LMU LUDWIG-MAXIMILIANS-UNIVERSITÄT MÜNCHEN

TWEAKING DEMOCRACY: INNOVATIONS IN DEMOCRATIC DECISION MAKING

MATCHING

HOW TO FIND A PARTNER THAT WON'T LEAVE YOU

Adrian Haret
a.haret@lmu.de

January 10, 2024

Suppose we have some resources that need to be distributed to several people who want them.

What’s the best way to do this?

Suppose we have some resources that need to be distributed to several people who want them.

What’s the best way to do this?

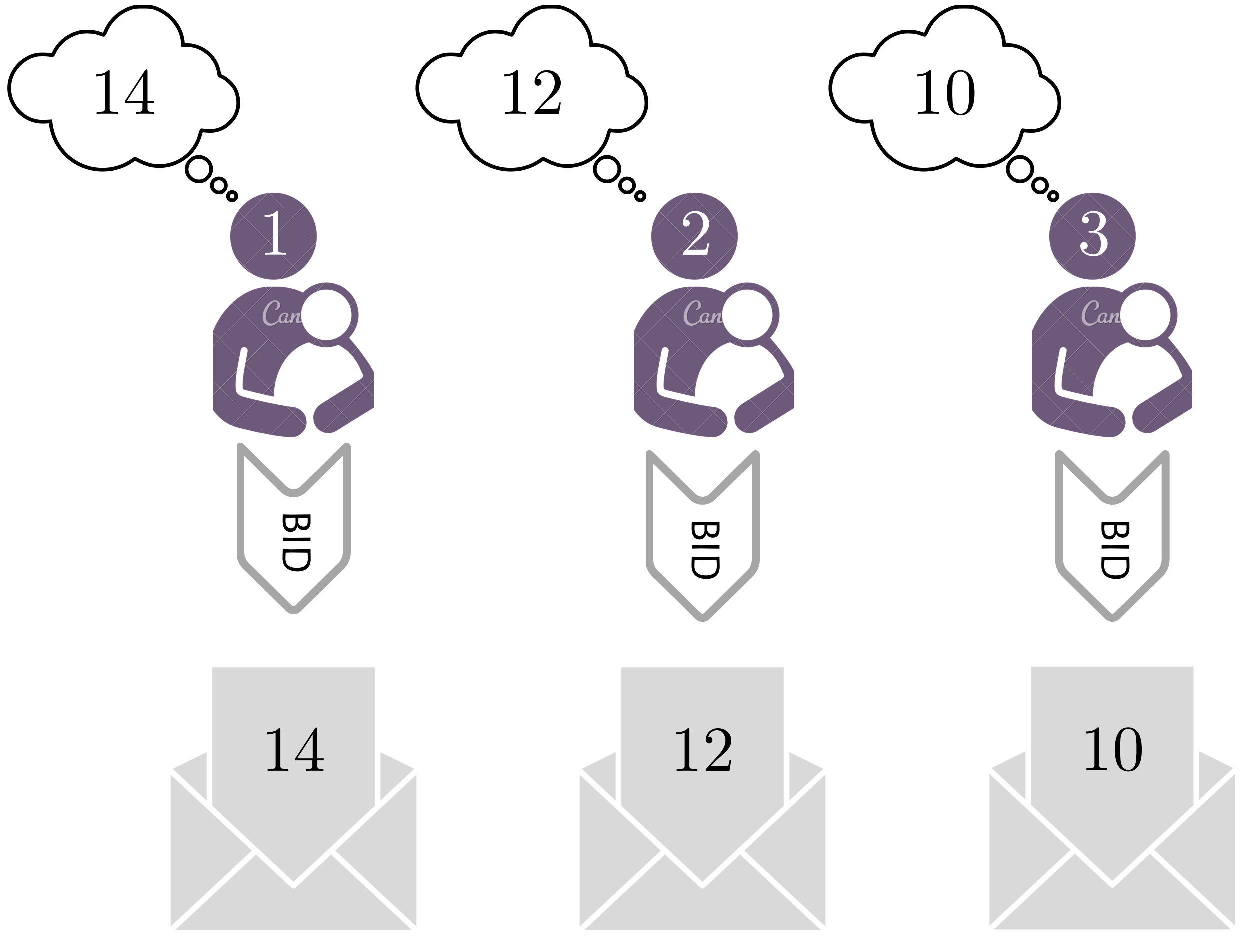
Often the best way is through sell the items through an auction.

But consider these scenarios.

ASSIGNING PLACES IN KINDERGARTEN

A kindergarten decides to auction off its available places.

