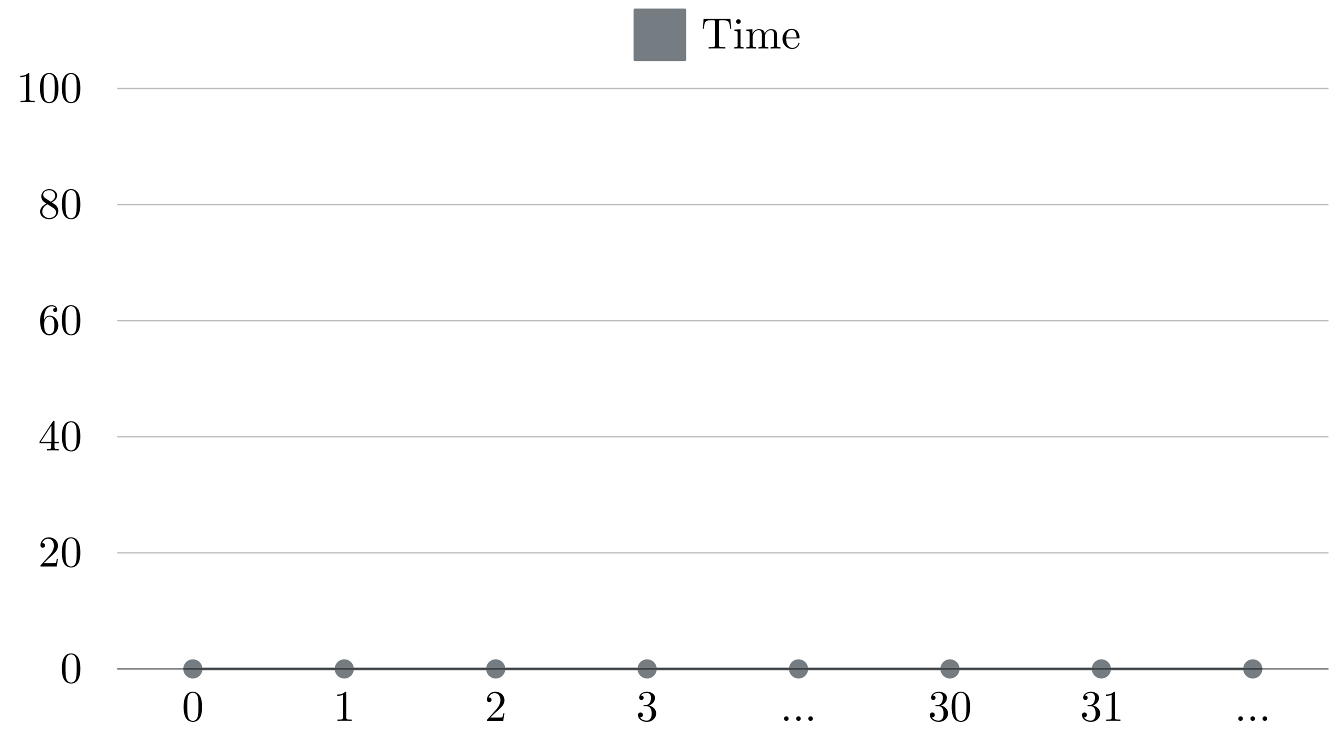
1, 2, 3

**Agents**

1, 2, 3

**Time**

$t = 0, 1, 2, \ldots$

## Agents

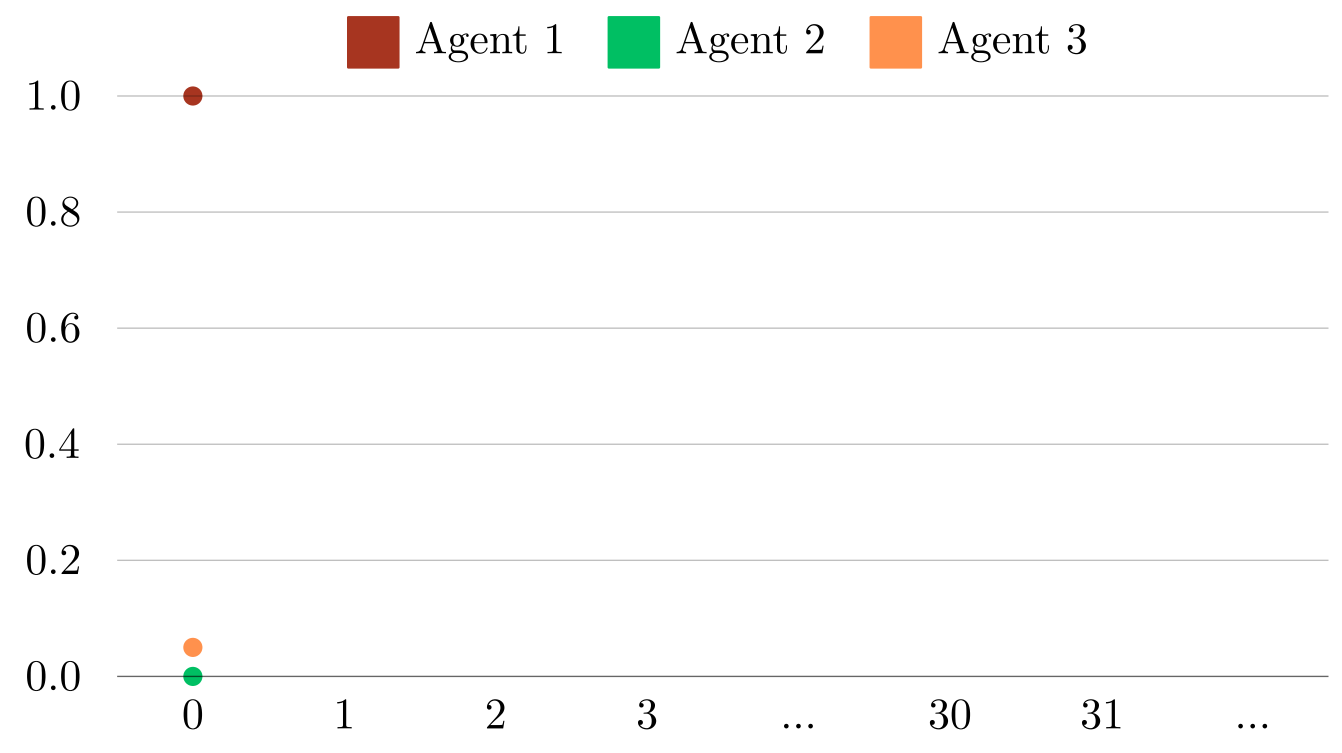
1, 2, 3

## Time

$t = 0, 1, 2, \ldots$

## Beliefs

Agent 1 starts out at 1, agents 2 and 3 start out at 0.

