



LUDWIG-  
MAXIMILIANS-  
UNIVERSITÄT  
MÜNCHEN

OCTOBER 29, 2024

# MODEL AGENTS: SOCIAL BEHAVIOR THROUGH THE FORMAL LENS

# SEGREGATION

Adrian Haret  
a.haret@lmu.de

Segregation is surprisingly  
(worryingly?) common. For instance,  
ethnic groups in cities...





CHOOSE A YEAR

1990

2000

2010

2016\*

SELECT

Race

Diversity

FILTER

Black

White

Asian

Hispanic

Native

Other

Dots represent population density

\* 2016 figures are estimates

# NEW YORK CITY

Williams, A., Eamamdjomeh, A. (2018). America is more diverse than ever — but still segregated. *The Washington Post*.



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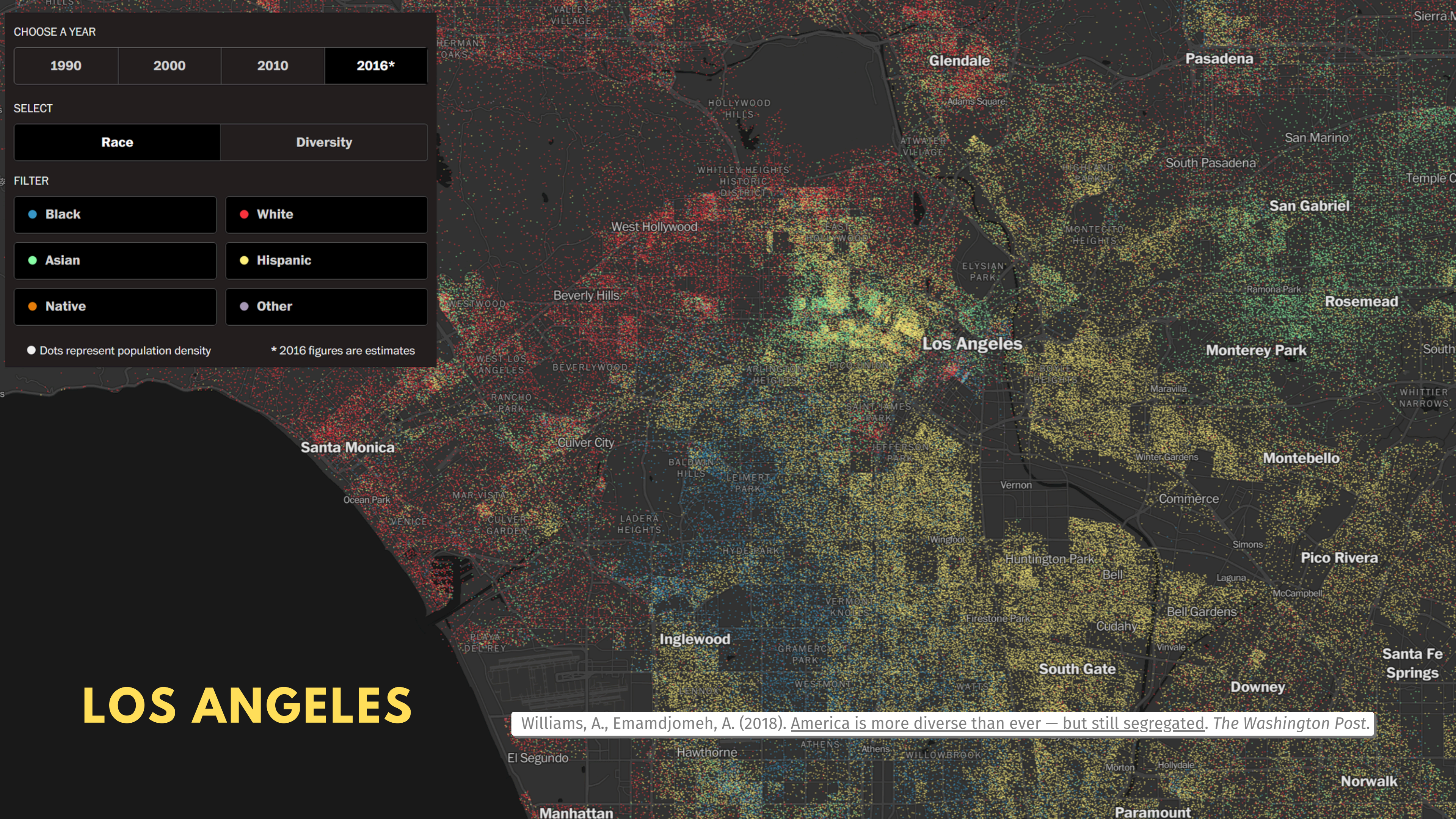
● Other

● Dots represent population density

\* 2016 figures are estimates

# LOS ANGELES

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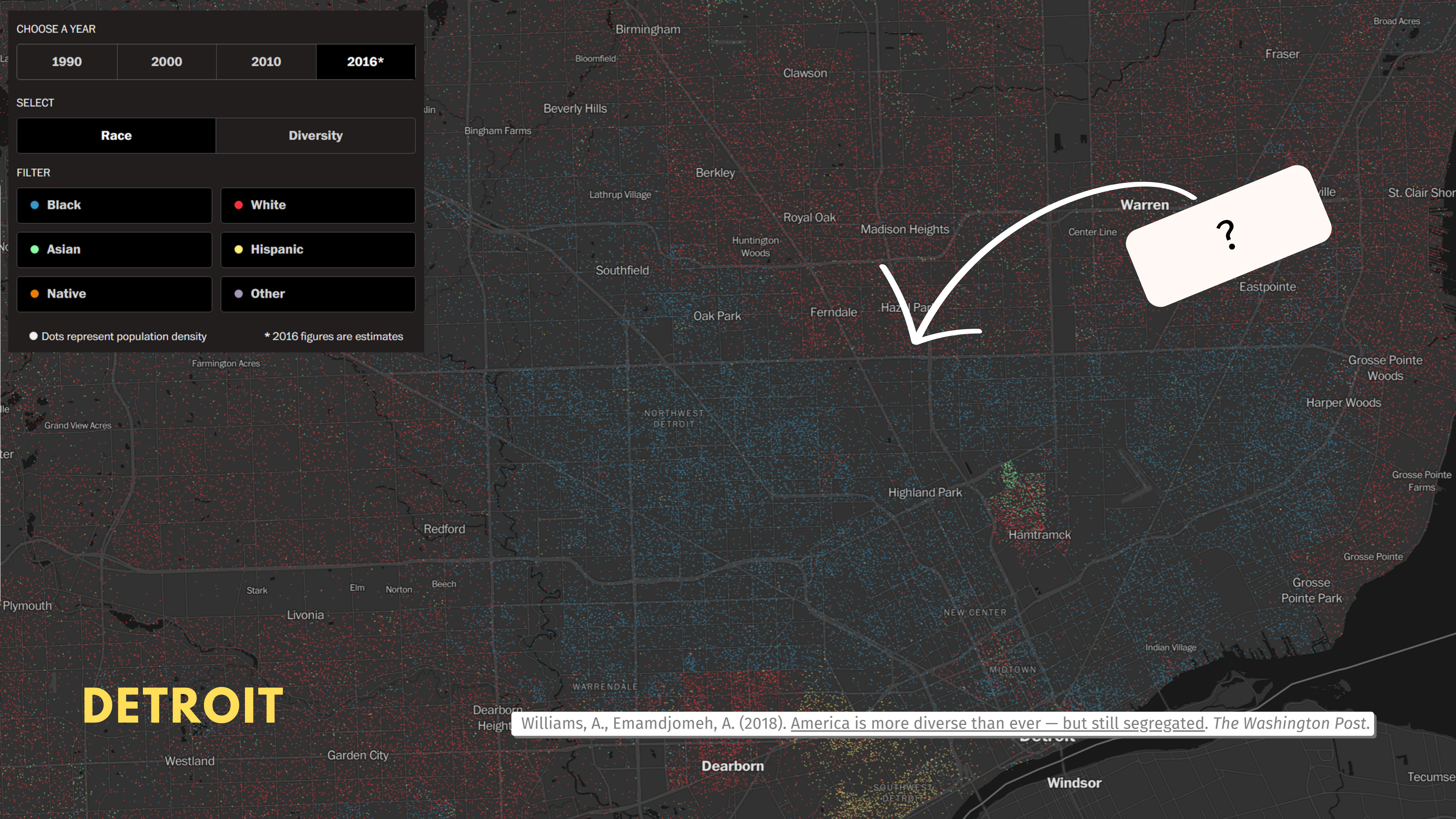
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# DETROIT

EMINEM

His palms are sweaty, knees weak, arms are heavy  
There's vomit on his sweater already, mom's spaghetti...

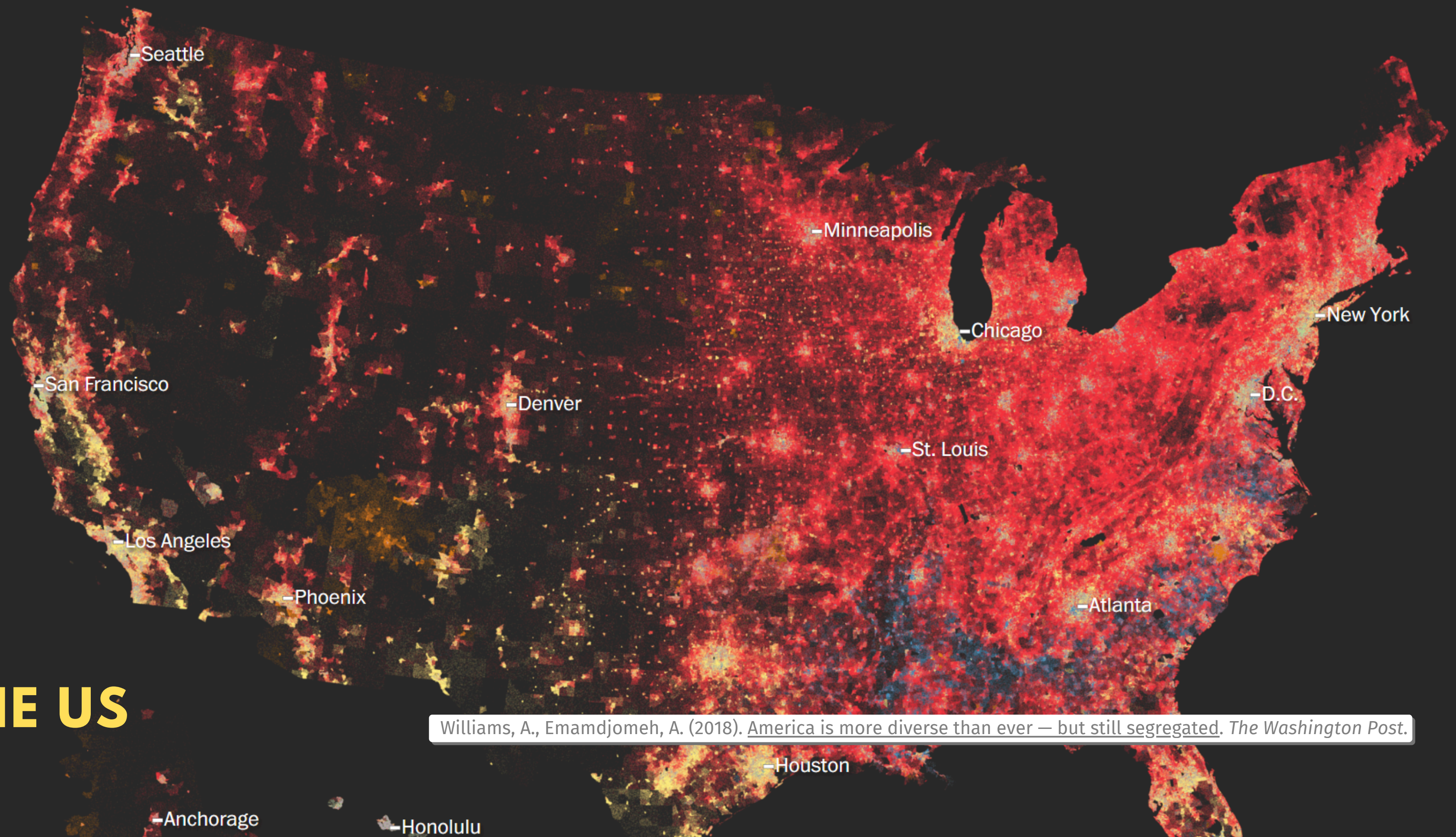


8 Mile

Williams, A., Eamandjomeh, A. (2018). America is more diverse than ever — but still segregated. *The Washington Post*.