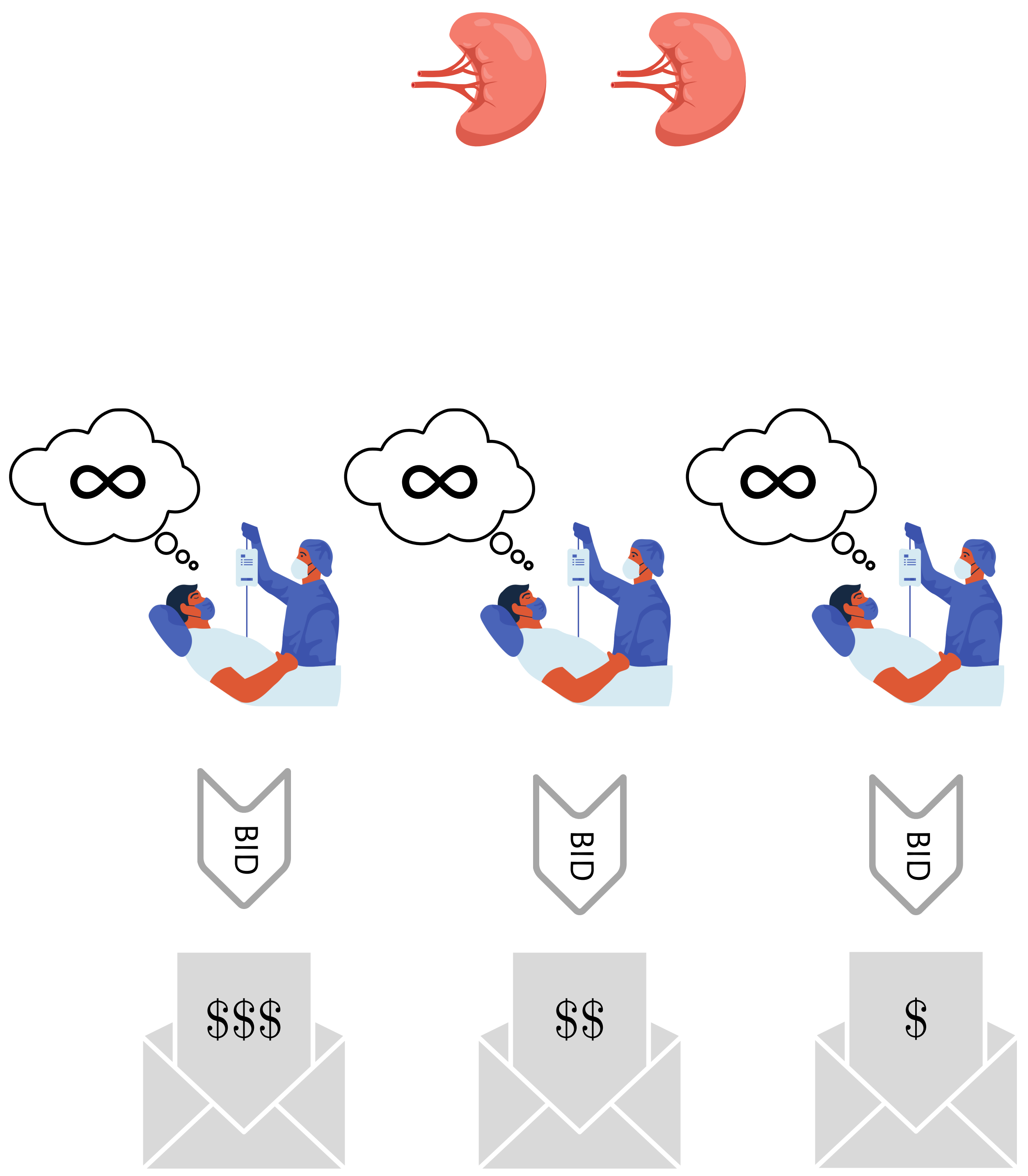
It invites an auction specialist to help it implement a mechanism that will squeeze out the most money from the interested parents.



WHO GETS A KIDNEY

There are three patients in dire need of a kidney transplant,
and two altruistic donors.

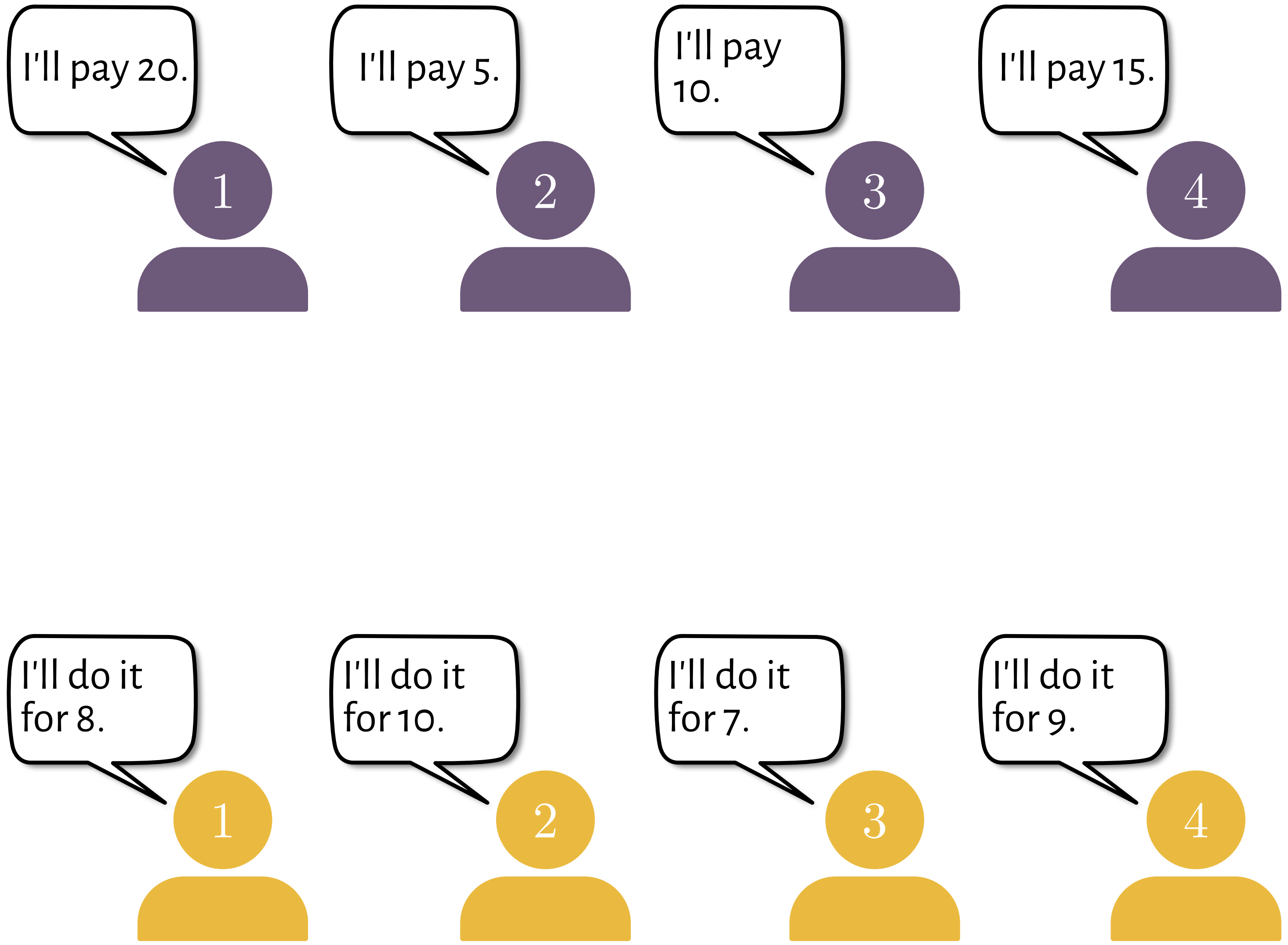
The hospital asks the patients to submit bids, and gives out the kidneys to the highest bidders...