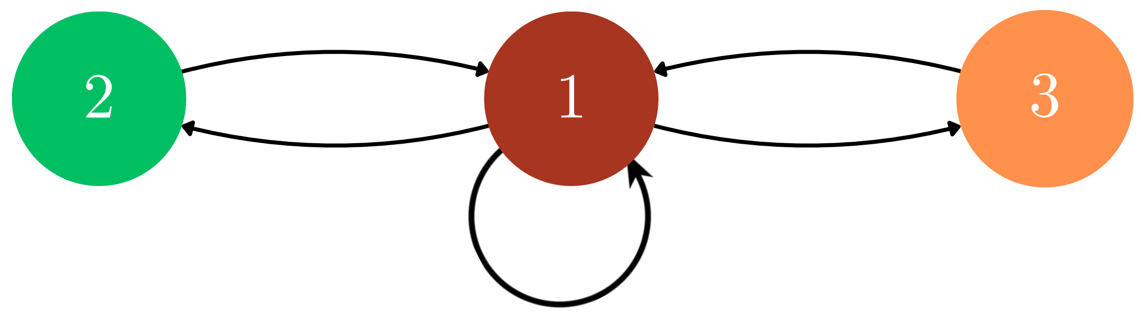
## Agents
$1, 2, 3$

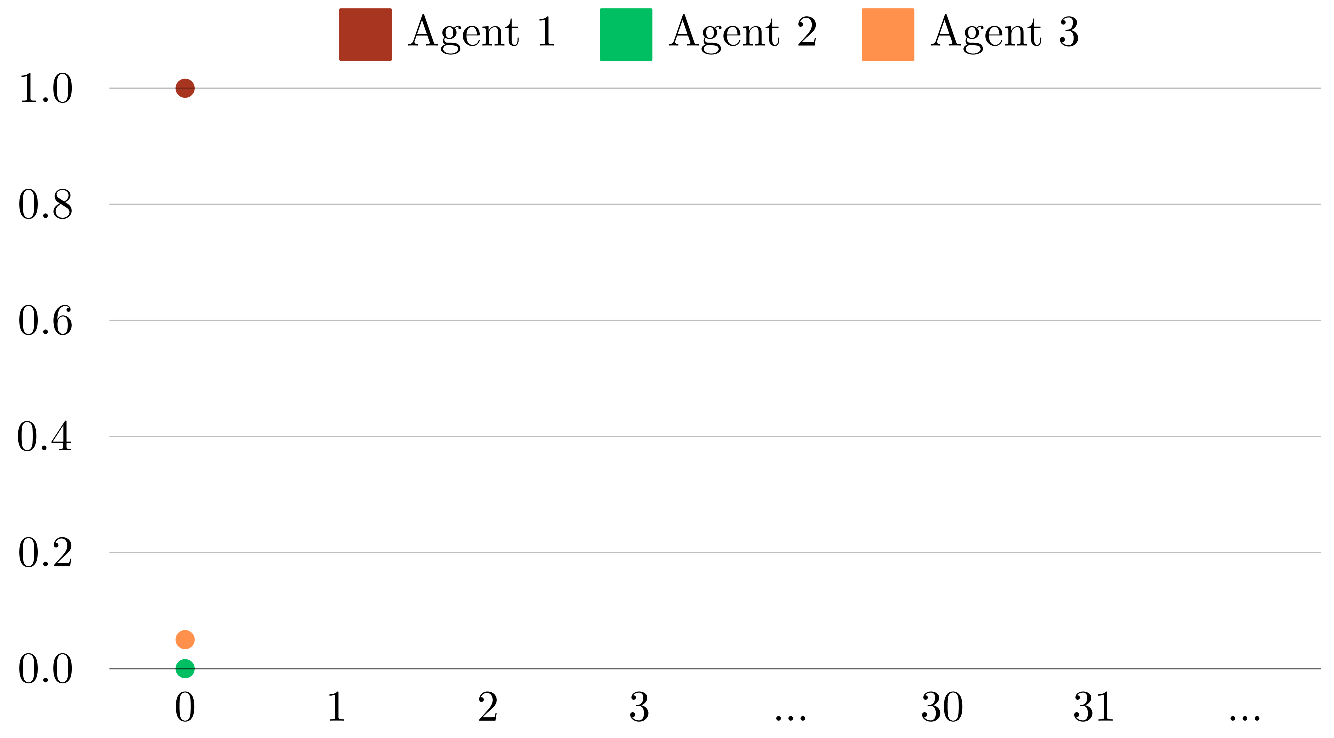
## Time
$t = 0, 1, 2, \ldots$

## Beliefs
Agent 1 starts out at 1, agents 2 and 3 start out at 0.

## Social network
Agent 1 pays attention to everyone, 2 and 3 pay attention only to 1.

## Neighborhoods
Agent 1's neighborhood is {1, 2, 3}, etc.



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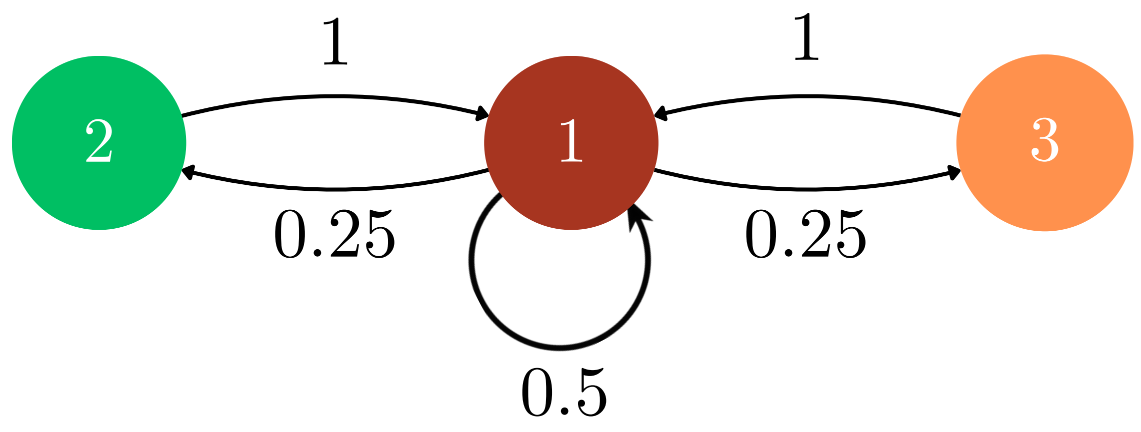
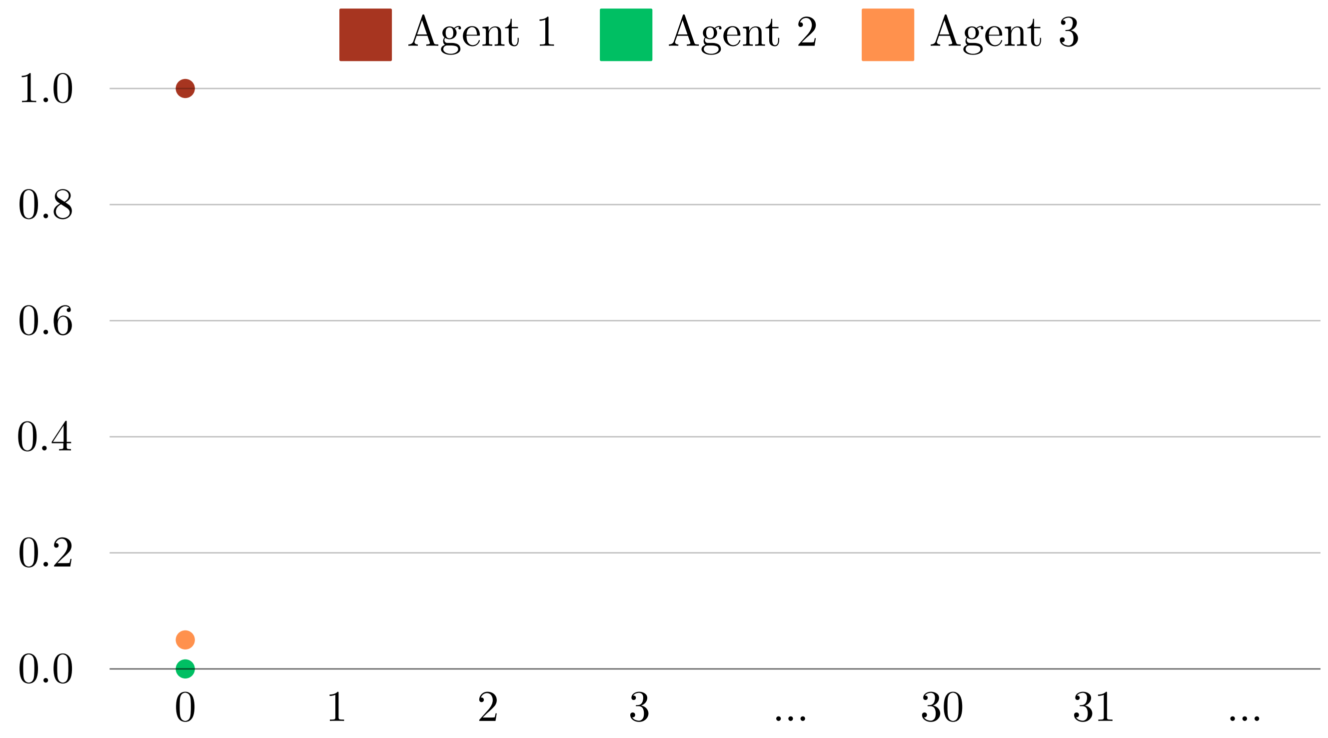
## Weights
Agent 1 puts 0.5 weight on themselves and 0.25 on 2 and 3 each, and so on.



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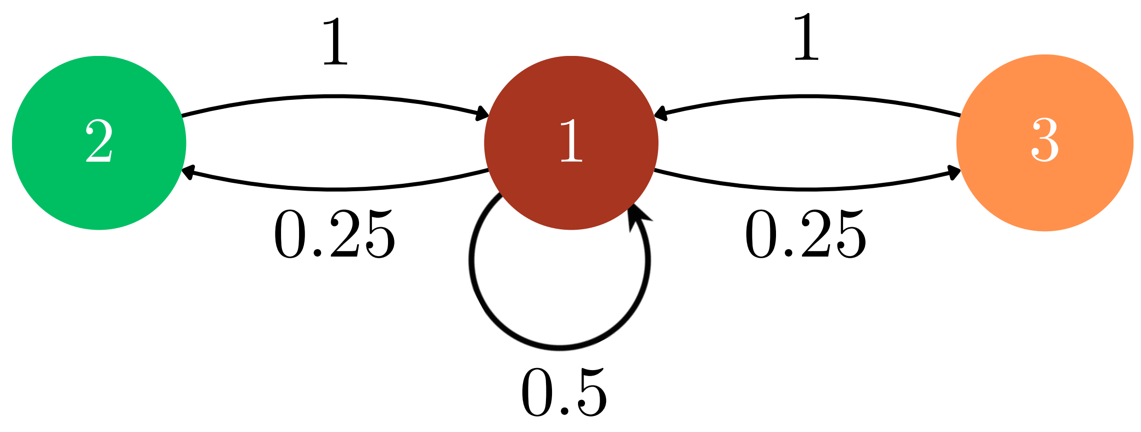
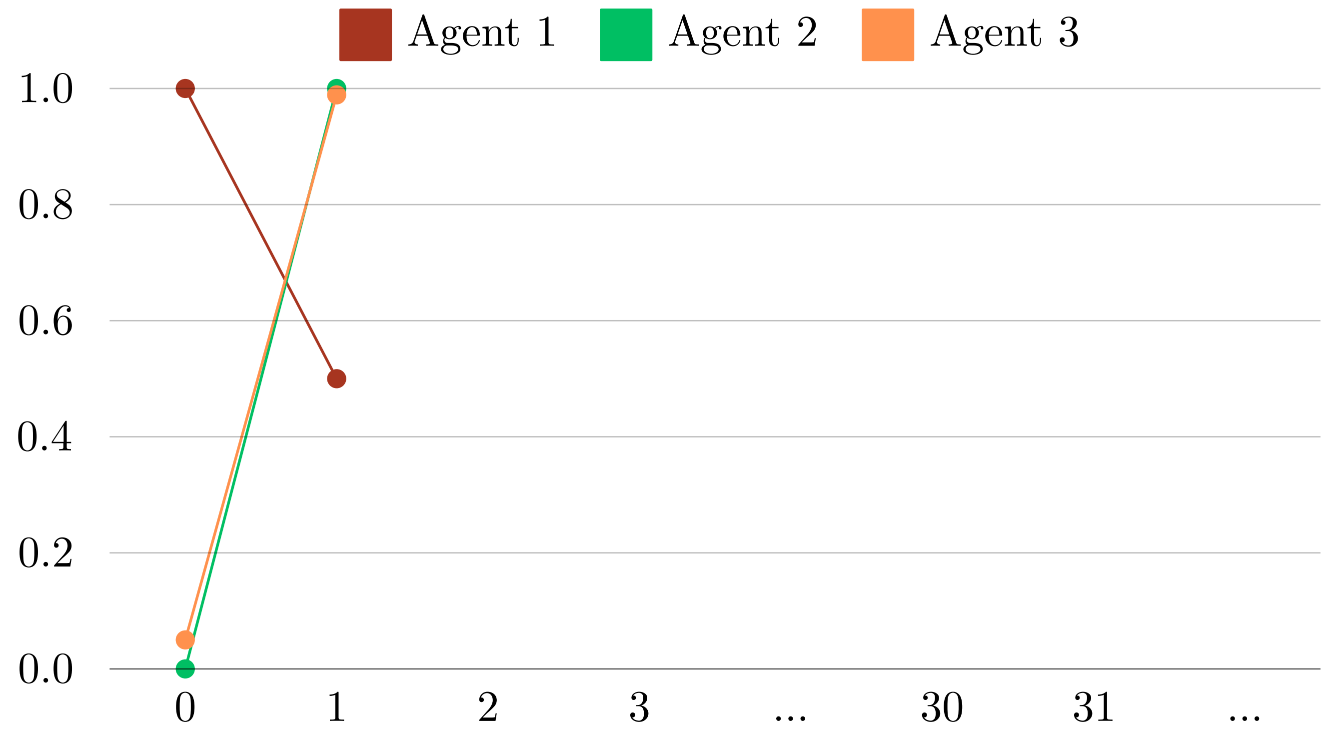
$$0.5 \cdot 1 + 0.25 \cdot 0 + 0.25 \cdot 0 = 0.5$$