● Black ● White ● Hispanic  
● Asian/Pacific Islander ● Native American ● Multi-race and other



THE US

Williams, A., Emamdjomeh, A. (2018). America is more diverse than ever — but still segregated. *The Washington Post*.

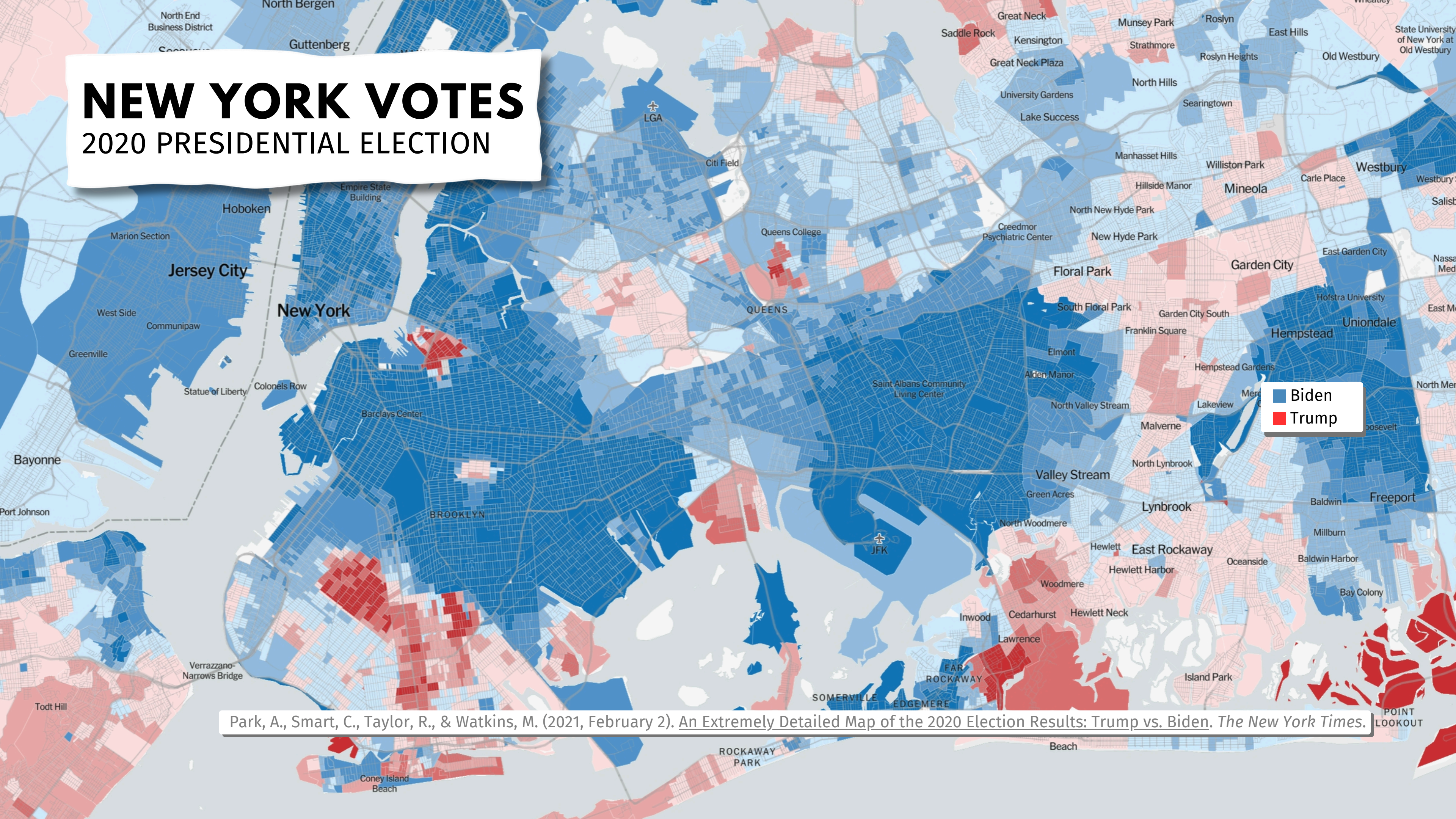


Not just ethnic groups...



# NEW YORK VOTES

## 2020 PRESIDENTIAL ELECTION



Park, A., Smart, C., Taylor, R., & Watkins, M. (2021, February 2). An Extremely Detailed Map of the 2020 Election Results: Trump vs. Biden. *The New York Times*.



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# LONDON

## TOTAL ANNUAL INCOME

Office for National Statistics.

Why does segregation occur?



JOSHUA M. EPSTEIN

If you can't grow it, you don't understand it.

PAUL E. SMALDINO  
Ok, let's grow it.



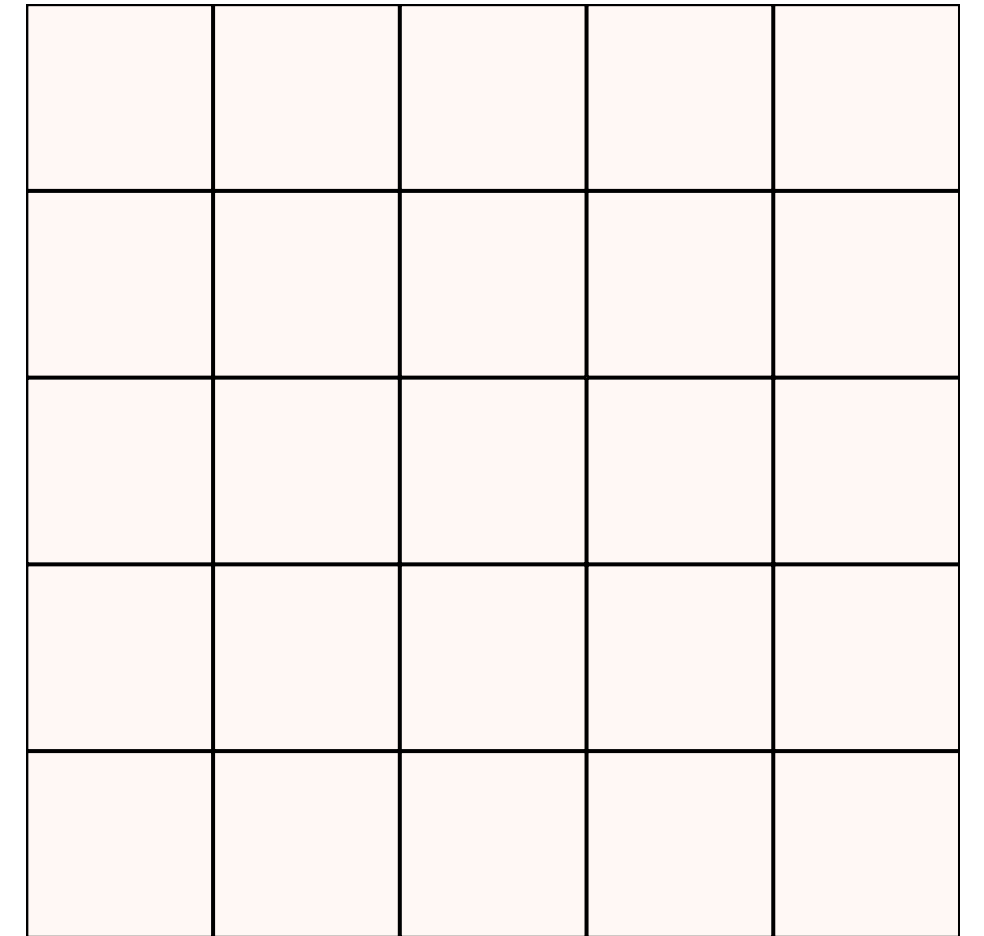
This will be a *spatial model*. So  
let's gather the ingredients.

# SPATIAL MODELS

## ENVIRONMENT

Some type of *space*, typically a square grid.

This is the default in NetLogo, where the black canvas is a square grid; the cells are called *patches*, and they take commands just like turtles do.



# SPATIAL MODELS

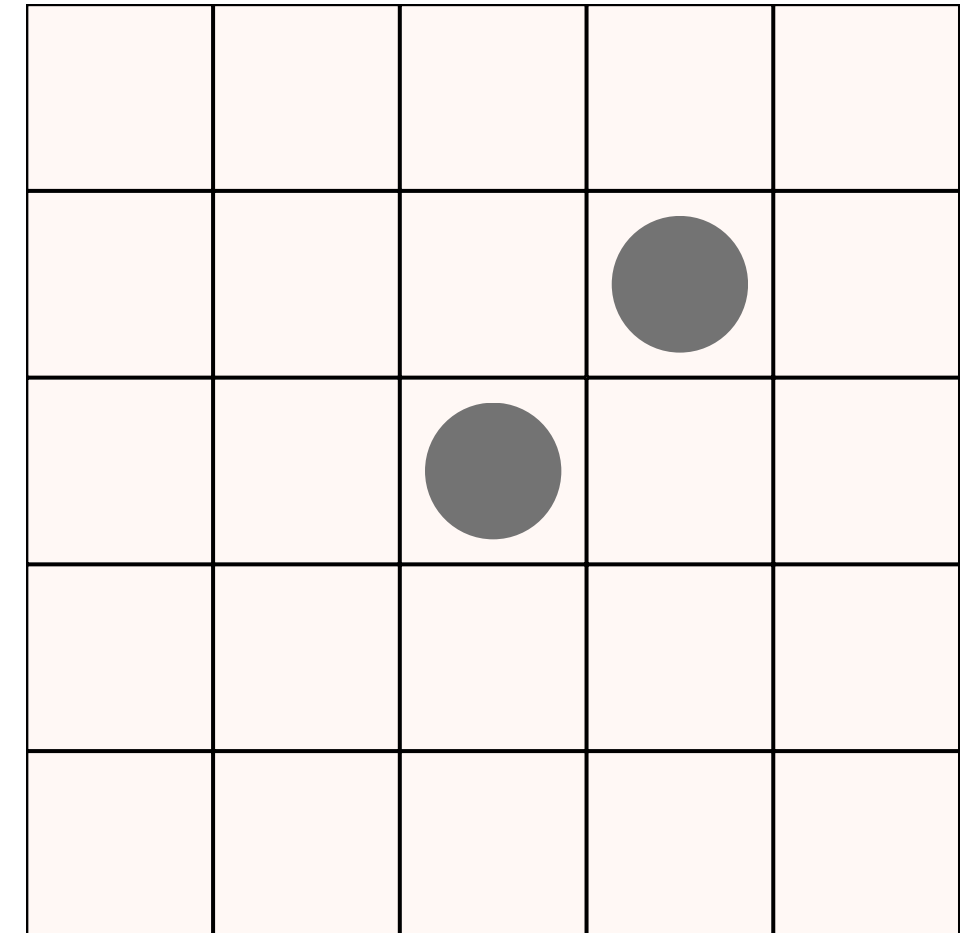
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## AGENTS

Agents are placed on different locations in space.