PAIRING LECTURERS WITH STUDENTS

Every course needs to have a TA assigned to it.

Lecturers and TAs announce their prices and decide through a double auction.



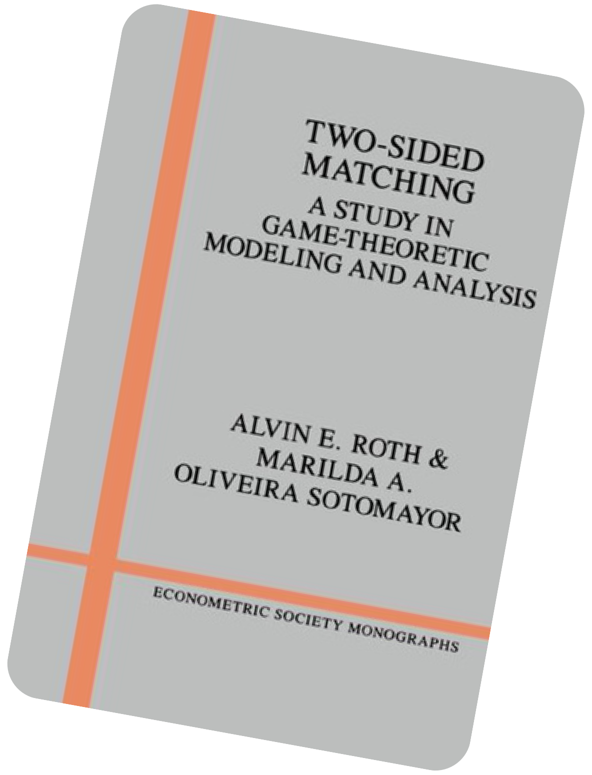
Ew.

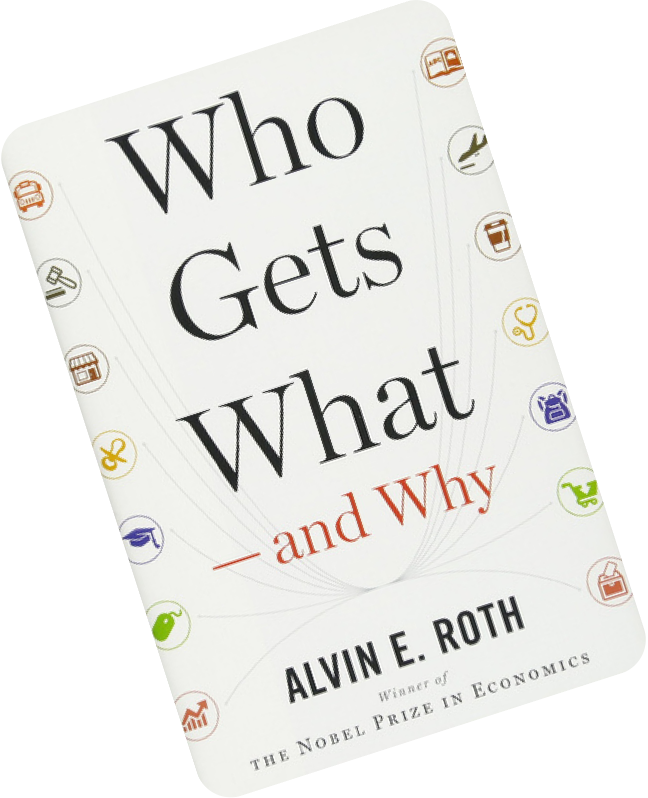
ALVIN E. ROTH

There are many situations where the use of payments is, for various reasons, *repugnant*.

MARILDA SOTOMAYOR

These are situations where you not only have to choose, but also *be chosen*.





Roth, A. E. & Sotomayor, M. (1990). *Two-Sided Matching: A Study in Game-Theoretic Modeling and Analysis*. Cambridge University Press.
Roth, A. E. (2015). *Who Gets What — and Why: The New Economics of Matchmaking and Market Design*. Houghton Mifflin Harcourt.

ALVIN E. ROTH

There are many situations where the use of payments is, for various reasons, *repugnant*.

MARILDA SOTOMAYOR

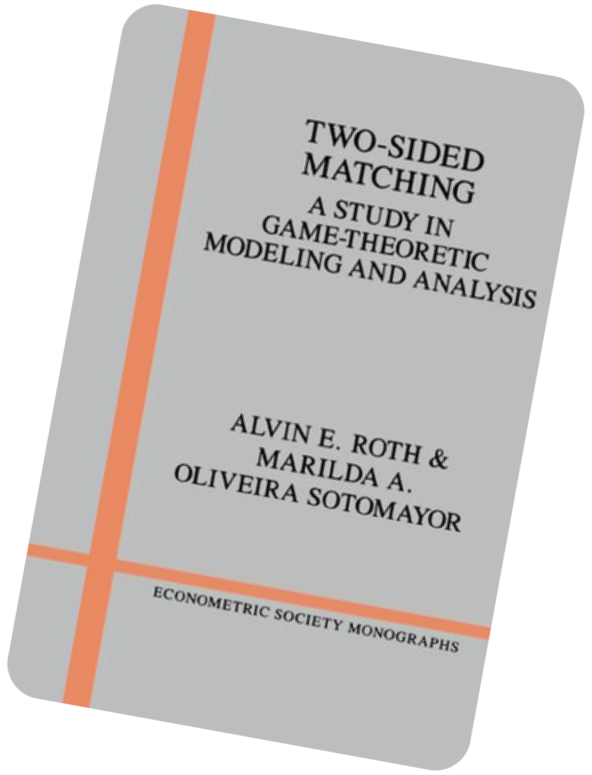
These are situations where you not only have to choose, but also *be chosen*.