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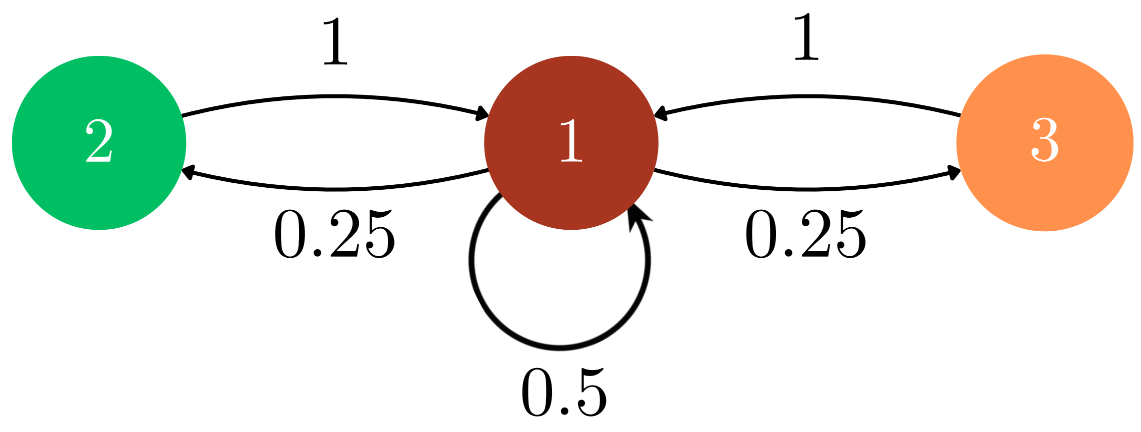


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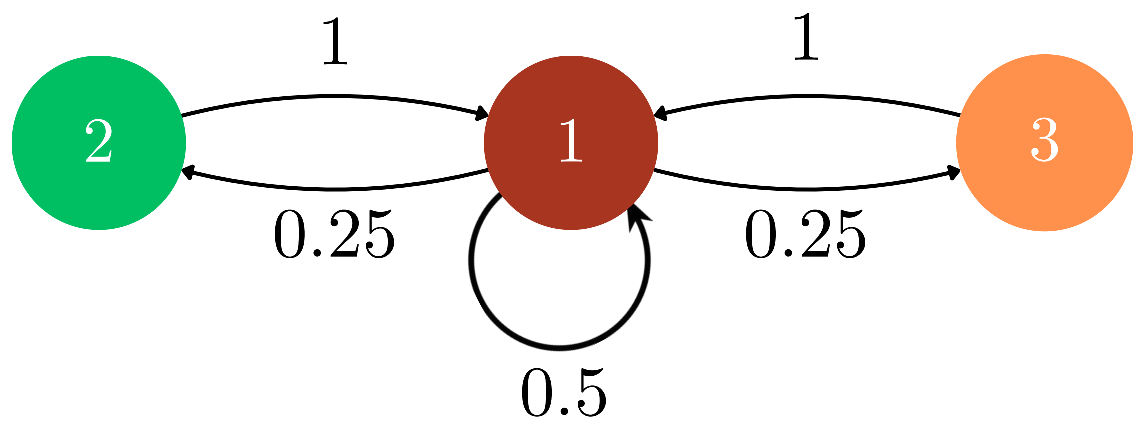