# SPATIAL MODELS

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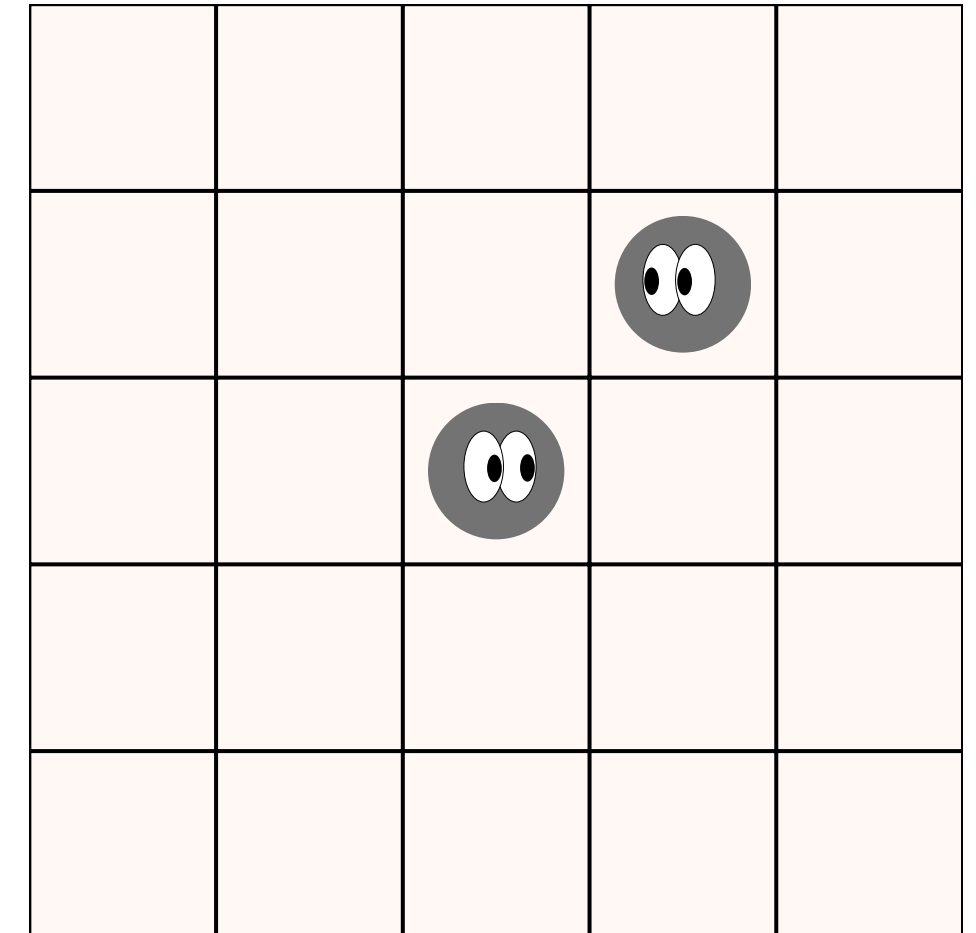
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## AGENTS

Agents are placed on different locations in space.

They have some information, e.g., about themselves and their *neighbors*.



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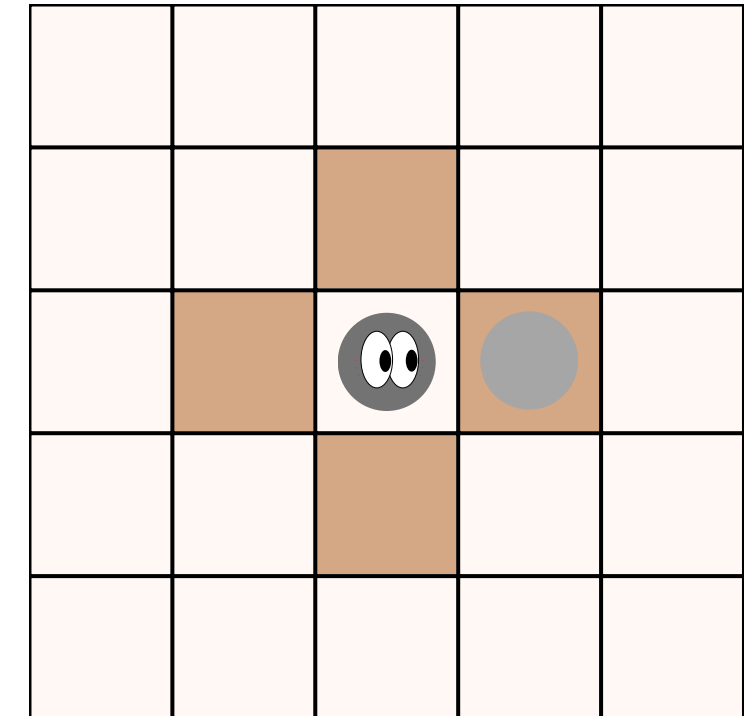
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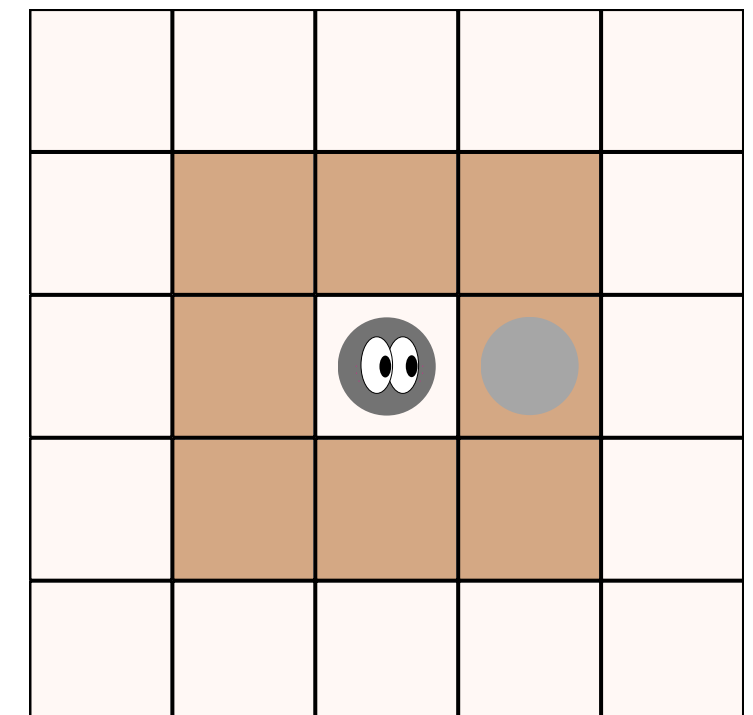
Agents are placed on different locations in space.

They have some information, e.g., about themselves and their *neighbors*.

Two types of neighborhoods: von Neumann and Moore.



VON NEUMANN NEIGHBORHOOD



MOORE NEIGHBORHOOD



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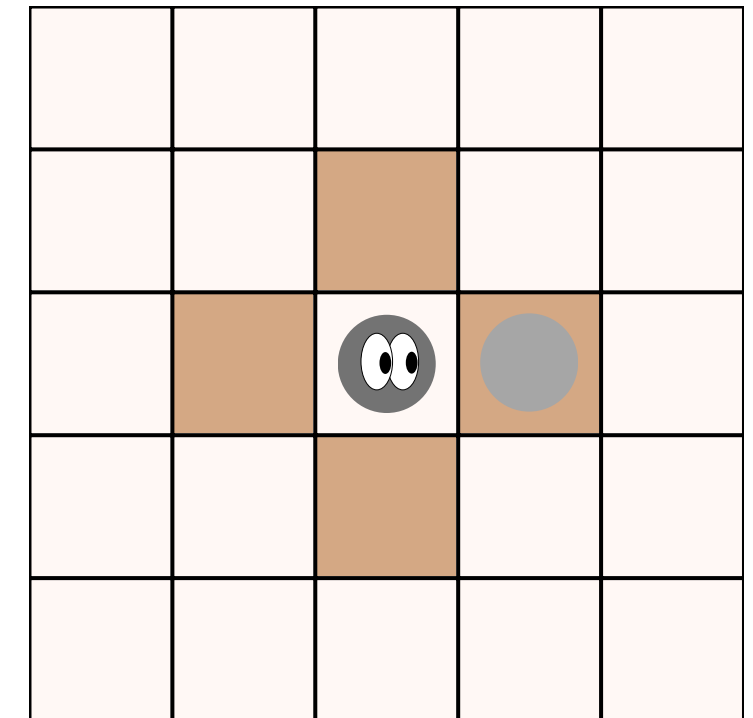
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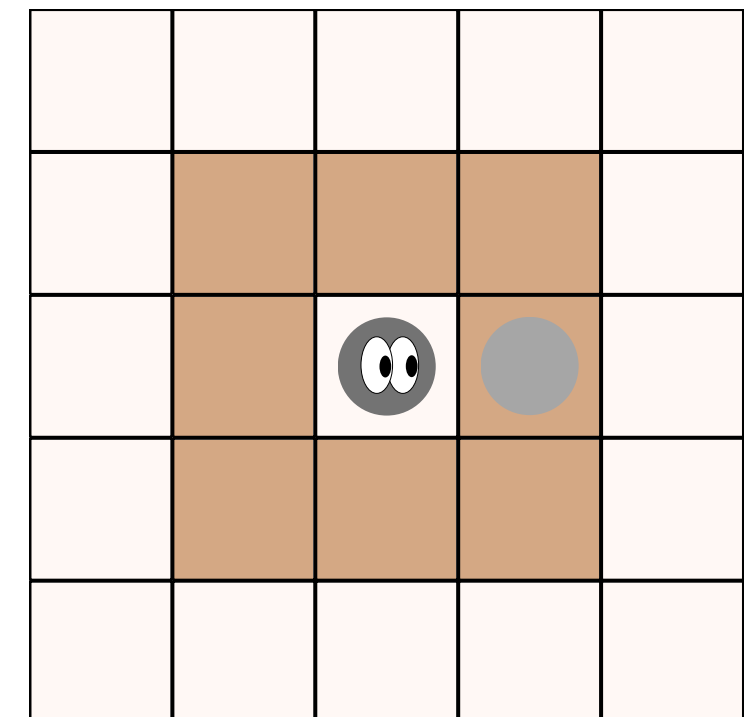
## DYNAMICS

Agents follow some simple (?) rules about how and where to move around.

They follow these rules over and over, and hilarity ensues.



VON NEUMANN NEIGHBORHOOD



MOORE NEIGHBORHOOD



JOHN H. CONWAY

This is kind of like *The Game of Life*!

# GAME OF LIFE

## SURVIVAL

Any live cell with two or three live neighbours lives on to the next generation.

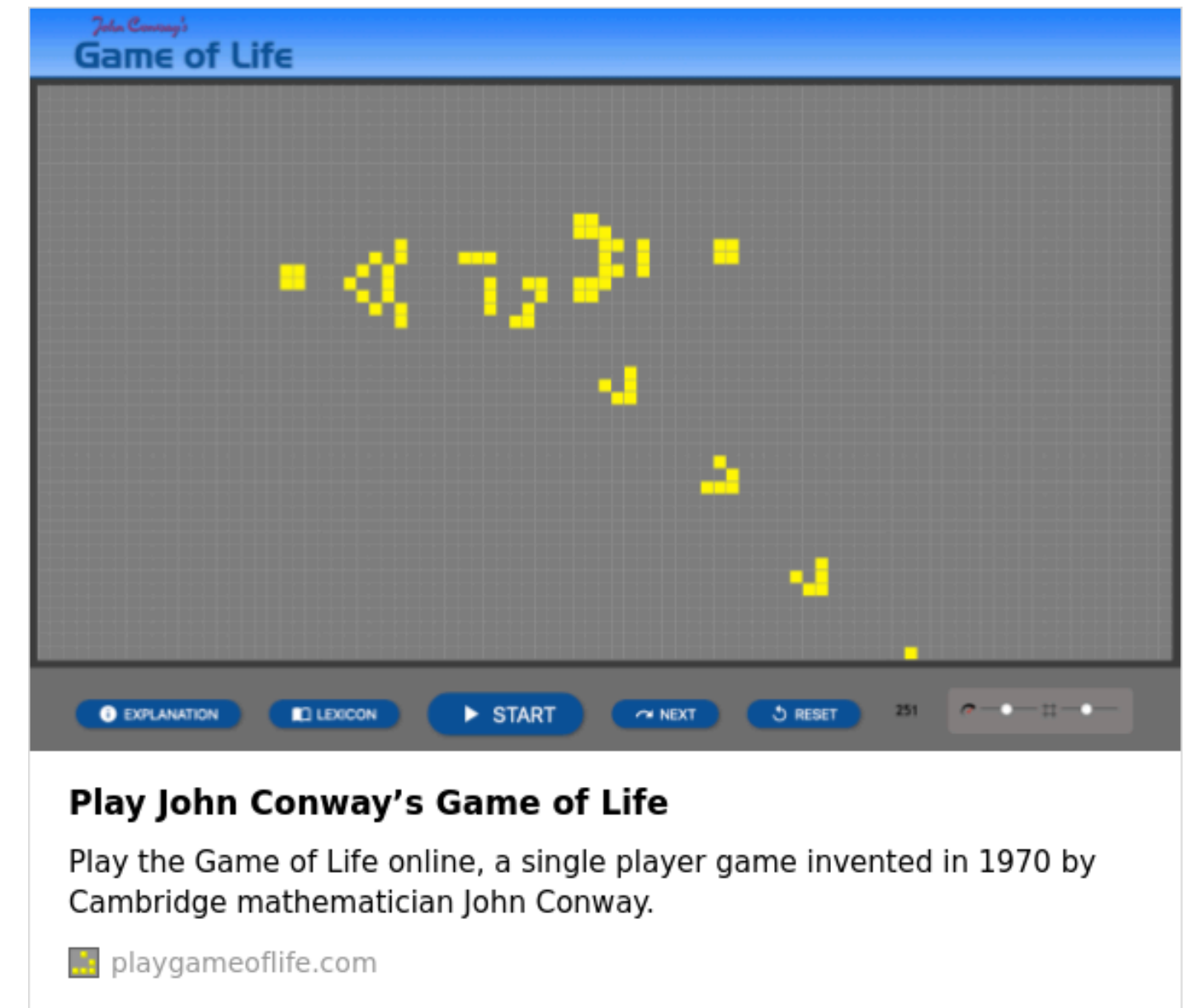
## DEATH

Any live cell with fewer than two live neighbours dies, as if by underpopulation.