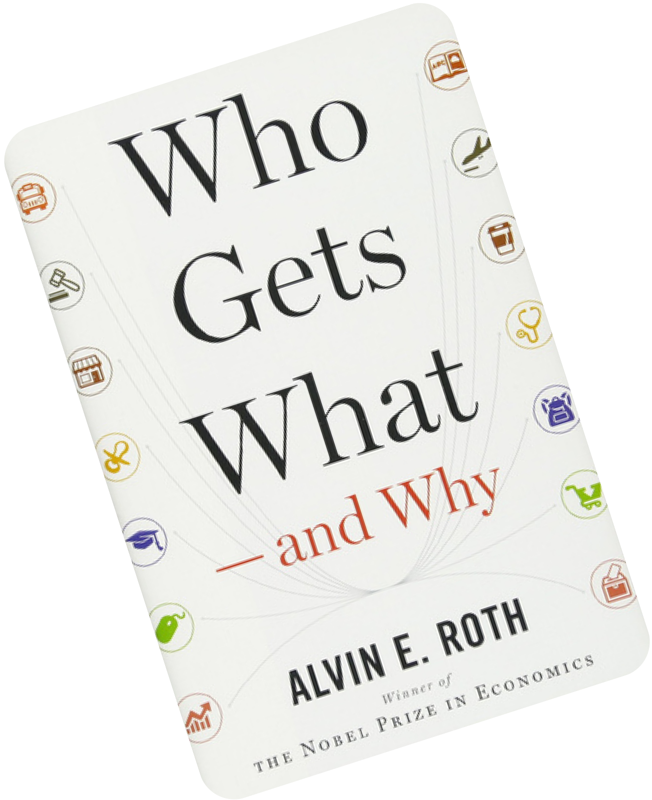
ALVIN E. ROTH

To be sure, exchanges still need to happen.

MARILDA SOTOMAYOR

But doing it with money goes against legal, ethical or societal norms.





Roth, A. E. & Sotomayor, M. (1990). *Two-Sided Matching: A Study in Game-Theoretic Modeling and Analysis*. Cambridge University Press.
Roth, A. E. (2015). *Who Gets What — and Why: The New Economics of Matchmaking and Market Design*. Houghton Mifflin Harcourt.

Assignment problems (e.g., matching schools to students, or patients to donors) arise where money does not–and, we feel, *should not*–play a role.

ALVIN E. ROTH

A salient example is assigning medical students to residency positions in hospitals.

From the turn of the century until 1945, the market suffered from a Prisoner's Dilemma problem in which competition by hospitals for interns manifested itself in a race to sign employment contracts earlier and earlier in a medical student's career.

Roth, A. E. (1984). The Evolution of the Labor Market for Medical Interns and Residents: A Case Study in Game Theory. *The Journal of Political Economy*, 92(6), 991–1016.

ALVIN E. ROTH

A salient example is assigning medical students to residency positions in hospitals.

From the turn of the century until 1945, the market suffered from a Prisoner's Dilemma problem in which competition by hospitals for interns manifested itself in a race to sign employment contracts earlier and earlier in a medical student's career.

This problem was successfully resolved in 1945 [by limiting information available about the students], but the market then suffered for several years from a "recontracting problem," [...] that put a premium on strategic behavior by market participants.

Roth, A. E. (1984). The Evolution of the Labor Market for Medical Interns and Residents: A Case Study in Game Theory. *The Journal of Political Economy*, 92(6), 991–1016.

ALVIN E. ROTH

A salient example is assigning medical students to residency positions in hospitals.

From the turn of the century until 1945, the market suffered from a Prisoner's Dilemma problem in which competition by hospitals for interns manifested itself in a race to sign employment contracts earlier and earlier in a medical student's career.

This problem was successfully resolved in 1945 [by limiting information available about the students], but the market then suffered for several years from a "recontracting problem," [...] that put a premium on strategic behavior by market participants.

The problem was that a student who was offered an internship at, say, [their] third-choice hospital, and who was informed [they] were an alternate (i.e., on a waiting list) at their second-choice hospital, would be inclined to wait as long as possible before accepting the position [they] had been offered, in the hope of eventually being offered a preferable position.

Roth, A. E. (1984). The Evolution of the Labor Market for Medical Interns and Residents: A Case Study in Game Theory. *The Journal of Political Economy*, 92(6), 991–1016.

The solution came by asking med students and hospitals to rank each other, then finding a matching through a centralized market mechanism.

The solution came by asking med students and hospitals to rank each other, then finding a matching through a centralized market mechanism.

And using a clever algorithm…

TWO-SIDED MATCHING

we can think of these as
hospitals and medical residents,
or student TAs and teachers

In the classic matching scenario there are disjoint sets L and R, of equal size, whose elements have preferences over each other, and need to be matched one to one.

we can think of these as
hospitals and medical residents,
or student TAs and teachers

In the classic matching scenario there are disjoint sets L and R, of equal size, whose elements have preferences over each other, and need to be matched one to one.

Ideally, matches are such that no two people would rather be matched with each other than with their current pairs.

DEFINITION (BLOCKING PAIRS)

A pair of elements (l, r), with l from L and r from R is a *blocking pair* for a prospective matching μ if l and r would rather be matched with each other than with their assigned matches.

DEFINITION (BLOCKING PAIRS)

A pair of elements (l, r), with l from L and r from R is a *blocking pair* for a prospective matching μ if l and r would rather be matched with each other than with their assigned matches.

In other words, (l, r) is a blocking pair if l and r prefer each other to their current matches.

$$L = \{a,\ b\},\ R = \{x,\ y\}.$$

FIND THE BLOCKING PAIRS

a prefers x to y

a	x	y		x	a	b
b	y	x		y	b	a

FIND THE BLOCKING PAIRS

$L = \{a,\ b\},\ R = \{x,\ y\}.$

Consider the matching
$\mu = \{(a,\ y),\ (b,\ x)\}.$

Blocking pairs?