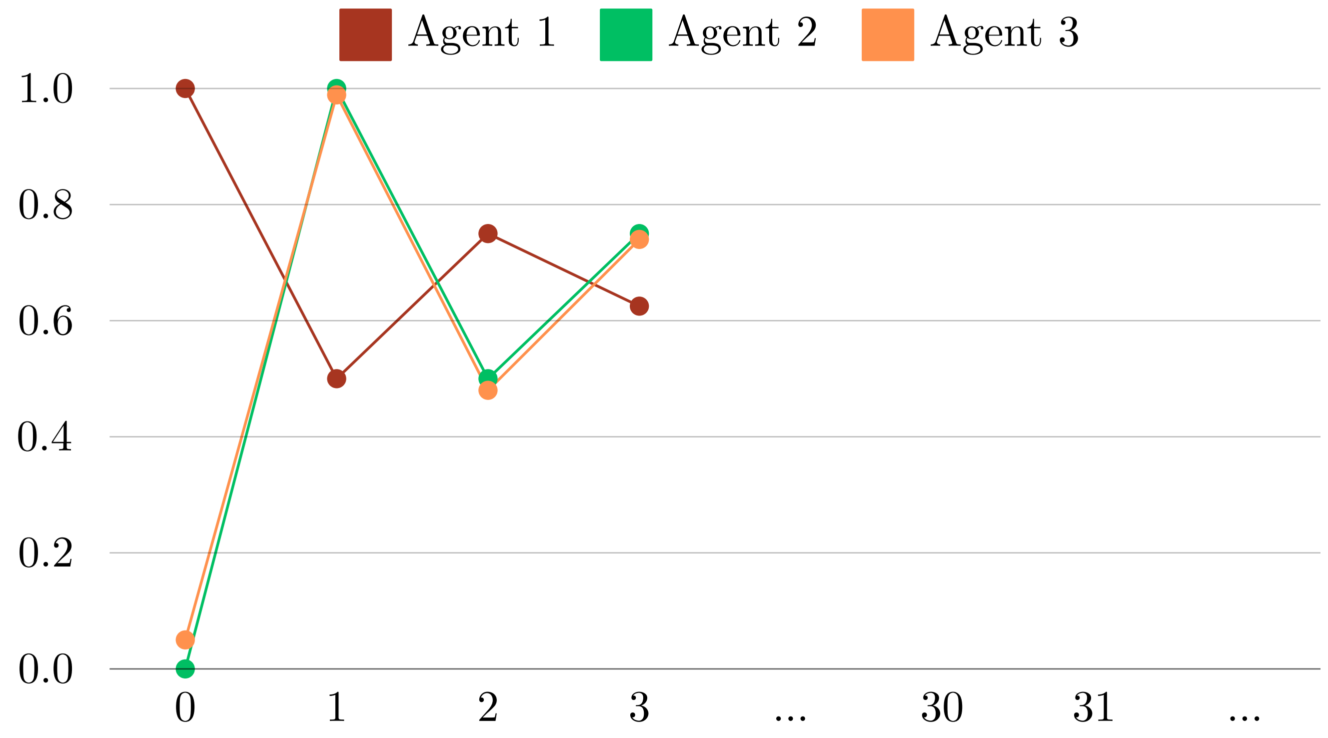

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$1, 2, 3$

**Time**
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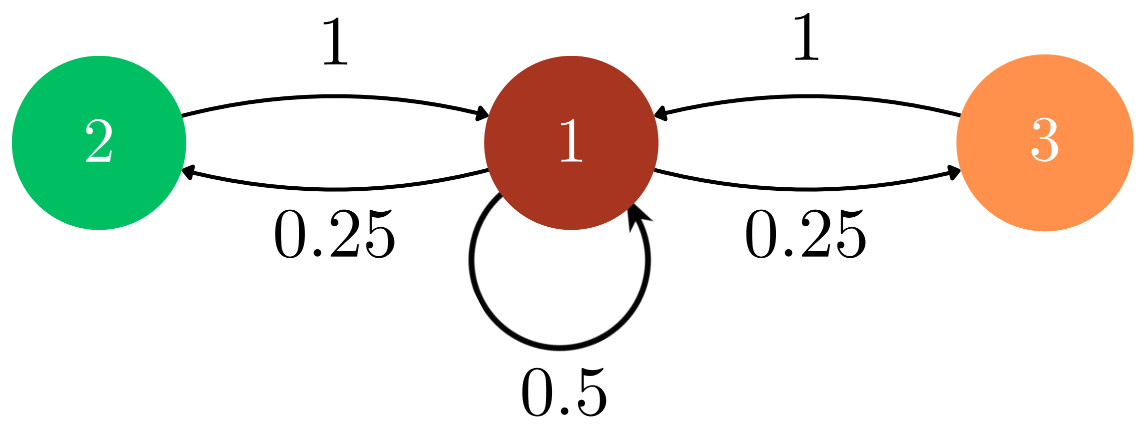


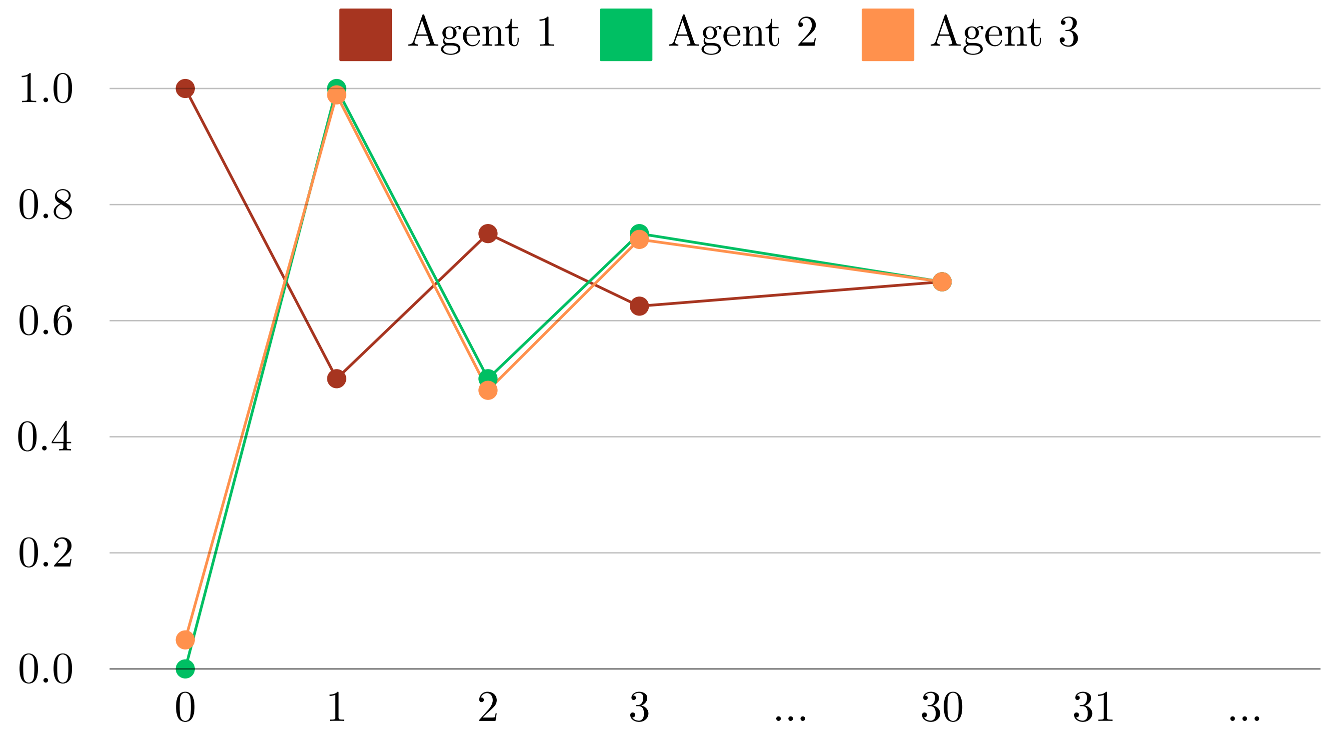

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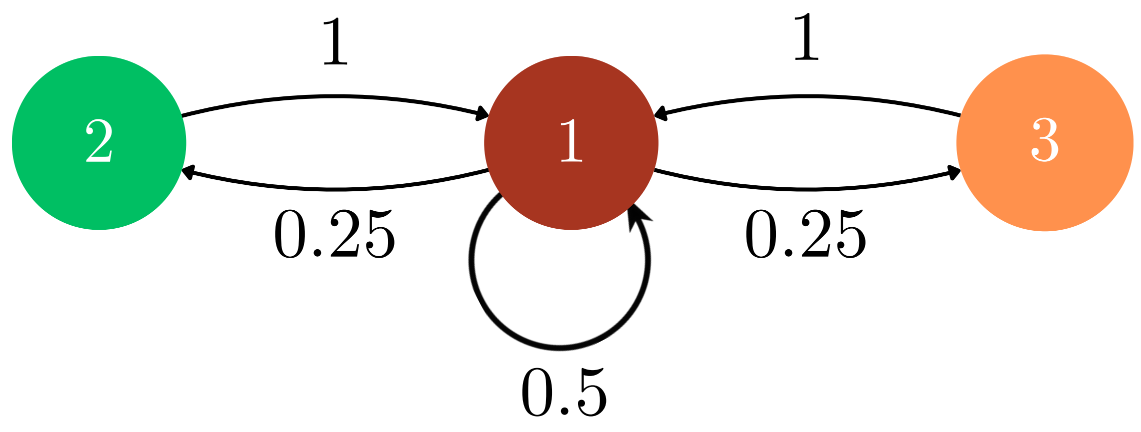


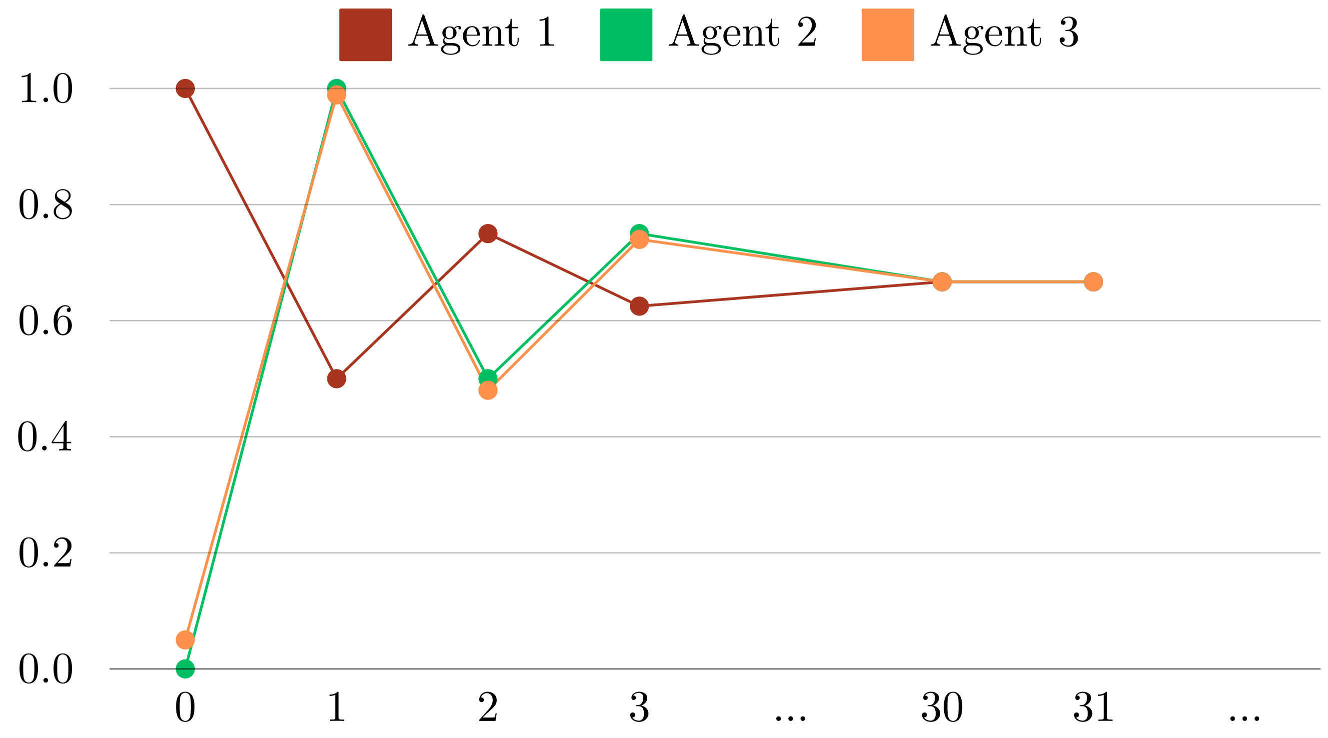

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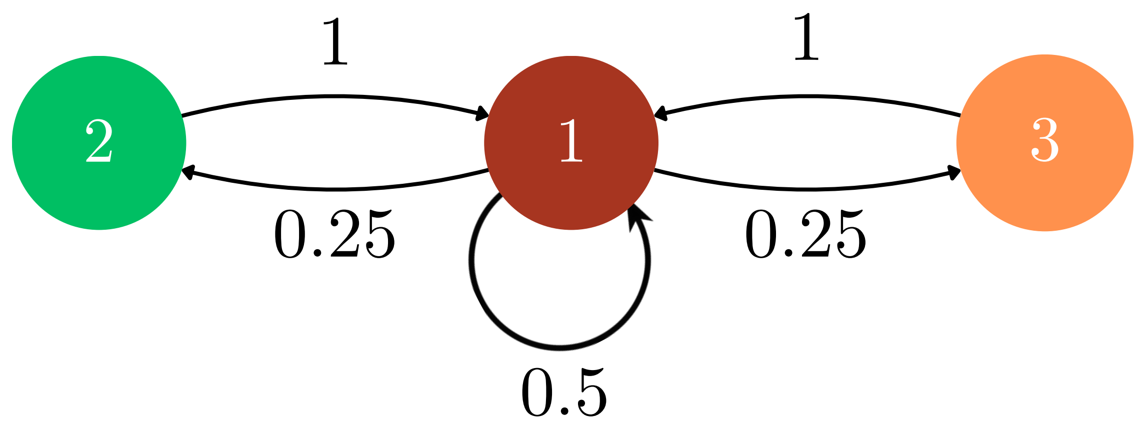
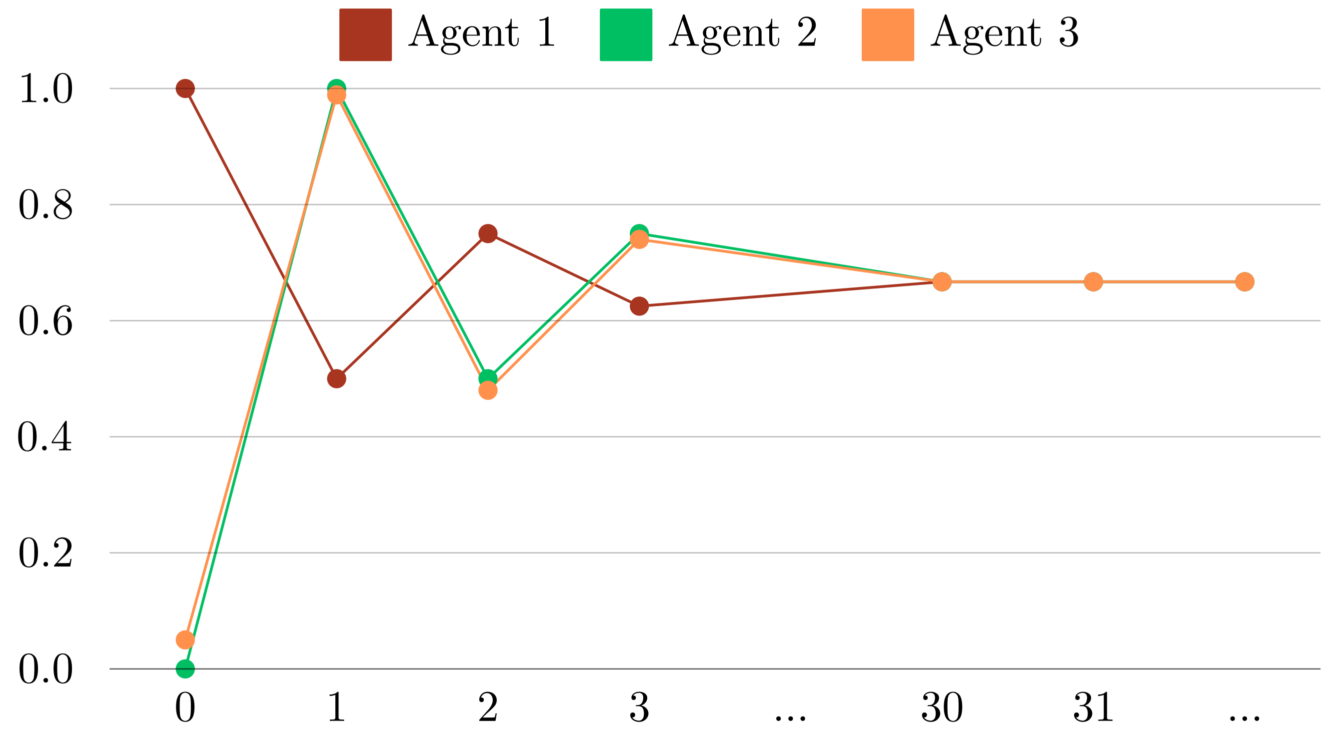
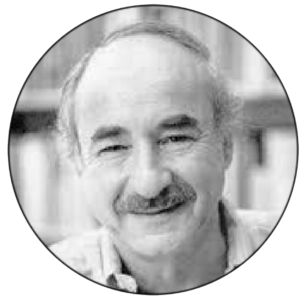


Do the beliefs of each agent *converge*, i.e., reach a point after which they do not change anymore?