Any live cell with more than three live neighbours dies, as if by overpopulation.

## BIRTH

Any dead cell with exactly three live neighbours becomes a live cell, as if by reproduction.



Back to segregation...



THOMAS SCHELLING

People get separated along many lines and in many ways.

There is segregation by sex, age, income, language, religion, color, taste, comparative advantage and the accidents of historical location.

Some segregation results from the practices of organizations; some is deliberately organized.

And some results from the interplay of individual choices that discriminate.

Schelling, T.C. (1969). Models of Segregation. *The American Economic Review*, 59(2), 488–493.

Schelling, T.C. (1971). Dynamic models of segregation. *The Journal of Mathematical Sociology*, 1(2), 143–186.

Schelling, T.C. (1978). *Micromotives and Macrobehavior*. Norton.

What kind of individual choices?



THOMAS SCHELLING

Perhaps people are rather intolerant of diversity and prefer to be in overwhelmingly similar surroundings.

Or maybe *small* biases turn out to have outsized effects.

Schelling, T.C. (1969). Models of Segregation. *The American Economic Review*, 59(2), 488–493.

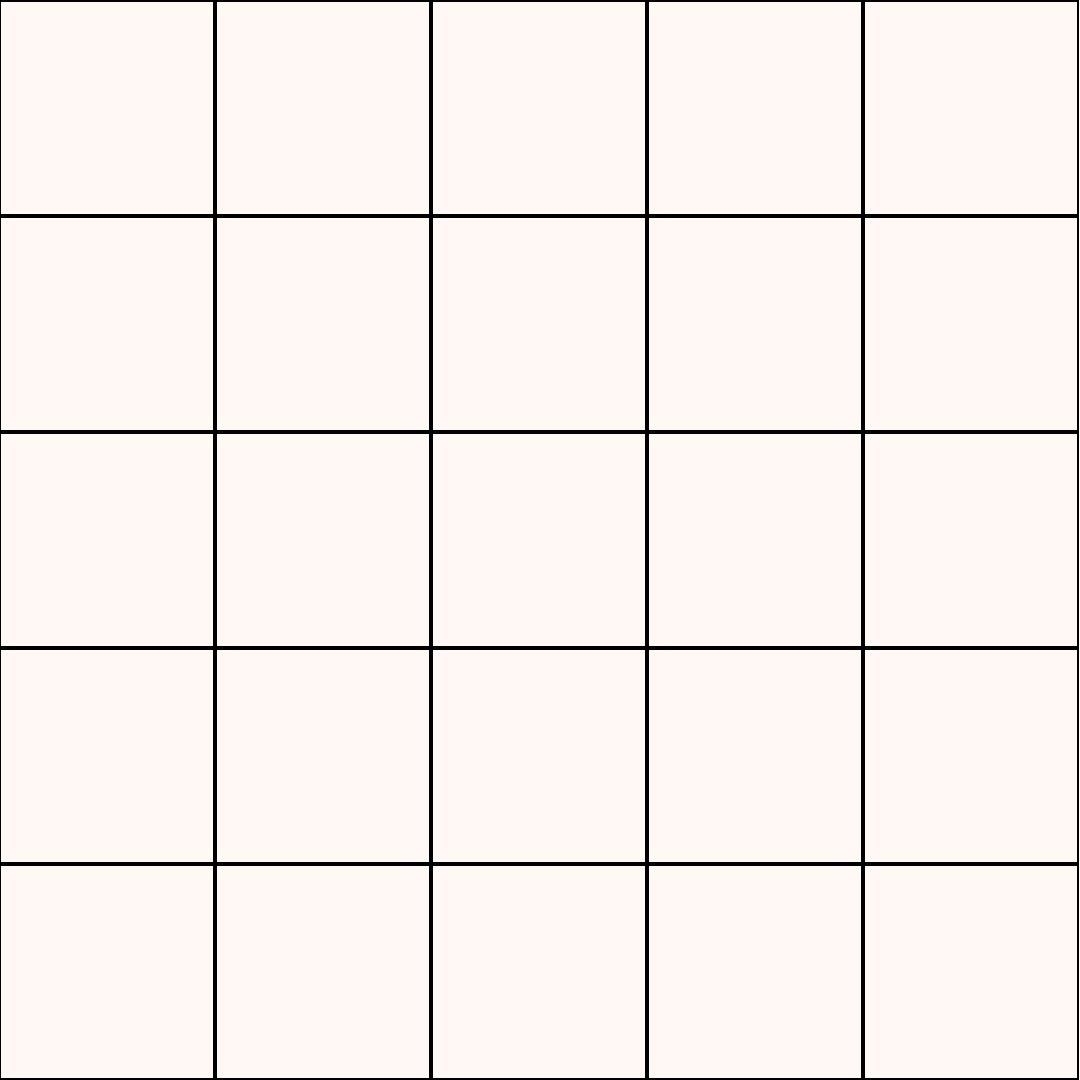
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# THE SCHELLING MODEL

## SETUP

The environment is an  $L \times L$  square grid.



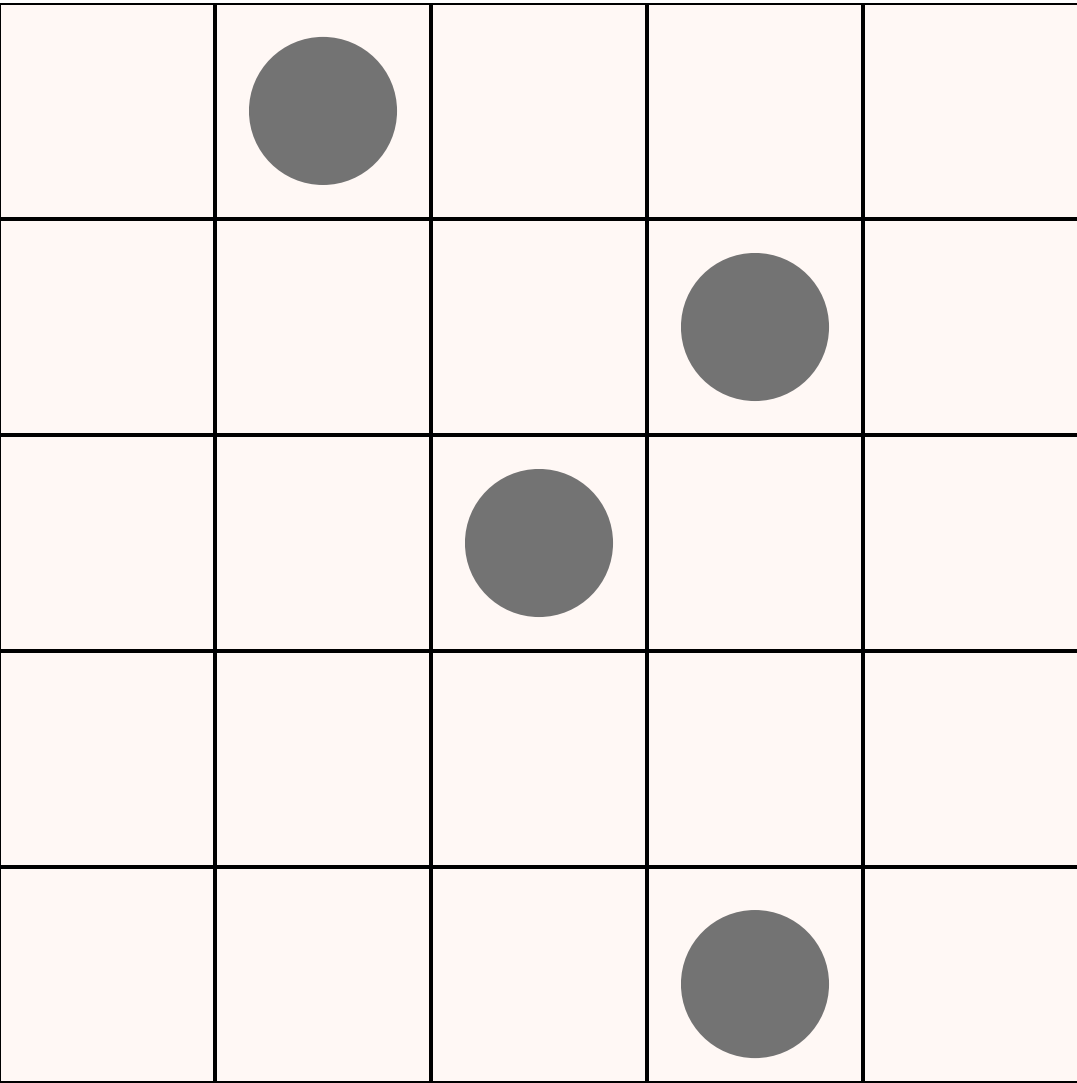


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The environment is an  $L \times L$  square grid.

Initially, each cell contains an agent with a probability  $p$ , the *density*.



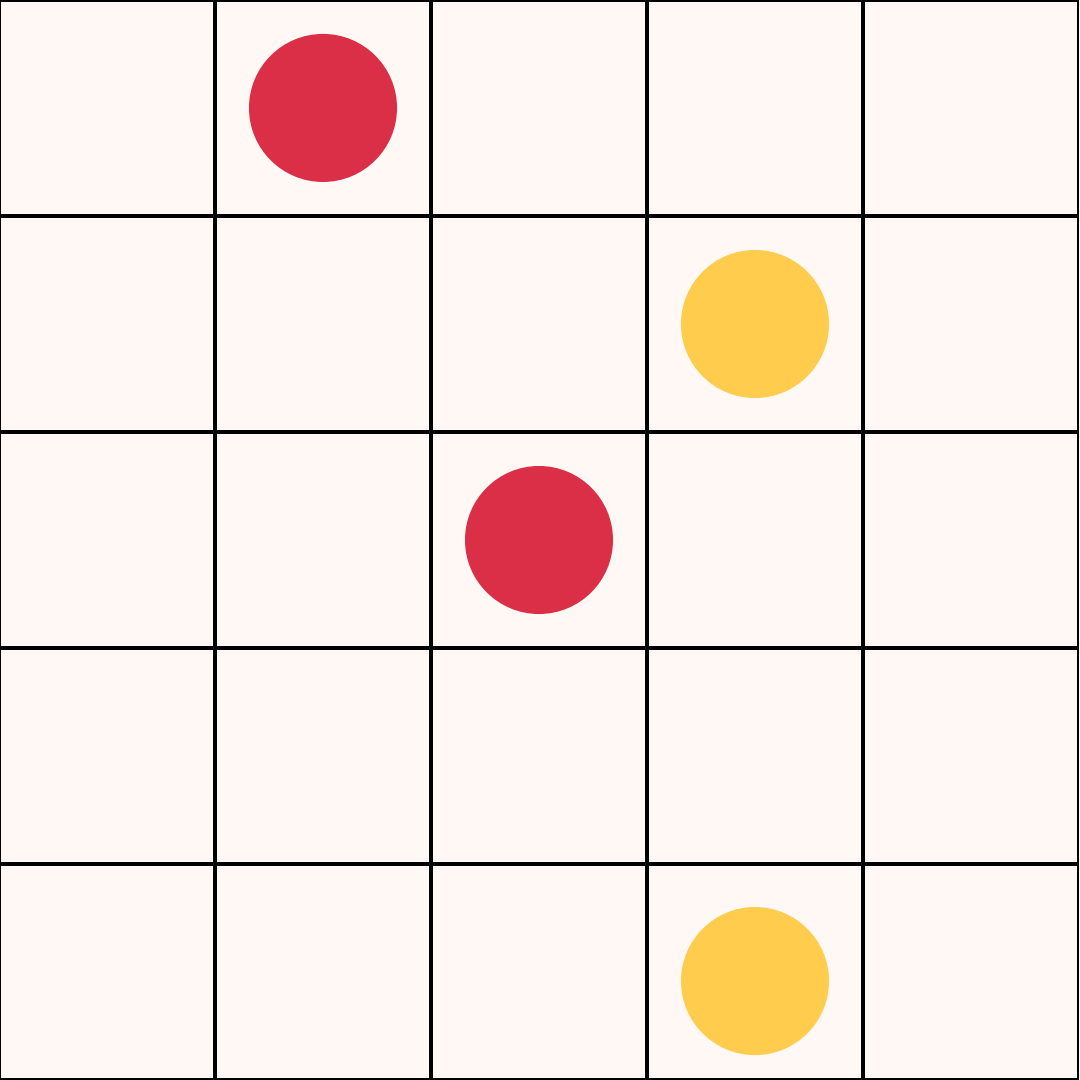
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Agents are of two *types*, Red and Yellow.