a prefers *x* to *y*

a	x	y
b	y	x

x	a	b
y	b	a

FIND THE BLOCKING PAIRS

$L = \{a, b\}, R = \{x, y\}.$

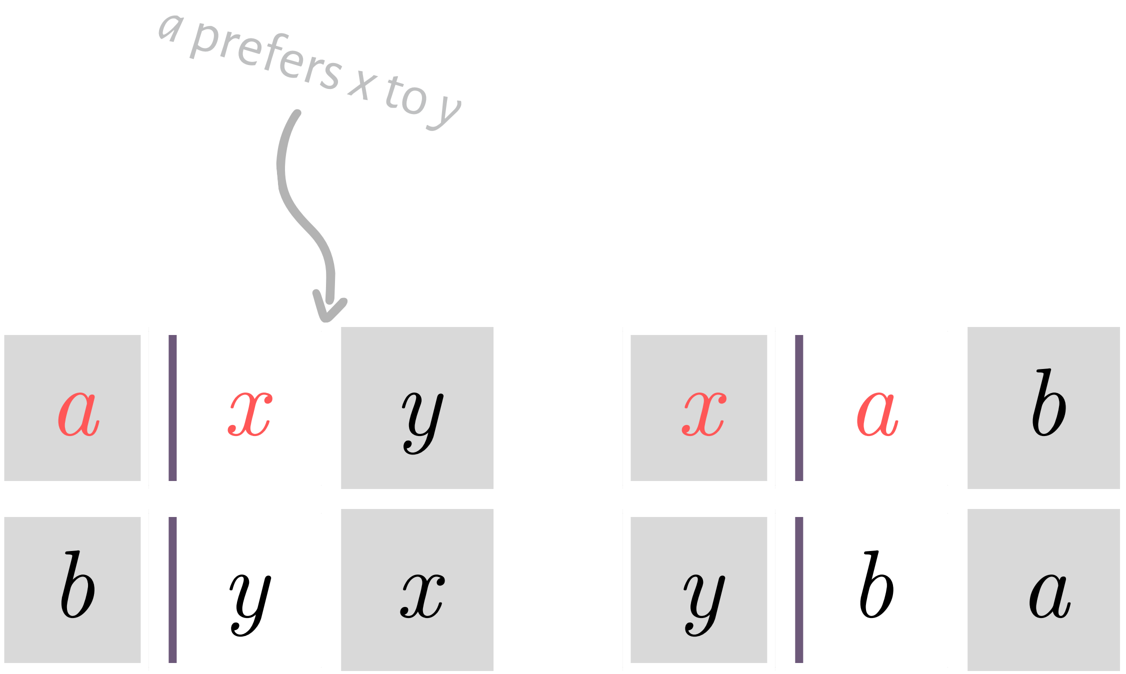
Consider the matching
$\mu = \{(a, y), (b, x)\}.$

Blocking pairs?

Yes!

For instance, (a, x).

a would rather be matched with *x* than with its current match *y*, and *x* would rather be matched with *a* than with its current match *b*.



FIND THE BLOCKING PAIRS

$L = \{a, b\}, R = \{x, y\}.$

Consider the matching
$\mu = \{(a, y), (b, x)\}$

Blocking pairs?

Yes!

For instance, (a, x).

And also: (b, y).

a would rather be matched with *x* than with its current match *y*, and *x* would rather be matched with *a* than with its current match *b*.

a prefers *x* to *y*

a	x	y
b	y	x

x	a	b
y	b	a

If there are blocking pairs markets can unravel, as agents create their own matchings outside the rules of the mechanism.

there are no participants who
would rather be matched to each
other than to their current matches

DEFINITION (STABLE MATCHING)

A matching μ is *stable* if there are no blocking pairs.

FIND THE STABLE MATCHING

$L = \{a,\ b,\ c\},\ R = \{x,\ y,\ z\}.$

$$\begin{array}{c|ccc} a & y & x & z \\ b & x & z & y \\ c & x & y & z \end{array} \qquad \begin{array}{c|ccc} x & a & c & b \\ y & c & a & b \\ z & a & c & b \end{array}$$

FIND THE STABLE MATCHING

$L = \{a,\ b,\ c\},\ R = \{x,\ y,\ z\}.$

Consider the matching
$\mu = \{(a,\ y),\ (b,\ z),\ (c,\ x)\}.$

Blocking pairs?

a	y	x	z
b	x	z	y
c	x	y	z

x	a	c	b
y	c	a	b
z	a	c	b

FIND THE STABLE MATCHING

$L = \{a,\ b,\ c\},\ R = \{x,\ y,\ z\}.$

Consider the matching
$\mu = \{(a,\ y),\ (b,\ z),\ (c,\ x)\}.$

Blocking pairs?

No! μ is stable.

a	y	x	z
b	x	z	y
c	x	y	z

x	a	c	b
y	c	a	b
z	a	c	b

Stability is a basic form of safety for participating in the market.

ALVIN E. ROTH

But do stable matchings always exist?

MARILDA SOTOMAYOR

And can we figure out if they do efficiently?

ALVIN E. ROTH

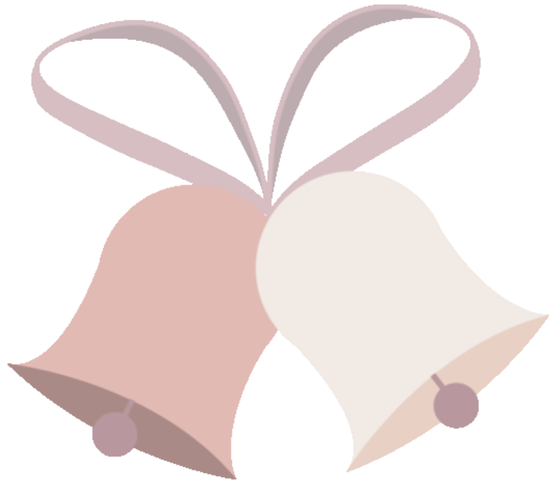
But do stable matchings always exist?

MARILDA SOTOMAYOR

And can we figure out if they do efficiently?

DAVID GALE

Yes.



LLOYD SHAPLEY

And yes.

here, *L*-agents are proposers; the *R*-proposing version is analogous

DEFINITION (DEFERRED ACCEPTANCE ALGORITHM)

A matching is constructed iteratively, over a number of rounds.

At round 1, each *L*-agent approaches their favorite *R*-agent and proposes a match. *R*-agents tentatively accept the best offer received, and reject all other offers.

At every round $k > 1$, each *L*-agent who got rejected approaches the next preferred *R*-agent who has not rejected them yet. Each *R*-agent tentatively accepts the best offer received so far and rejects inferior offers.

Stop when no new proposals are made.