If yes, then is there *consensus*, i.e., do beliefs converge to the same value?

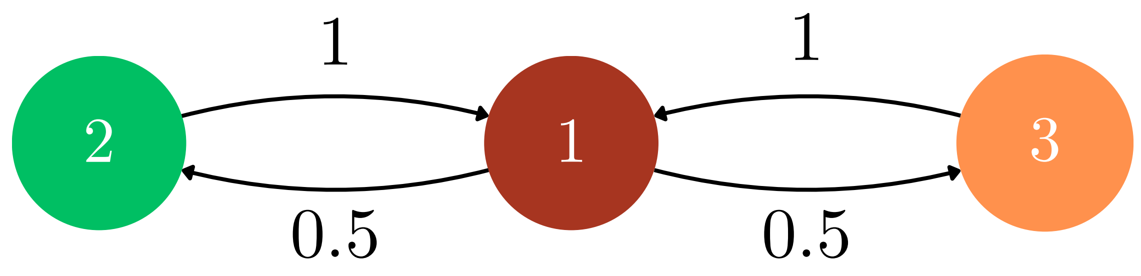


MORRIS DEGROOT

Yes!

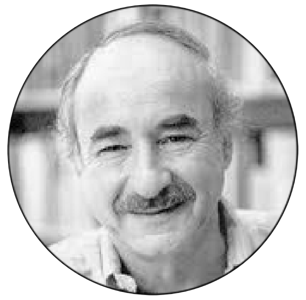
Under certain conditions...

Certain types of cycles are bad news.




**DEFINITION**

A network is *aperiodic* if the greatest common divisor of any two cycle lengths is 1.



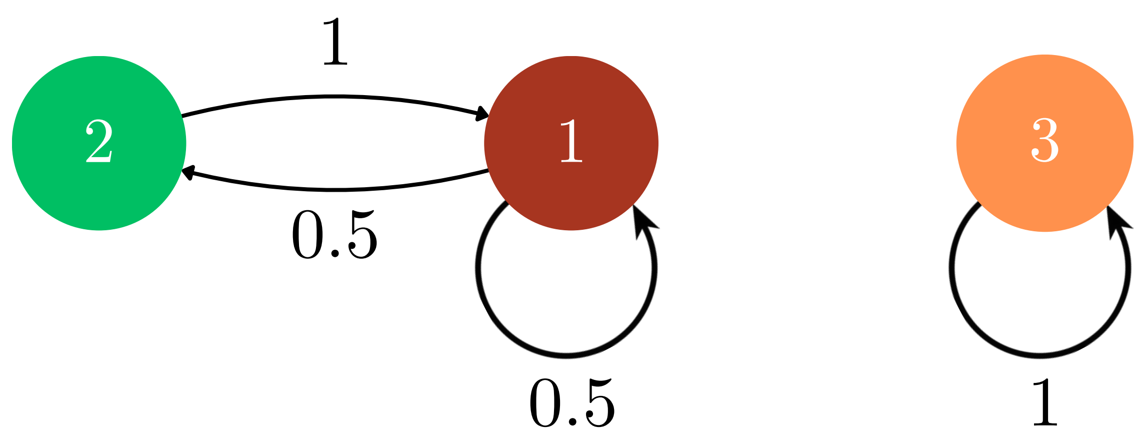
MORRIS DEGROOT

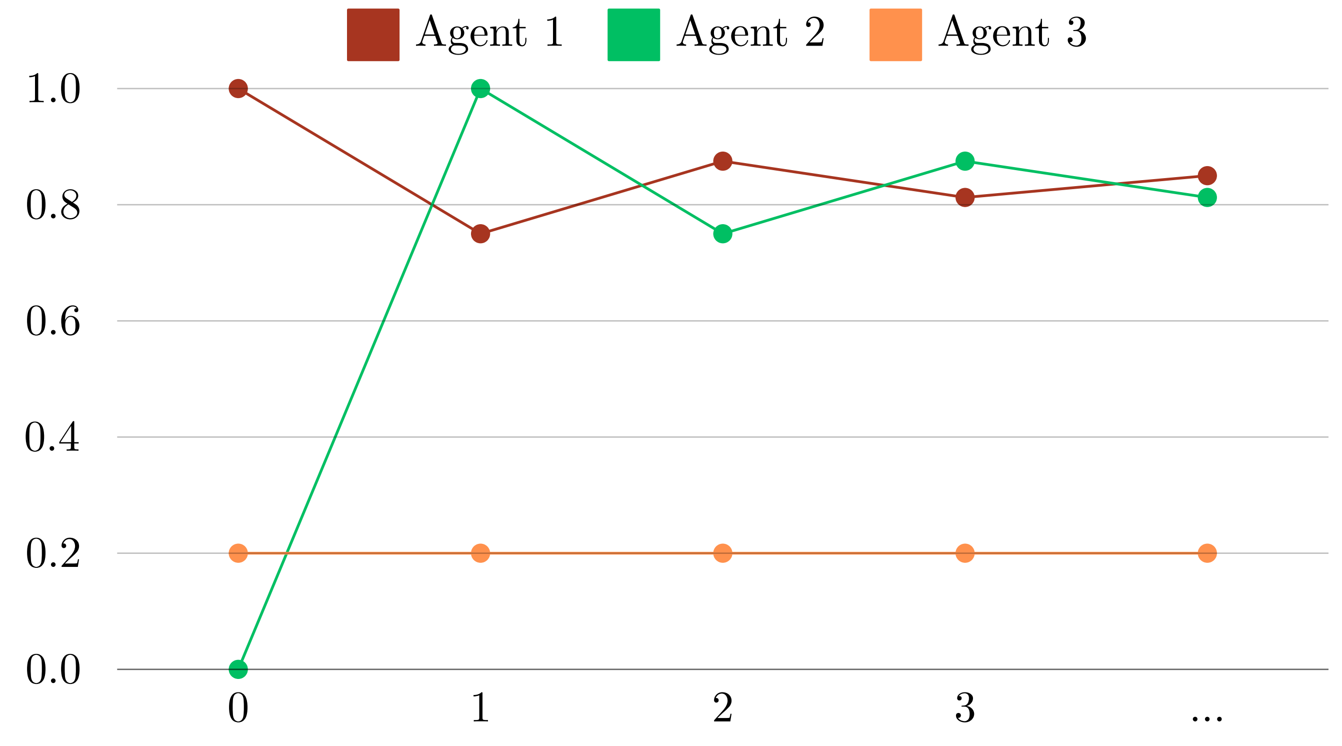
Cycles of length 2, 3 and 4 are fine.

Also cycles of length 1, i.e., self-loops.

But cycles of length 2 and 4, or 3 and 6... no bueno.

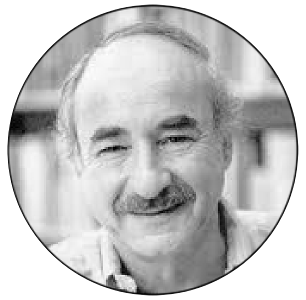
Isolated nodes are also bad news.



**DEFINITION**

A network is *strongly connected* if there is a path from any node to any other node.



MORRIS DEGROOT

Strong connectedness and aperiodicity do the trick!

**THEOREM (DEGROOT, 1974)**

If the social network is strongly connected and aperiodic, then, for any initial beliefs, agents converge in the limit to the same belief.

DeGroot, M. H. (1974). Reaching a Consensus. *Journal of the American Statistical Association*, 69(345), 118–121.

Nice!

But what needs to happen for agents in the DeGroot model to arrive at a consensus that is also *correct*?

# THE DEGROOT MODEL WITH TRUTH

| | |
|---|---|
| agents | $1, 2, \ldots, n$ |
| time | $t \in \{0, 1, 2, \ldots\}$ |
| true state | $\mu \in (0, 1)$ NEW |
| belief of agent $i$ at $t$ | number between $0$ and $1$<br>drawn from a distribution with mean $\mu$<br>and finite variance above a threshold $\delta > 0$ NEW |
| social network | aperiodic, strongly connected directed graph with agents as vertices,<br>and who-pays-attention-to-who as edges |
| agent $i$'s neighborhood | agents that $i$ pays attention to |
| weight on edge from $i$ to $j$ | number that indicates how much weight $i$ places on $j$'s opinion;<br>we assume $i$ distributes a total weight of $1$ across $i$'s neighborhood |
| update rule | at time $t+1$ every agent updates their belief<br>to a weighted average over the beliefs of neighbors |

BENJAMIN GOLUB

We want to speak now of wise *networks*.

MATTHEW O. JACKSON

As with the Condorcet Jury Theorem, this is a limit condition as the network grows larger and larger.

Golub, B., & Jackson, M. O. (2010). Naïve Learning in Social Networks and the Wisdom of Crowds. *American Economic Journal: Microeconomics*, 2(1), 112–149.

**DEFINITION**

We write $G_n$ for a network with $n$ vertices.

A sequence $G_1, G_2, \ldots, G_n, \ldots$ of (strongly connected and aperiodic) networks of increasing size is *wise* if the consensus belief approaches the true state $\mu$ asymptotically, as $n$ goes to infinity.

BENJAMIN GOLUB

There’s a really cool way of thinking about the consensus belief.

Golub, B., & Jackson, M. O. (2010). Naïve Learning in Social Networks and the Wisdom of Crowds. *American Economic Journal: Microeconomics*, 2(1), 112–149.