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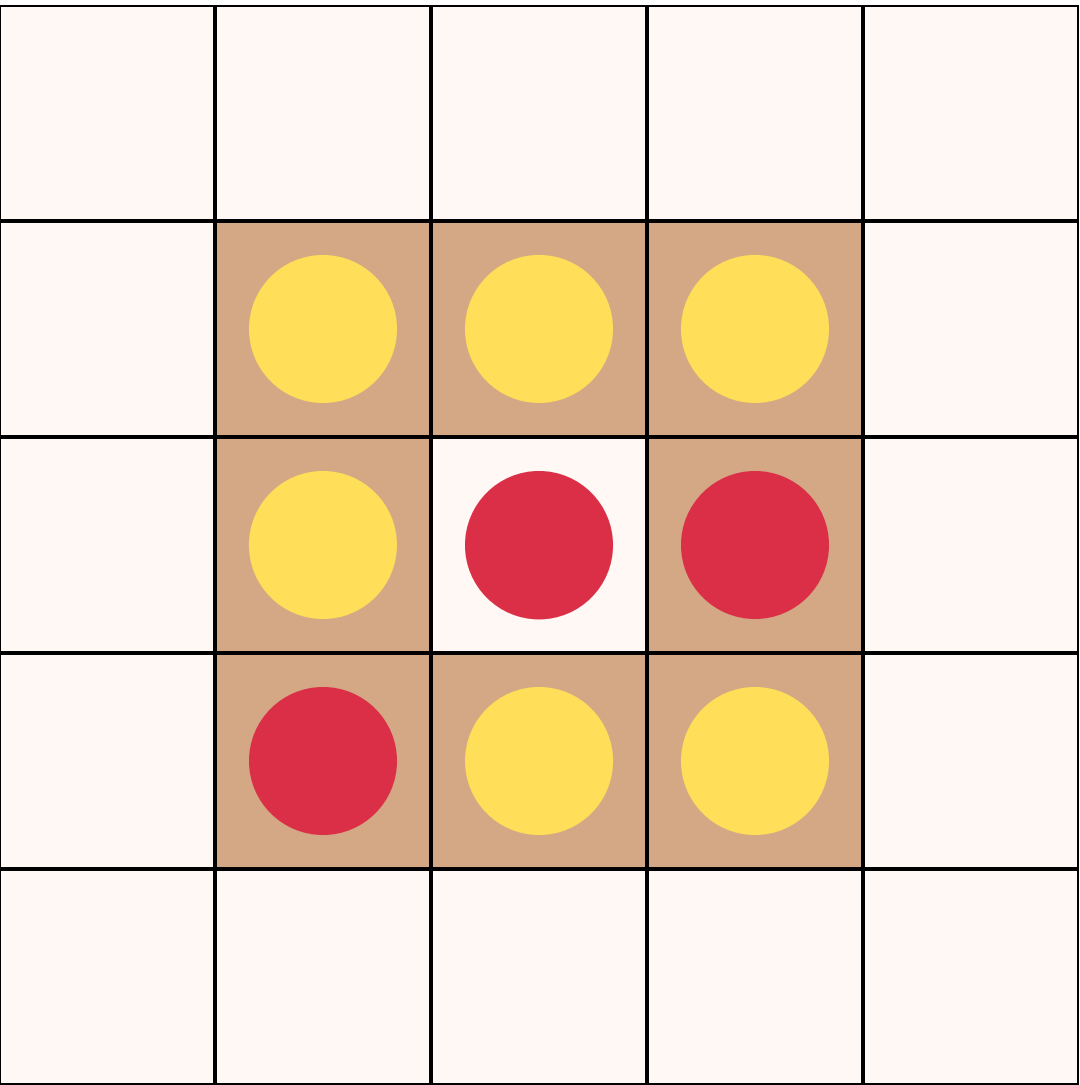
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Agents are of two *types*, Red and Yellow.

Each agent has a *similarity threshold*  $\theta$  in  $[0, 1]$ .

At every time-step, the *similarity ratio*  $s_i$  of agent  $i$  is the proportion of agents in the Moore neighborhood of  $i$  of the same type as  $i$ .



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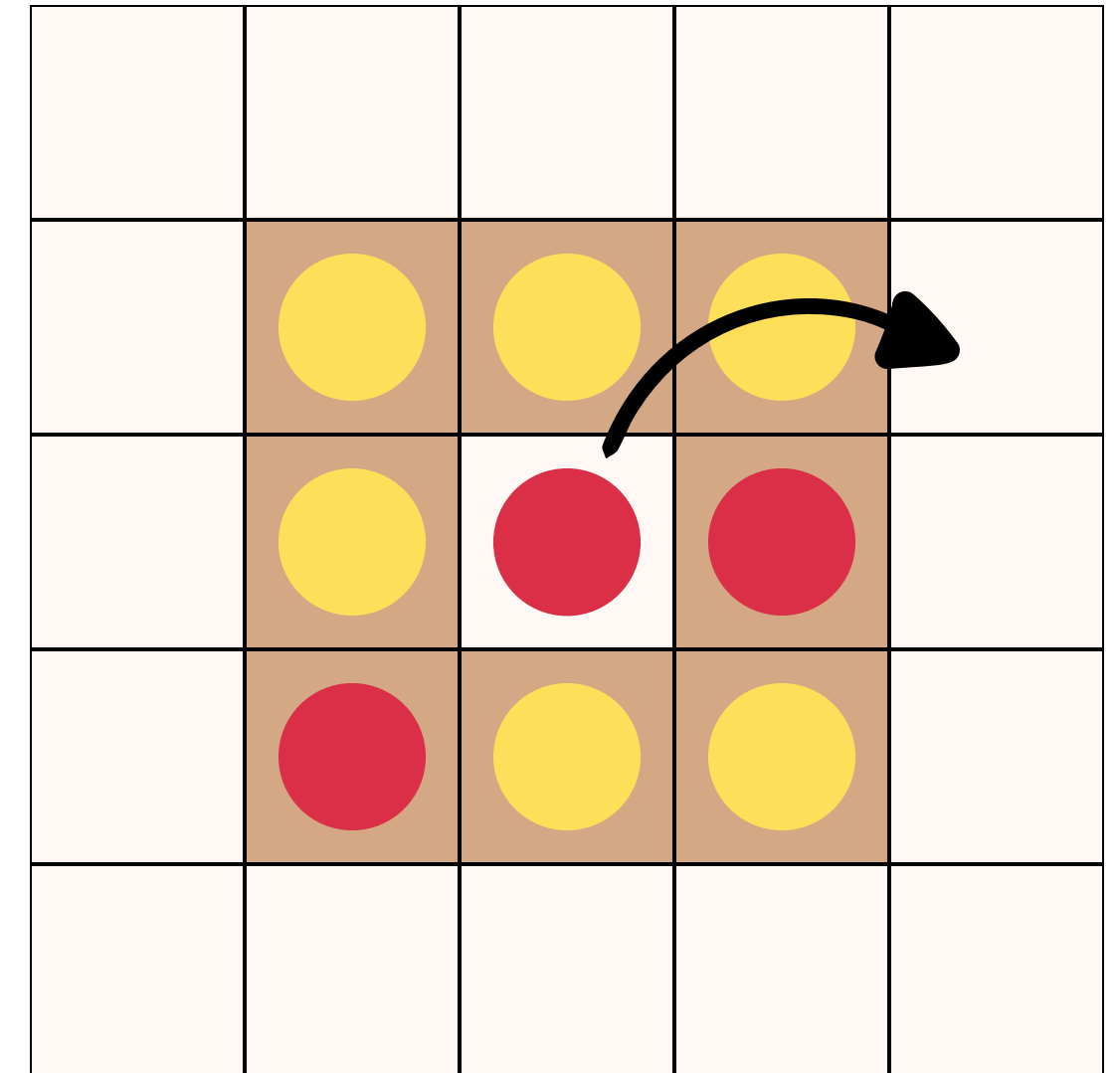
## DYNAMICS

If  $s_i < \theta$ , agent  $i$  is *unhappy*; otherwise,  $i$  is *happy*.

If agent  $i$  is happy, they do nothing.

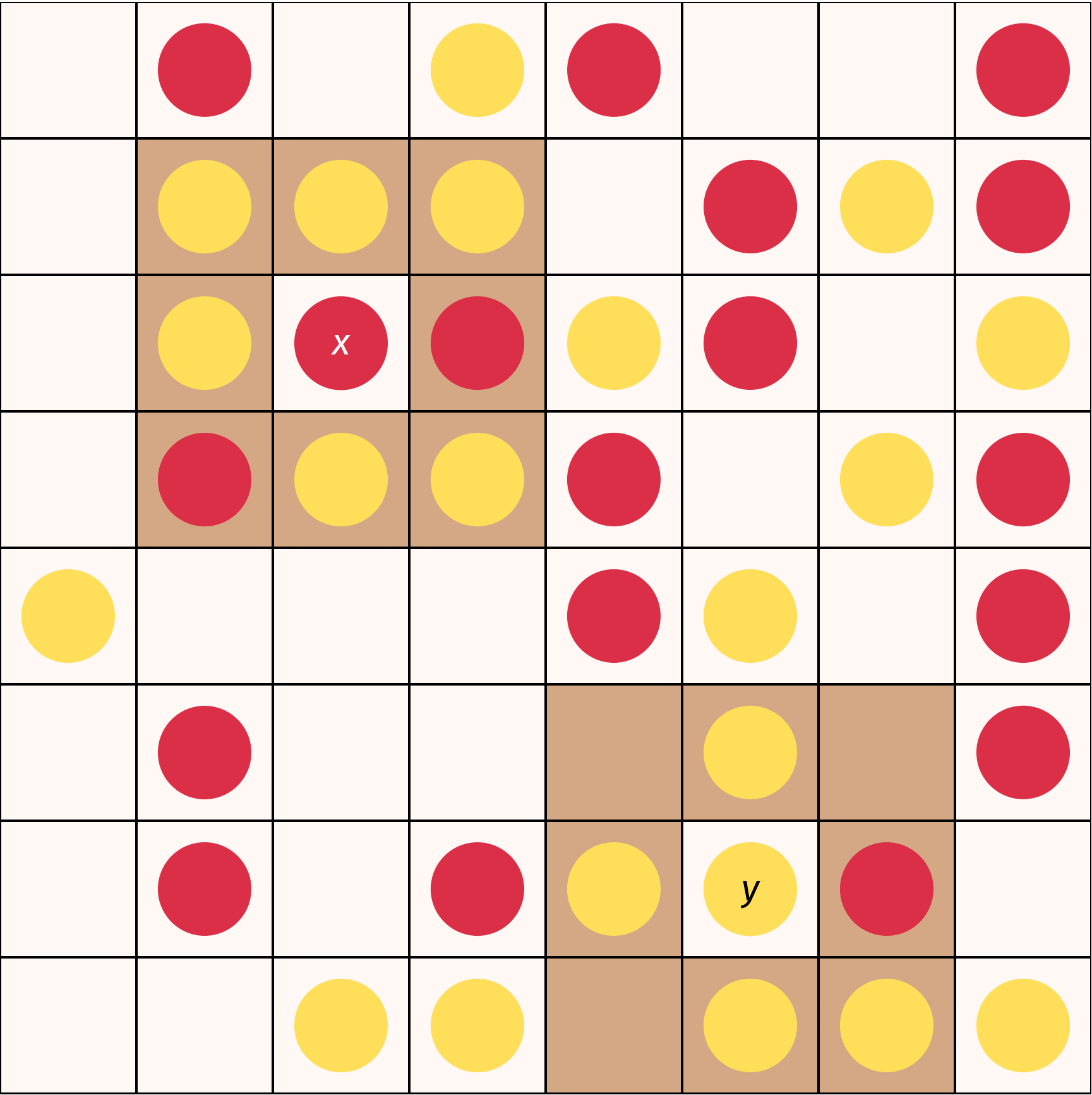
If agent  $i$  is unhappy, they move to a random empty cell.