DEFERRED ACCEPTANCE IN ACTION

$$
\begin{array}{c|ccc}
a & y & x & z \\
b & x & z & y \\
c & x & y & z
\end{array}
\qquad
\begin{array}{c|ccc}
x & a & c & b \\
y & c & a & b \\
z & a & c & b
\end{array}
$$

DEFERRED ACCEPTANCE IN ACTION

Round 1

a proposes to *y*

$$\begin{array}{c|ccc} a & y & x & z \\ b & x & z & y \\ c & x & y & z \end{array} \qquad \begin{array}{c|ccc} x & a & c & b \\ y & c & a & b \\ z & a & c & b \end{array}$$

Round 1

a proposes to *y*

y says yes → tentative pairing (*a*, *y*)

DEFERRED ACCEPTANCE IN ACTION

a	y	x	z	x	a	c	b
b	x	z	y	y	c	a	b
c	x	y	z	z	a	c	b

DEFERRED ACCEPTANCE IN ACTION

Round 1

a proposes to *y*

y says yes → tentative pairing (*a*, *y*)

b proposes to *x*

x says yes → tentative pairing (*b*, *x*)

a	y	x	z
b	x	z	y
c	x	y	z

x	a	c	b
y	c	a	b
z	a	c	b

DEFERRED ACCEPTANCE IN ACTION

Round 1

a proposes to *y*

y says yes → tentative pairing (*a*, *y*)

b proposes to *x*

x says yes → tentative pairing (*b*, *x*)

a	y	x	z		x	a	c	b
b	x	z	y		y	c	a	b
c	x	y	z		z	a	c	b

DEFERRED ACCEPTANCE IN ACTION

Round 1

a proposes to *y*

y says yes → tentative pairing (*a*, *y*)

b proposes to *x*

x says yes → tentative pairing (*b*, *x*)

c proposes to *x*

a	y	x	z	x	a	c	b
b	x	z	y	y	c	a	b
c	x	y	z	z	a	c	b

DEFERRED ACCEPTANCE IN ACTION

Round 1

a proposes to *y*

y says yes → tentative pairing (*a*, *y*)

b proposes to *x*

x says yes → tentative pairing (*b*, *x*)

c proposes to *x*

x says yes → tentative pairing (*c*, *x*)

a	y	x	z	x	a	c	b
b	x	z	y	y	c	a	b
c	x	y	z	z	a	c	b

DEFERRED ACCEPTANCE IN ACTION

x drops previous pairing *b*, since they like *c* more than *b*!

Round 1

- *a* proposes to *y*
  - *y* says yes → tentative pairing (*a*, *y*)
- *b* proposes to *x*
  - *x* says yes → tentative pairing (*b*, *x*)
- *c* proposes to *x*
  - *x* says yes → tentative pairing (*c*, *x*)

a	y	x	z	x	a	c	b
b	x	z	y	y	c	a	b
c	x	y	z	z	a	c	b

DEFERRED ACCEPTANCE IN ACTION

Round 1

a proposes to *y*

y says yes → tentative pairing (*a*, *y*)

b is now out of a match

b proposes to *x*

x says yes → tentative pairing (*b*, *x*)

c proposes to *x*

x drops previous pairing *b*, since they like *c* more than *b*!

x says yes → tentative pairing (*c*, *x*)

a	y	x	z	x	a	c	b
b	x	z	y	y	c	a	b
c	x	y	z	z	a	c	b

DEFERRED ACCEPTANCE IN ACTION

x drops previous pairing *b*, since they like *c* more than *b*!

Round 1

a proposes to *y*

y says yes → tentative pairing (*a*, *y*)

b proposes to *x*

x says yes → tentative pairing (*b*, *x*)

c proposes to *x*

x says yes → tentative pairing (*c*, *x*)

b is now out of a match

a	y	x	z
b	~~x~~	z	y
c	x	y	z

x	a	c	~~b~~
y	c	a	b
z	a	c	b

b has been rejected by *x*, so we remove *x* from *b*'s order and we need another round to find a match for *b*

DEFERRED ACCEPTANCE IN ACTION

x drops previous pairing *b*, since they like *c* more than *b*!

Round 1

a proposes to *y*

y says yes → tentative pairing (*a*, *y*)

b proposes to *x*

b is now out of a match

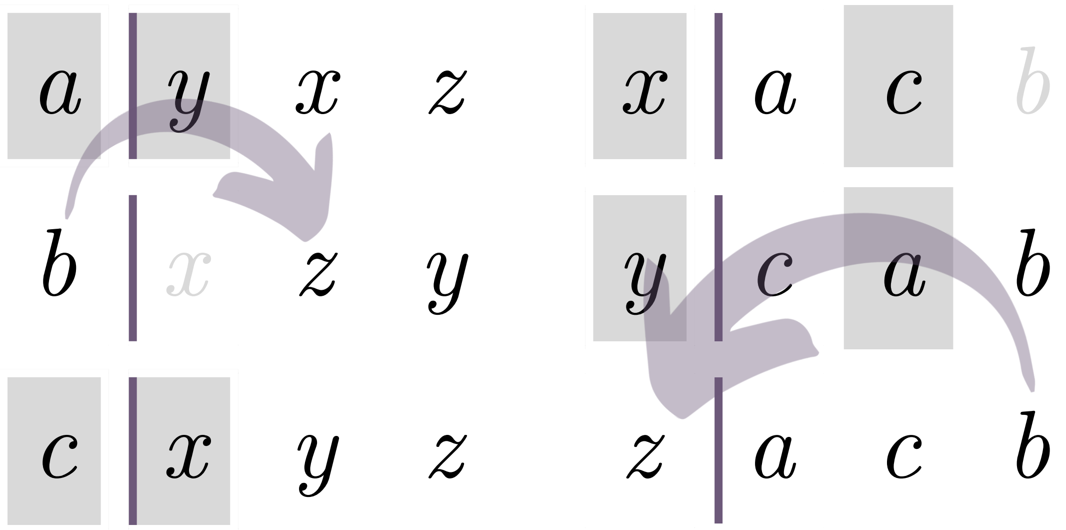
x says yes → tentative pairing (*b*, *x*)

c proposes to *x*

x says yes → tentative pairing (*c*, *x*)

Round 2

b proposes to *z*



DEFERRED ACCEPTANCE IN ACTION

x drops previous pairing b, since they like c more than b!

z has nothing better going on

Round 1

a proposes to *y*

y says yes → tentative pairing (*a*, *y*)

b proposes to *x*

x says yes → tentative pairing (*b*, *x*)

c proposes to *x*

x says yes → tentative pairing (*c*, *x*)

Round 2

b proposes to *z*

z says yes → tentative pairing (*b*, *z*)

b is now out of a match

a	y	x	z
b	x	z	y
c	x	y	z

x	a	c	b
y	c	a	b
z	a	c	b

DEFERRED ACCEPTANCE IN ACTION

x drops previous pairing *b*, since they like *c* more than *b*!