BENJAMIN GOLUB

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MATTHEW O. JACKSON

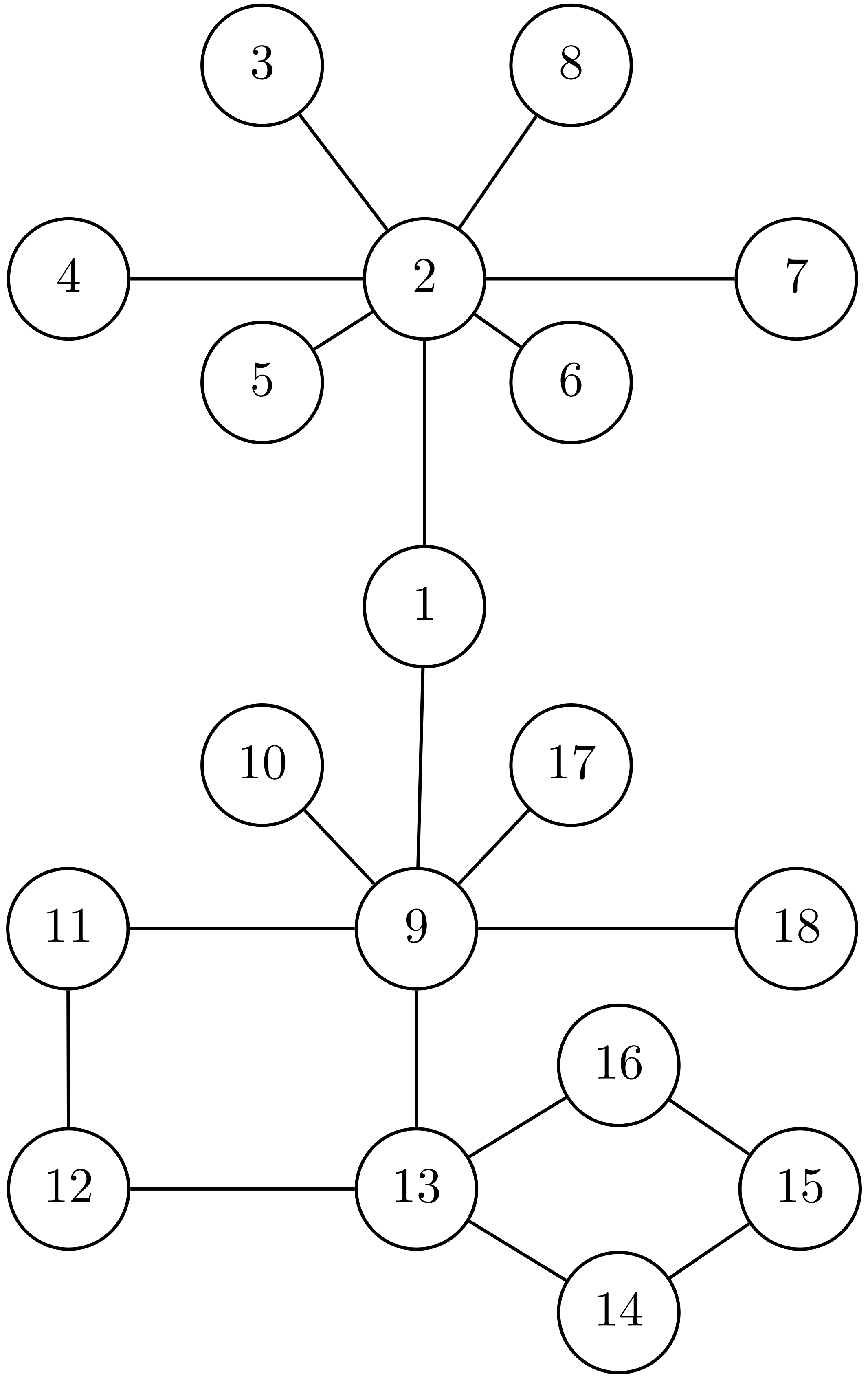
The consensus belief is a linear combination of the initial beliefs and the *eigenvector centralities* of the nodes.

Golub, B., & Jackson, M. O. (2010). Naïve Learning in Social Networks and the Wisdom of Crowds. *American Economic Journal: Microeconomics*, 2(1), 112–149.

The *centrality* of a node in a network is a measure of how influential that node is.

# EIGENVECTOR CENTRALITY

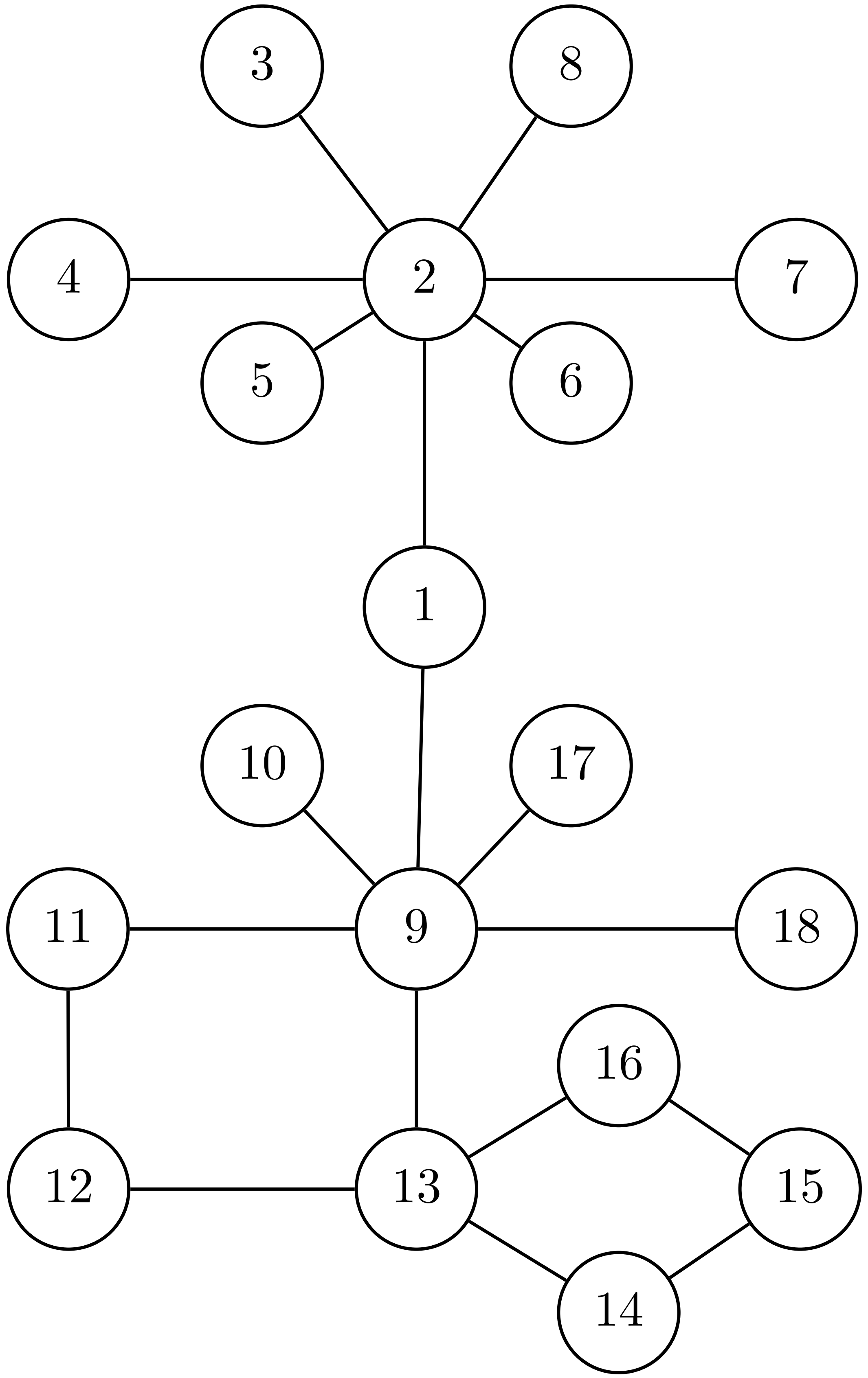
A node is influential if it is connected to an influential node.



# EIGENVECTOR CENTRALITY

A node is influential if it is connected to an influential node.

The centrality of a neighbor is proportional to the sum of neighbors’ centralities.



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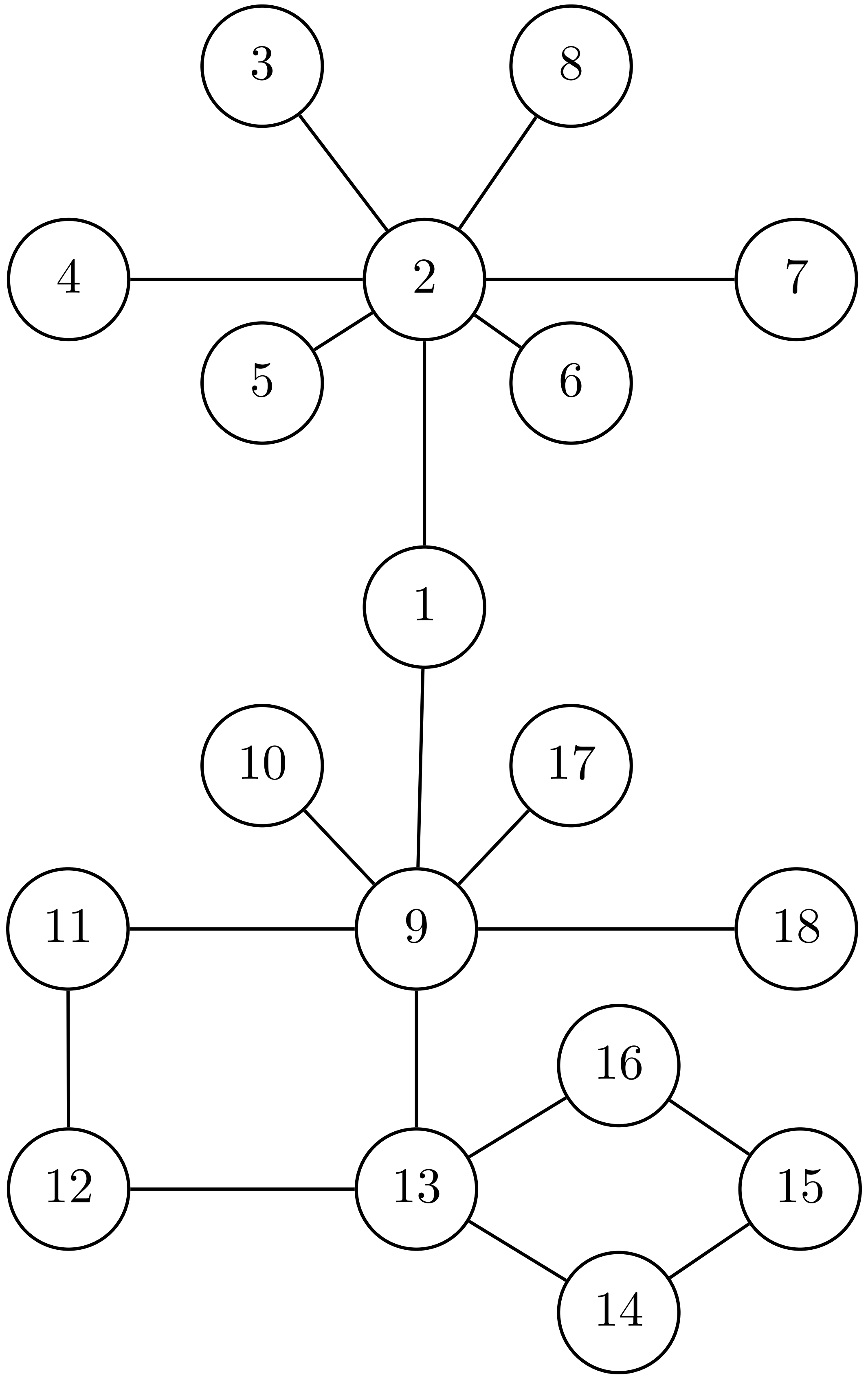
Obtained by solving a system of linear equations.