Stop when all agents are happy, or when you get bored.



# EXAMPLE

Suppose agents have a similarity threshold of 0.3.

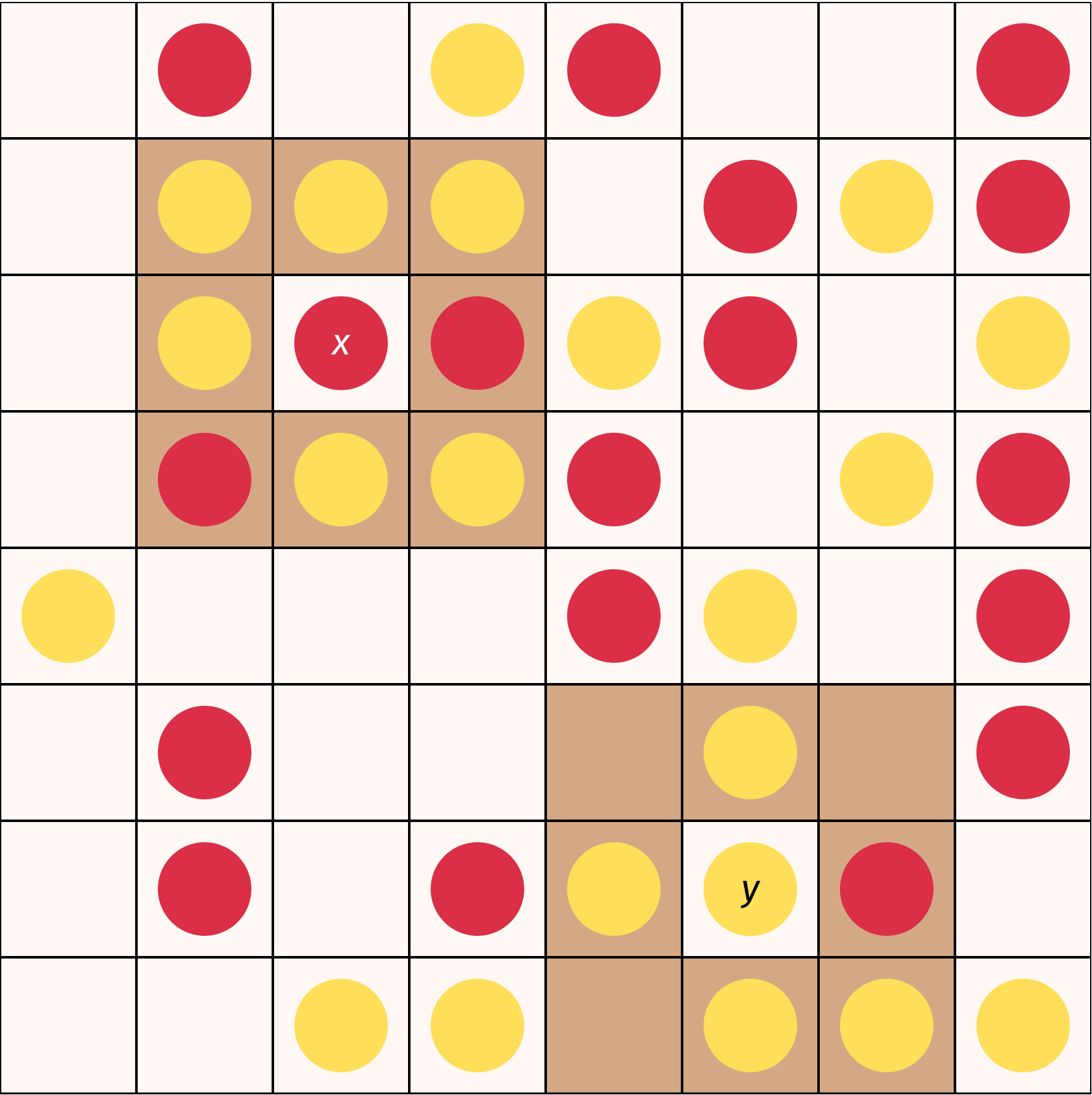


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## FIRST AGENT

The similarity index for agent  $x$  is  $2/8 = 0.25$ .



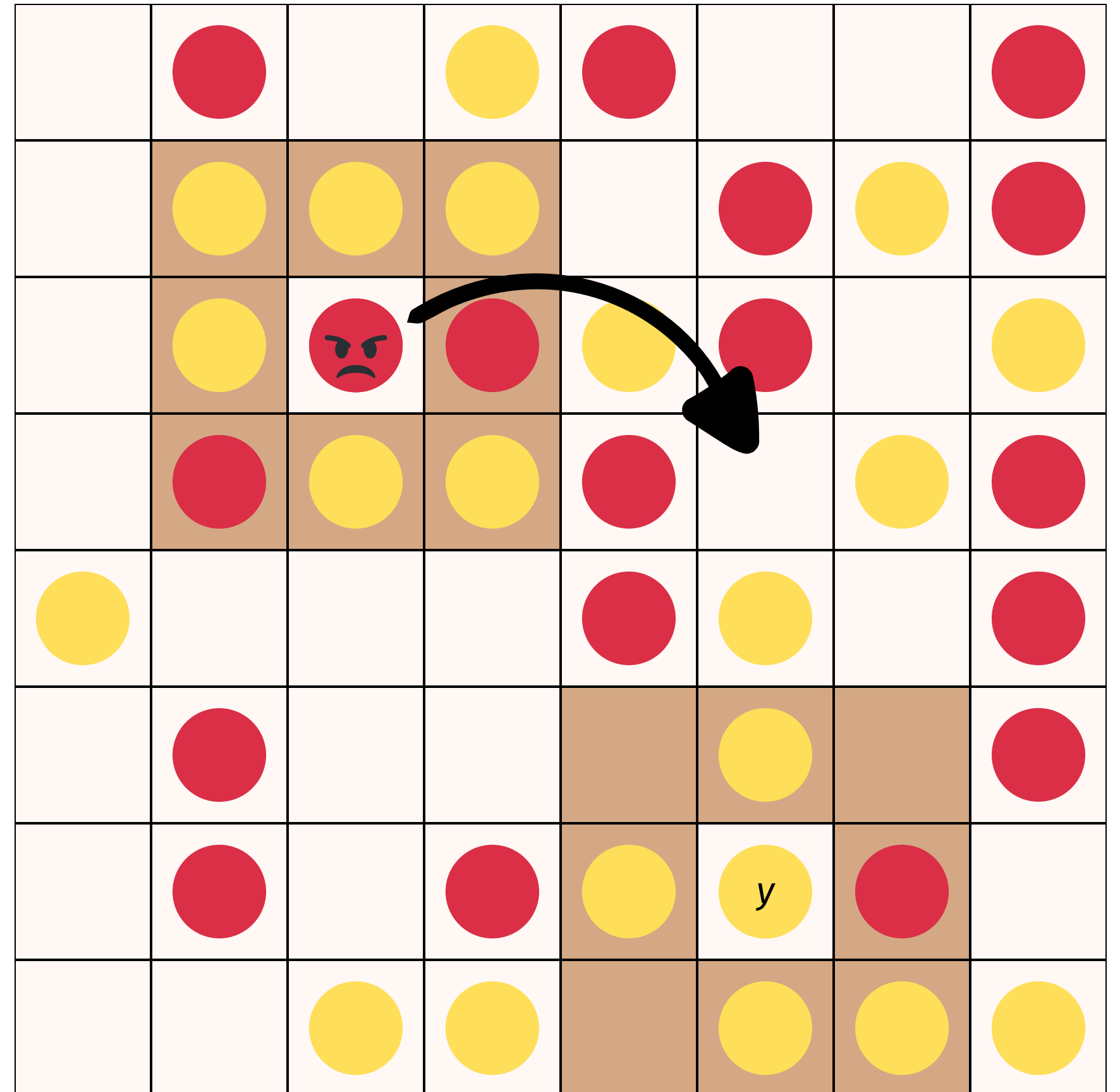
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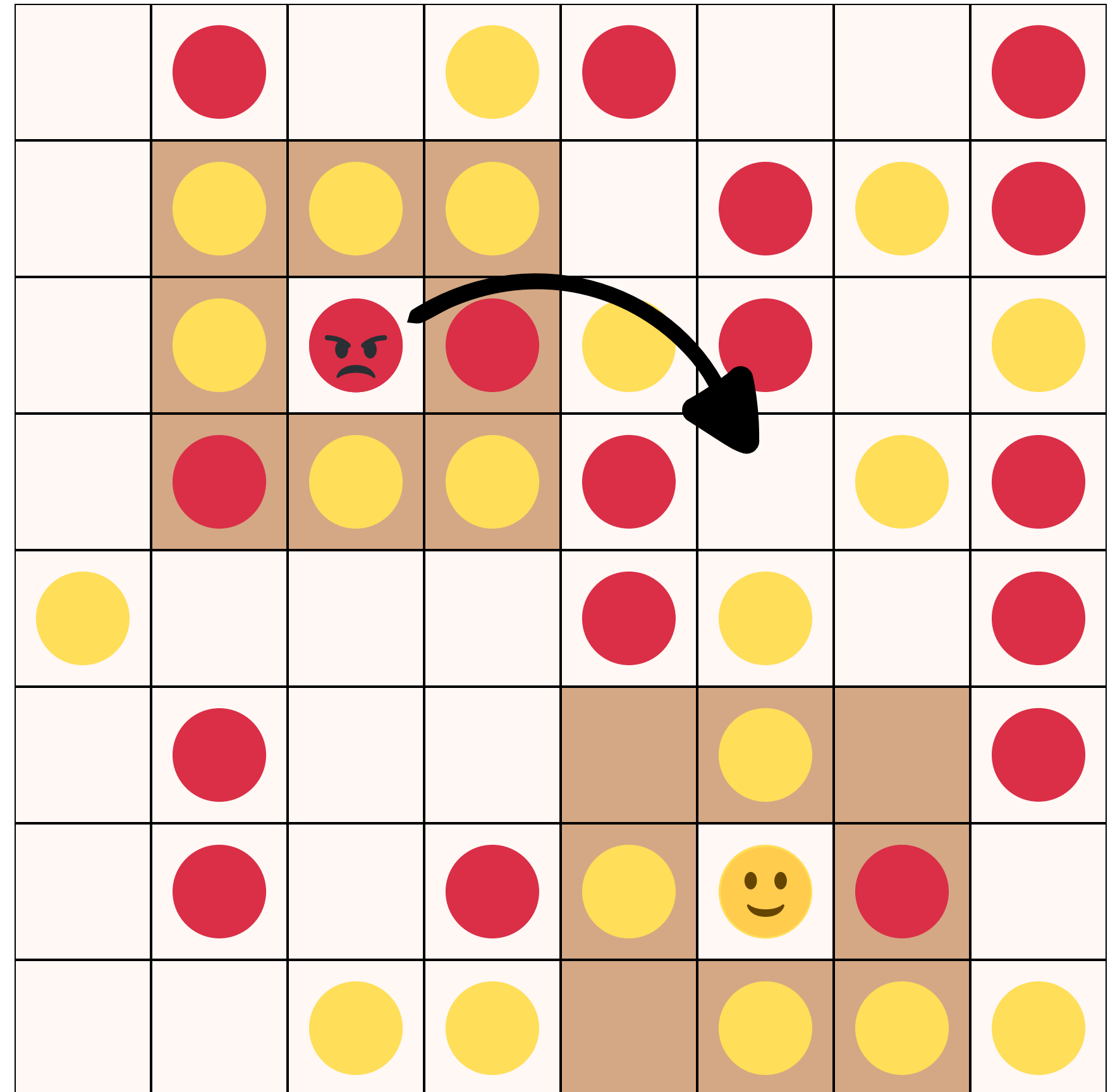
The similarity index for agent x is  $2/8 = 0.25$ .