Round 1

a proposes to *y*

y says yes → tentative pairing (*a*, *y*)

b proposes to *x*

x says yes → tentative pairing (*b*, *x*)

b is now out of a match

c proposes to *x*

x says yes → tentative pairing (*c*, *x*)

a	y	x	z
b	x	z	y
c	x	y	z

x	a	c	b
y	c	a	b
z	a	c	b

Round 2

b proposes to *z*

z says yes → tentative pairing (*b*, *z*)

z has nothing better going on

We're done!

The Deferred Acceptance algorithm is nice.

THEOREM (GALE & SHAPLEY, 1962)

The Deferred Acceptance algorithm terminates in a finite number of steps, and outputs a stable matching.

Gale, D., Shapley, L. S. (1962). College Admissions and the Stability of Marriage. *American Mathematical Monthly*. 69 (1): 9–14.

THEOREM (GALE & SHAPLEY, 1962)
The Deferred Acceptance algorithm terminates in a finite number of steps, and outputs a stable matching.

PROOF (TERMINATION)
The number of students and teachers is finite.

In non-terminating rounds there is at least on proposal rejected, and the proposing side does not repeat proposals.

Sooner or later we run out of proposals to be made.

At which point algorithm terminates.

Gale, D., Shapley, L. S. (1962). College Admissions and the Stability of Marriage. *American Mathematical Monthly*. 69 (1): 9–14.

In fact, if L and R both have n elements, the maximum number of proposals is n^2-2n+2.

THEOREM (GALE & SHAPLEY, 1962)

The Deferred Acceptance algorithm terminates in a finite number of steps, and outputs a stable matching.

PROOF (STABLE MATCHING)

Consider the matching on which the algorithm terminates.

Gale, D., Shapley, L. S. (1962). College Admissions and the Stability of Marriage. *American Mathematical Monthly*. 69 (1): 9–14.

THEOREM (GALE & SHAPLEY, 1962)

The Deferred Acceptance algorithm terminates in a finite number of steps, and outputs a stable matching.

PROOF (STABLE MATCHING)

Consider the matching on which the algorithm terminates.

Suppose there exists a blocking pair (ℓ, r'), where the matching matches (ℓ, r) and (ℓ', r').

Gale, D., Shapley, L. S. (1962). College Admissions and the Stability of Marriage. *American Mathematical Monthly*. 69 (1): 9–14.

THEOREM (GALE & SHAPLEY, 1962)

The Deferred Acceptance algorithm terminates in a finite number of steps, and outputs a stable matching.

PROOF (STABLE MATCHING)

Consider the matching on which the algorithm terminates.

Suppose there exists a blocking pair (ℓ, r'), where the matching matches (ℓ, r) and (ℓ', r').

We infer from this that $r' \succ_\ell r$.

Gale, D., Shapley, L. S. (1962). College Admissions and the Stability of Marriage. *American Mathematical Monthly*. 69 (1): 9–14.

THEOREM (GALE & SHAPLEY, 1962)

The Deferred Acceptance algorithm terminates in a finite number of steps, and outputs a stable matching.

PROOF (STABLE MATCHING)

Consider the matching on which the algorithm terminates.

Suppose there exists a blocking pair (ℓ, r'), where the matching matches (ℓ, r) and (ℓ', r').

We infer from this that $r' \succ_\ell r$.

But then ℓ must have proposed to r' and r' must have said no, which means $\ell' \succ_{r'} \ell$.

Gale, D., Shapley, L. S. (1962). College Admissions and the Stability of Marriage. *American Mathematical Monthly*. 69 (1): 9–14.

THEOREM (GALE & SHAPLEY, 1962)

The Deferred Acceptance algorithm terminates in a finite number of steps, and outputs a stable matching.

PROOF (STABLE MATCHING)

Consider the matching on which the algorithm terminates.

Suppose there exists a blocking pair (ℓ, r'), where the matching matches (ℓ, r) and (ℓ', r').

We infer from this that $r' \succ_\ell r$.

But then ℓ must have proposed to r' and r' must have said no, which means $\ell' \succ_{r'} \ell$.

But wait! We started from (ℓ, r') being a blocking pair, which implies that $\ell \succ_{r'} \ell'$.

Gale, D., Shapley, L. S. (1962). College Admissions and the Stability of Marriage. *American Mathematical Monthly*. 69 (1): 9–14.

THEOREM (GALE & SHAPLEY, 1962)

The Deferred Acceptance algorithm terminates in a finite number of steps, and outputs a stable matching.

PROOF (STABLE MATCHING)

Consider the matching on which the algorithm terminates.

Suppose there exists a blocking pair (ℓ, r'), where the matching matches (ℓ, r) and (ℓ', r').

We infer from this that $r' \succ_\ell r$.

But then ℓ must have proposed to r' and r' must have said no, which means $\ell' \succ_{r'} \ell$.

But wait! We started from (ℓ, r') being a blocking pair, which implies that $\ell \succ_{r'} \ell'$.

Contradiction.

Gale, D., Shapley, L. S. (1962). College Admissions and the Stability of Marriage. *American Mathematical Monthly*. 69 (1): 9–14.

ALVIN E. ROTH

So the proposing side keeps making offers, and the other side rejects them when better offers come along.

MARILDA SOTOMAYOR

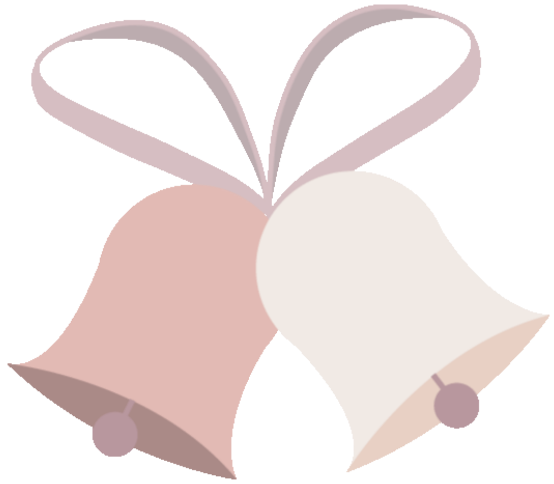
Sounds like the receiving side is getting the better deal here.