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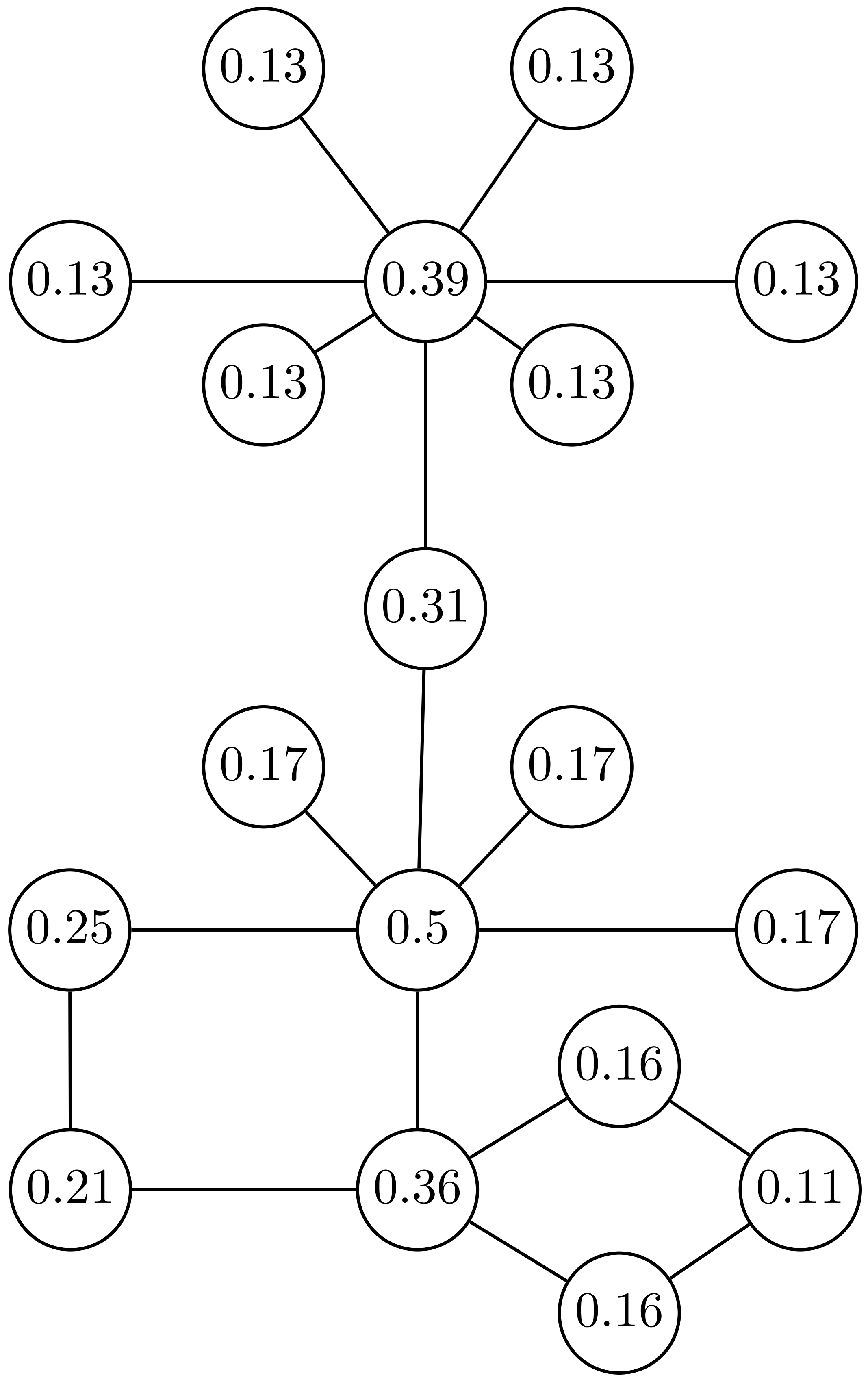


LARRY PAGE
Eigenvector centrality is behind PageRank.

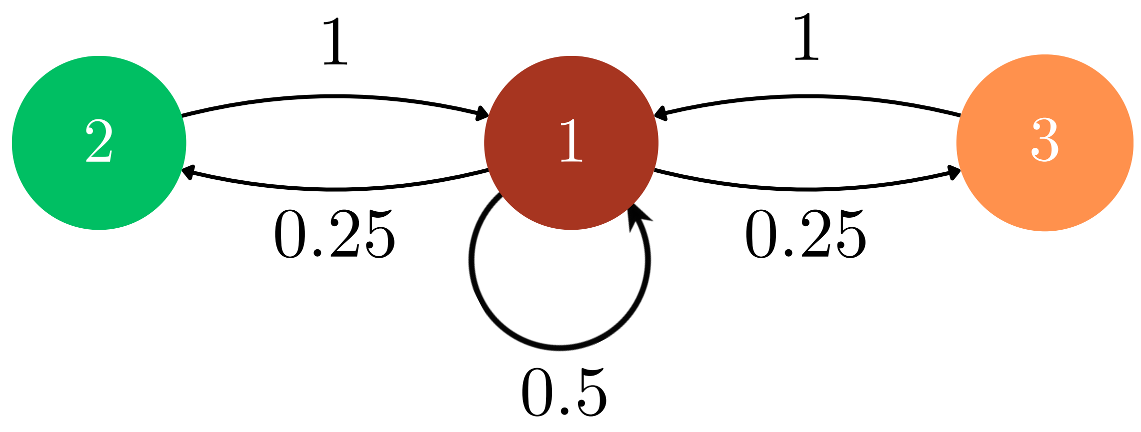
SERGEY BRIN
That is, the original Google algorithm for ranking webpages.

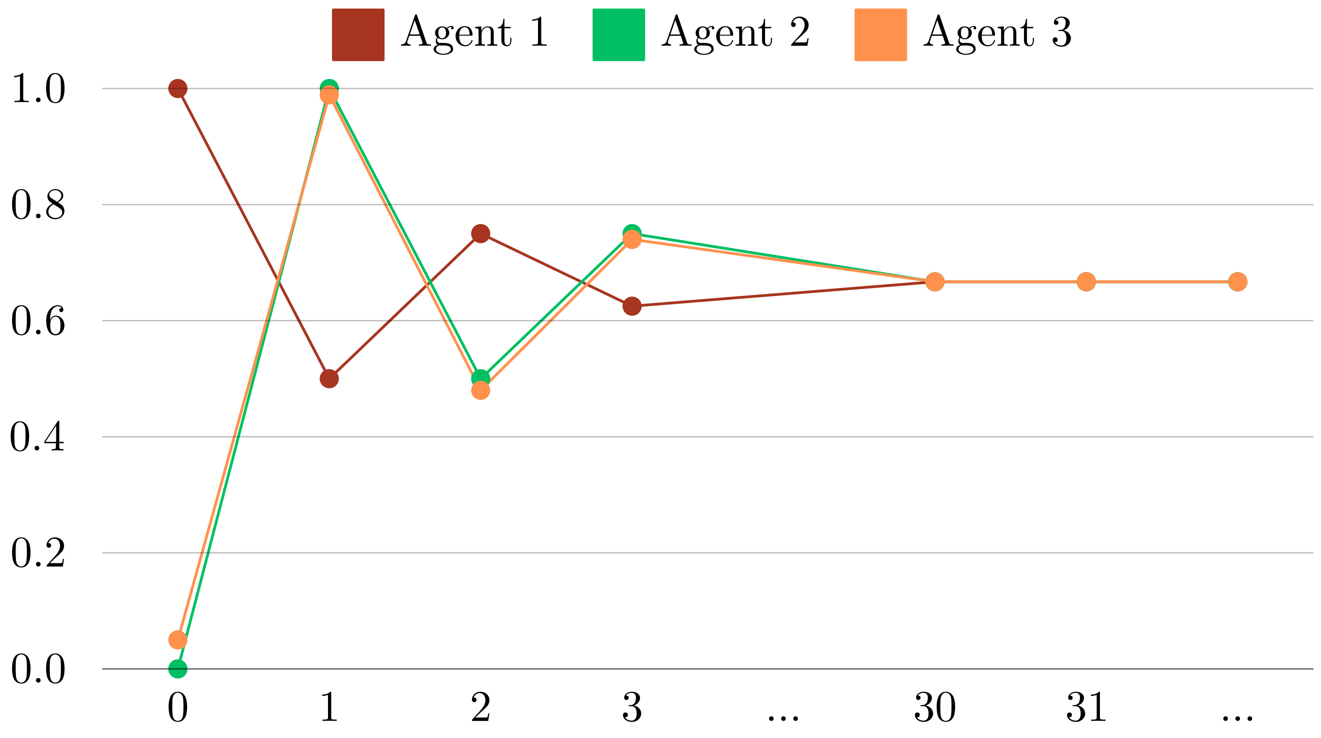
# EIGENVECTOR CENTRALITY & CONSENSUS BELIEFS

The eigenvector centralities are $c = (2/3, 1/6, 1/6)$.