Agent x is unhappy, hence moves.

## SECOND AGENT

The similarity index for agent y is  $3/5 = 0.6$ .

Agent y is happy, hence stays put.



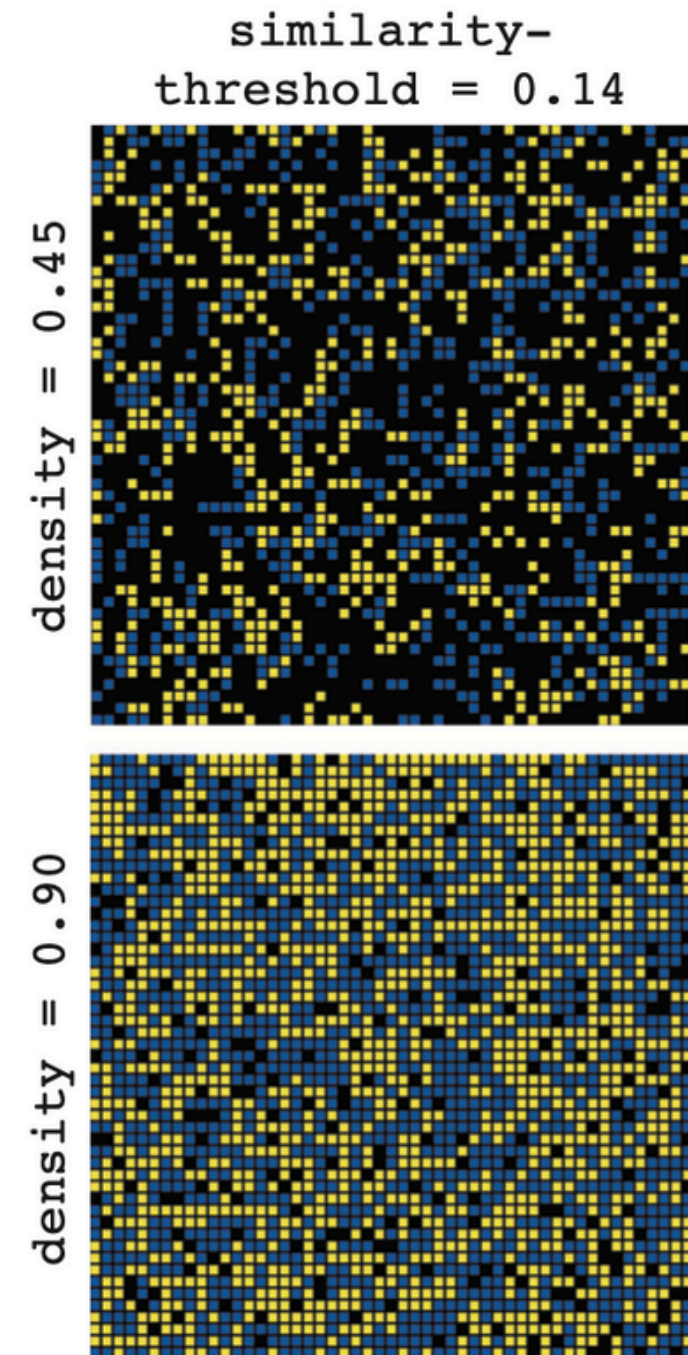


A threshold of 0.3 does not seem that discriminatory. Agents are not even keen on being in a majority!

# RUNNING THE MODEL UNDER VARIOUS PARAMETERS

## SMALL BIAS, BIG EFFECT

Interestingly, we see significant levels of segregation even for moderate values of the similarity threshold.

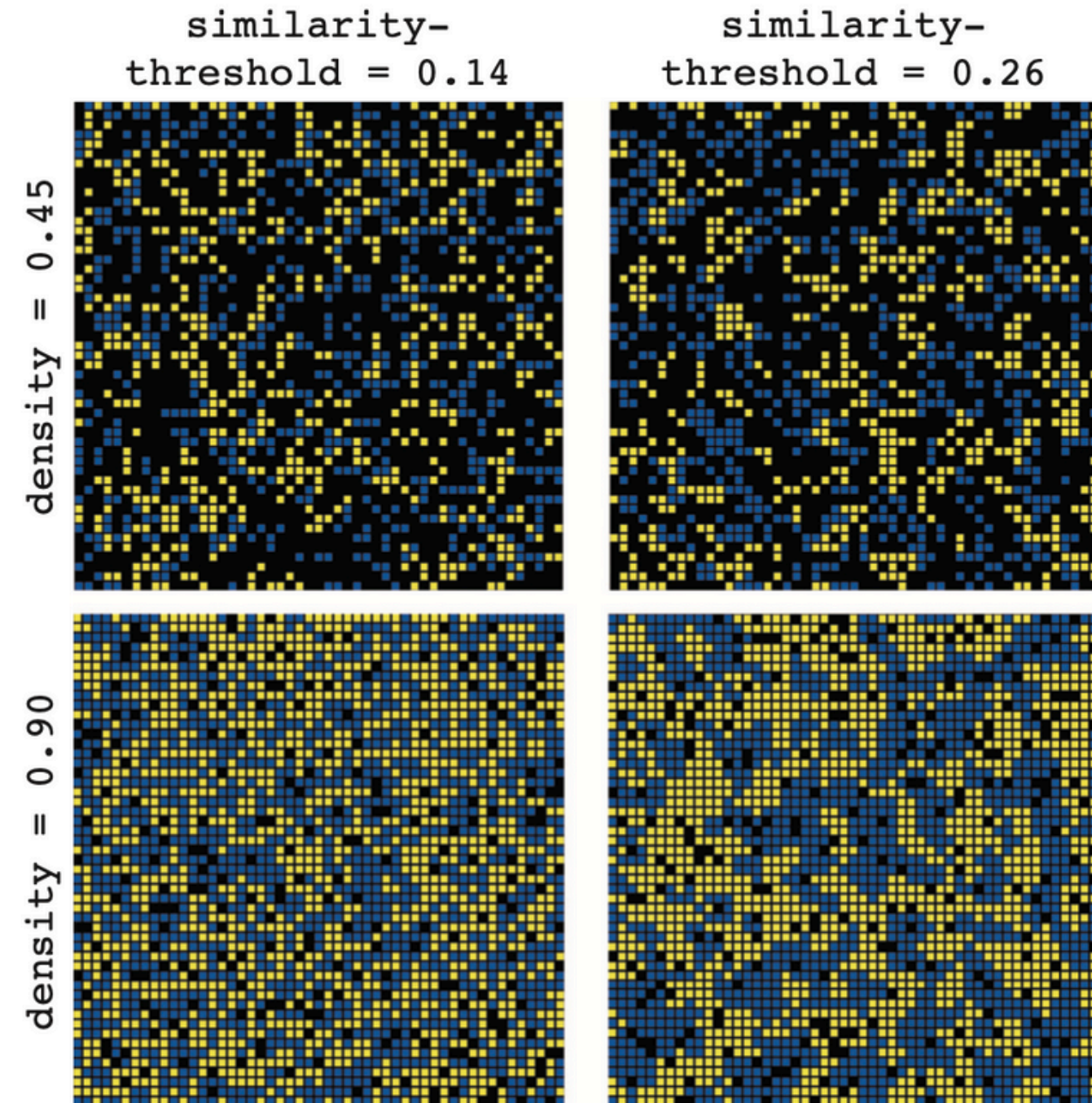


Smaldino, P. (2023). *Modeling Social Behavior. Mathematical and Agent-Based Models of Social Dynamics and Cultural Evolution*. Princeton University Press.

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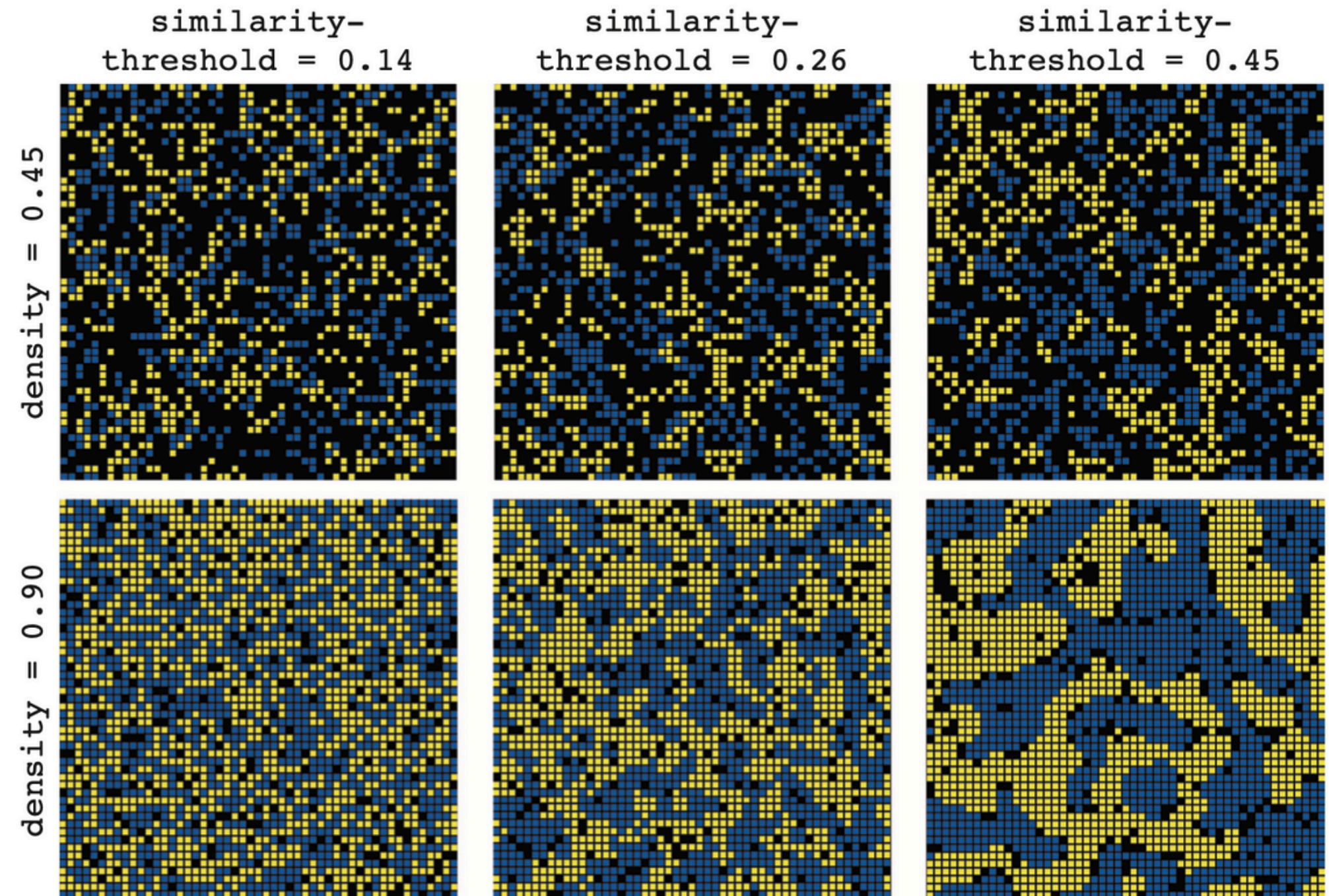
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NICKY CASE

I made a simulation of the Schelling model, which you can find online here:

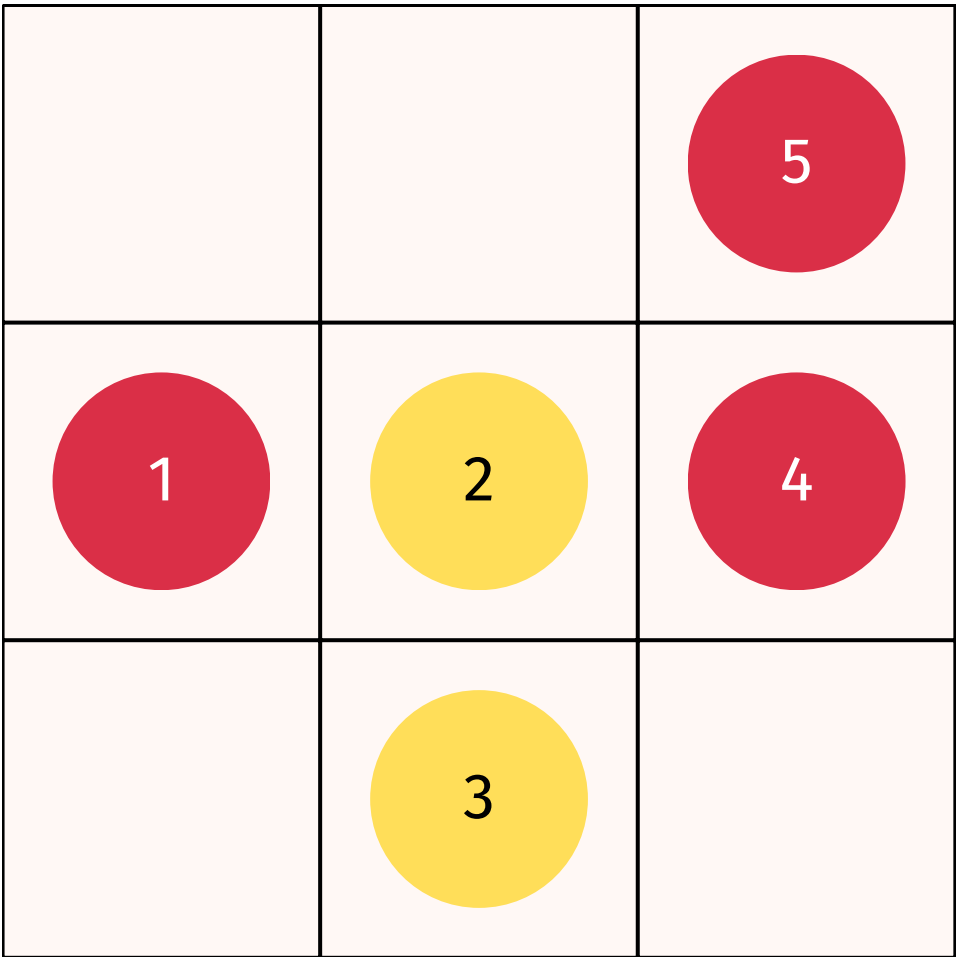
<https://ncase.me/polygons/>

Can we evaluate the results more  
precisely?

# QUANTITATIVE MEASURE OF SEGREGATION

## AVERAGE SIMILARITY

The *average similarity* is the average proportion of agents of the same type across the whole population.



AVERAGE SIMILARITY

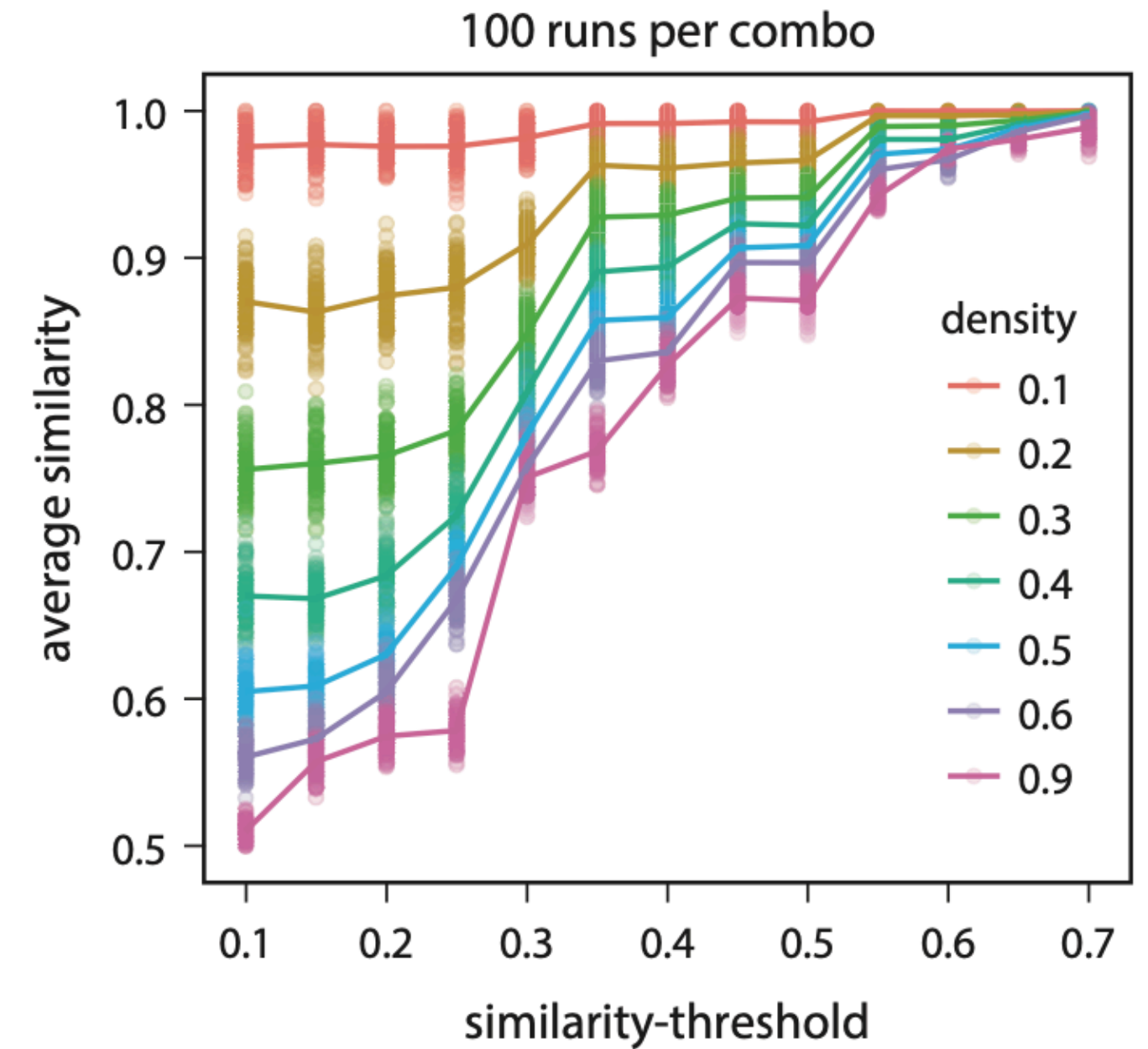
$$\frac{1}{5} \left( 0 + \frac{1}{4} + \frac{1}{3} + \frac{1}{3} + \frac{1}{2} \right) = 0.28$$



# BATCH EXPERIMENTS

## SMALL BIAS, BIG EFFECT

Segregation generally grows after a number of time-steps.



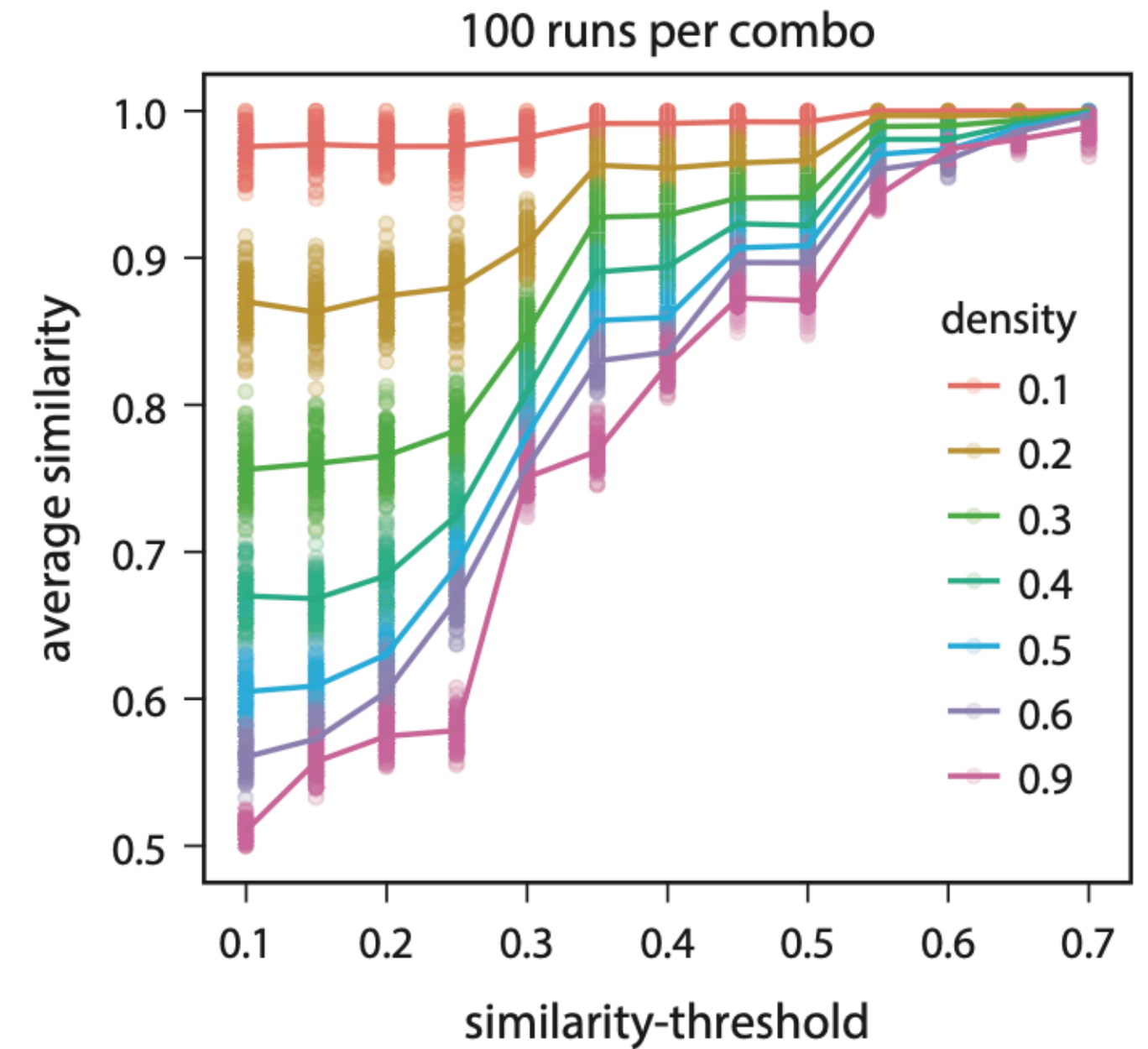


# BATCH EXPERIMENTS

## SMALL BIAS, BIG EFFECT

Segregation generally grows after a number of time-steps.

With increases more dramatic for higher densities.



Does the model always terminate?



H. PEYTON YOUNG

I studied the random process for the Schelling model on a line.

Peyton Young, H. (2001). *Individual Strategy and Social Structure: An Evolutionary Theory of Institutions*. Princeton University Press.

NICOLE IMMORLICA

We studied it on a ring.



Brandt, C., Immorlica, N., Kamath, G., & Kleinberg, R. (2012). An analysis of one-dimensional Schelling segregation. *Proceedings of STOC 12*.



LOUISE MOLITOR

We looked at a game-theoretic version of the model.

Chauhan, A., Lenzner, P., & Molitor, L. (2018). Schelling segregation with strategic agents. In *Algorithmic Game Theory* (pp. 137–149). Springer.

Any thoughts on this? Blindspots,  
ways to extend it?

Now that we've grown it, do we understand it?

Is segregation in the US a result of individual choices?



PAUL E. SMALDINO

No!

We've found only *one* mechanism that generates segregation.

But reality is always more complicated.

Smaldino, P. (2023). *Modeling Social Behavior. Mathematical and Agent-Based Models of Social Dynamics and Cultural Evolution*. Princeton University Press.