Centralities indicate the importance of the nodes for the limit consensus belief:

$$\left(\frac{2}{3}, \frac{1}{6}, \frac{1}{6}\right) \cdot (1, 0, 0) = \frac{2}{3} \cdot 1 + \frac{1}{6} \cdot 0 + \frac{1}{6} \cdot 0 = \frac{2}{3}.$$



## THEOREM (GOLUB & JACKSON, 2010)

A sequence $G_1, G_2, \ldots, G_n, \ldots$ of (strongly connected and aperiodic) networks of increasing size is *wise* if and only if the eigenvector centrality of every agent $i$ approaches $0$ asymptotically, as $n$ goes to infinity.

Golub, B., & Jackson, M. O. (2010). Naïve Learning in Social Networks and the Wisdom of Crowds. *American Economic Journal: Microeconomics*, 2(1), 112–149.

BENJAMIN GOLUB

For a network to be wise, there can’t be a node that, in the long run, retains positive influence.

MATTHEW O. JACKSON

As the network grows and grows, the influence of every node should go to 0.

Golub, B., & Jackson, M. O. (2010). Naïve Learning in Social Networks and the Wisdom of Crowds. *American Economic Journal: Microeconomics*, 2(1), 112–149.

# NETWORKS THAT ARE NOT WISE

The network grows by adding agents that listen to the central agent 1.

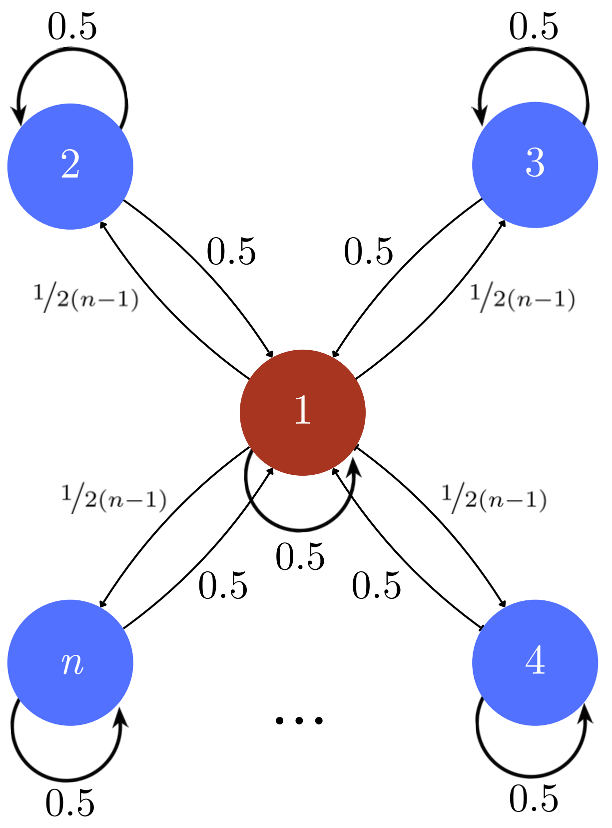
The eigenvector centralities are:

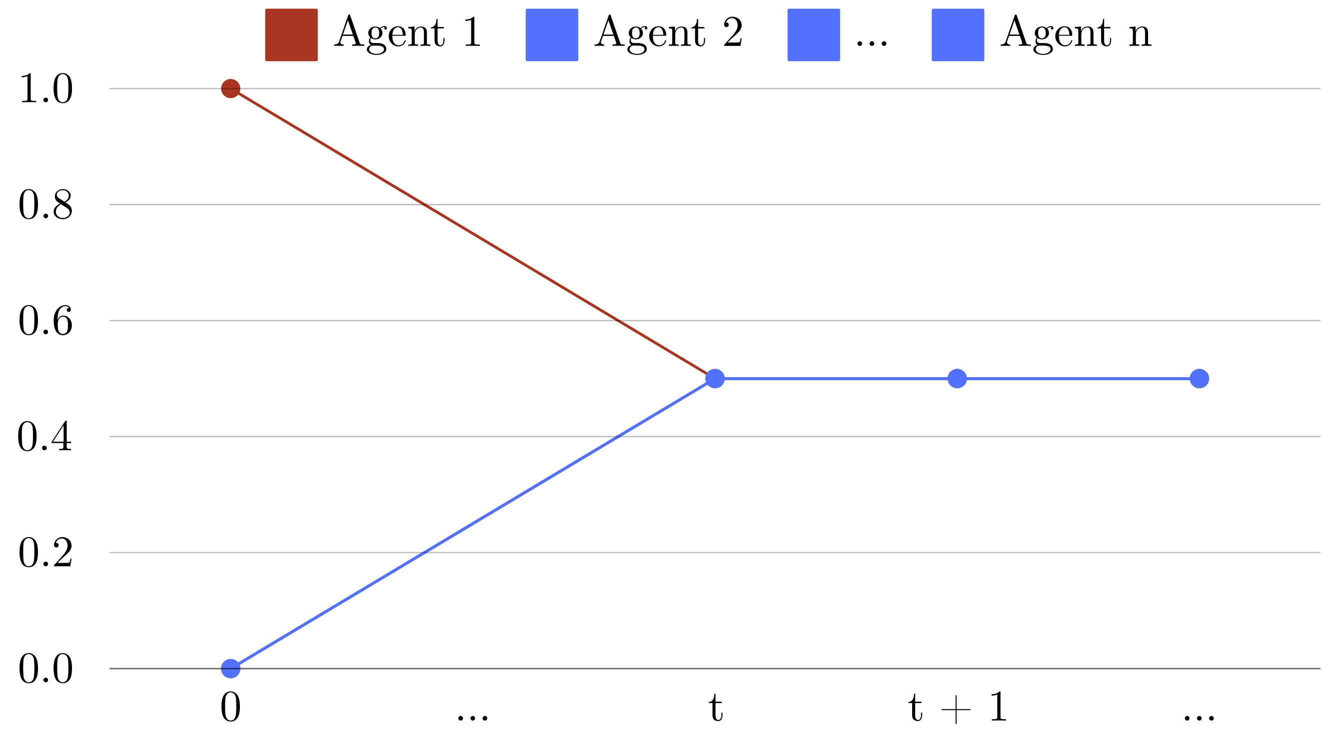
$$\boldsymbol{c} = \left(\frac{1}{2}, \frac{1}{2(n-1)}, \ldots, \frac{1}{2(n-1)}\right)$$

Agent 1 retains a constant share of (network) influence as $n$ grows.

And thus decides the consensus belief.

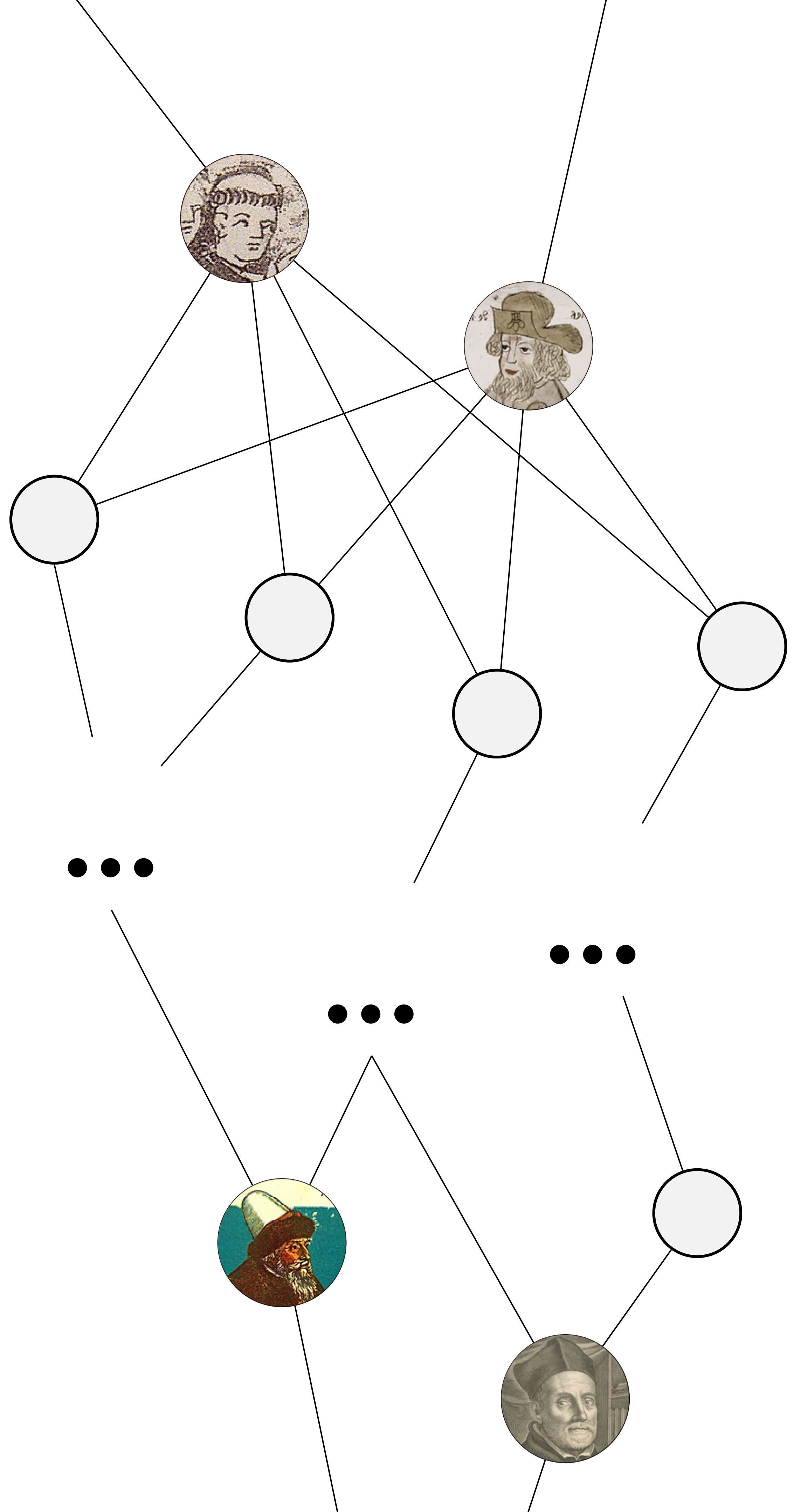
No bueno.



Influential nodes draw the collective opinion towards their own opinion, rather than the truth.

Maybe what happened with the vegetable lamb...



ELON MUSK

Free speech is the bedrock of a functioning democracy.

And X is the digital town square where matters vital to the future of humanity are debated.

But if some voices in this square have outsized influence, due to their position in the network, how democratic is this really?...