DAVID GALE

Actually, this arrangement favors the proposing side.

LLOYD SHAPLEY

They get the best matching they could possibly get.



R-optimal defined analogously

DEFINITION (OPTIMAL MATCHING)

A matching μ is *L-optimal* if every agent l in L ends up being matched with their most preferred achievable agent in R.

An agent r being *achievable for l* means, here, that there is some stable matching where l and r are matched.

OPTIMAL STABLE MATCHINGS

With L on the proposing side we get:

$$\mu = \{(a,\ y),\ (b,\ z),\ (c,\ x)\}.$$

a	y	x	z
b	x	z	y
c	x	y	z

x	a	c	b
y	c	a	b
z	a	c	b

OPTIMAL STABLE MATCHINGS

With L on the proposing side we get:

$$\mu = \{(a,\ y),\ (b,\ z),\ (c,\ x)\}.$$

a	y	x	z
b	x	z	y
c	x	y	z

x	a	c	b
y	c	a	b
z	a	c	b

With R on the proposing side we get:

$$\mu' = \{(a,\ x),\ (b,\ z),\ (c,\ y)\}.$$

a	y	x	z
b	x	z	y
c	x	y	z

x	a	c	b
y	c	a	b
z	a	c	b

OPTIMAL STABLE MATCHINGS

With L on the proposing side we get:

$$\mu = \{(a,\ y),\ (b,\ z),\ (c,\ x)\}.$$

a	y	x	z	x	a	c	b
b	x	z	y	y	c	a	b
c	x	y	z	z	a	c	b

With R on the proposing side we get:

$$\mu' = \{(a,\ x),\ (b,\ z),\ (c,\ y)\}.$$

a	y	x	z	x	a	c	b
b	x	z	y	y	c	a	b
c	x	y	z	z	a	c	b

Achievable for a: $\{x,\ y,\ \ldots\}$.

OPTIMAL STABLE MATCHINGS

With L on the proposing side we get:

$$\mu = \{(a,\ y),\ (b,\ z),\ (c,\ x)\}.$$

a	y	x	z	x	a	c	b
b	x	z	y	y	c	a	b
c	x	y	z	z	a	c	b

With R on the proposing side we get:

$$\mu' = \{(a,\ x),\ (b,\ z),\ (c,\ y)\}.$$

a	y	x	z	x	a	c	b
b	x	z	y	y	c	a	b
c	x	y	z	z	a	c	b

Achievable for a: $\{x,\ y,\ \ldots\}$.

Is μ' L-optimal?

OPTIMAL STABLE MATCHINGS

With L on the proposing side we get:

$$\mu = \{(a,\ y),\ (b,\ z),\ (c,\ x)\}.$$

a	y	x	z
b	x	z	y
c	x	y	z

x	a	c	b
y	c	a	b
z	a	c	b

With R on the proposing side we get:

$$\mu' = \{(a,\ x),\ (b,\ z),\ (c,\ y)\}.$$

a	y	x	z
b	x	z	y
c	x	y	z

x	a	c	b
y	c	a	b
z	a	c	b

Achievable for a: $\{x,\ y,\ \ldots\}$.

Is μ' L-optimal? Clearly not: a can do better!

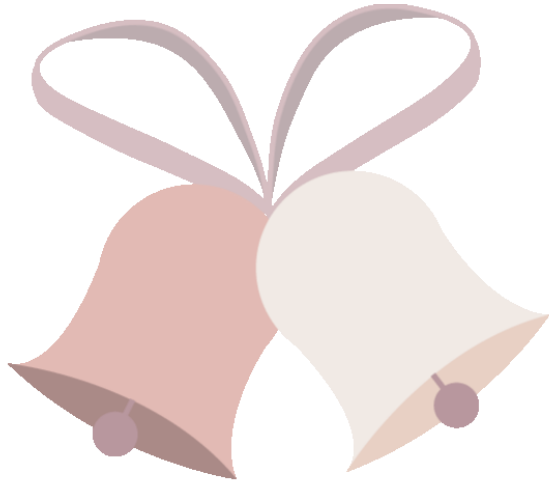
THEOREM (GALE & SHAPLEY, 1962)

The matching produced by the L-proposing version of the Deferred Acceptance algorithm is L-optimal.

Gale, D., Shapley, L. S. (1962). College Admissions and the Stability of Marriage. *American Mathematical Monthly*. 69 (1): 9–14.

ALVIN E. ROTH
What about the incentives?

MARILDA SOTOMAYOR
Is there any benefit to lying?



ALVIN E. ROTH

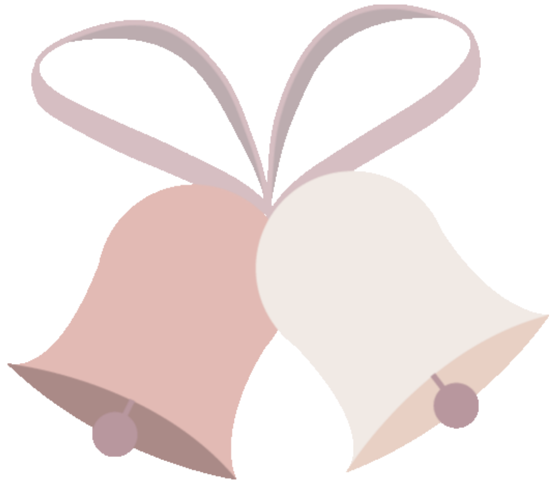
What about the incentives?

MARILDA SOTOMAYOR

Is there any benefit to lying?

DAVID GALE

It depends!



LLOYD SHAPLEY

Turns out the proposing side has no such incentive... but the other side does.

THEOREM (GALE & SHAPLEY, 1962)

There is no incentive for the proposing side in the Deferred Acceptance algorithm to lie.

This does not, however, hold for the other side.

Gale, D., Shapley, L. S. (1962). College Admissions and the Stability of Marriage. *American Mathematical Monthly*. 69 (1): 9–14.

DAVID GALE

Too bad.

LLOYD SHAPLEY

But maybe we can find some other mechanism that is stable and strategyproof for both sides.

DAVID GALE

Too bad.

LLOYD SHAPLEY

But maybe we can find some other mechanism that is stable and strategyproof for both sides.

ALVIN E. ROTH

Well, actually, we can't.

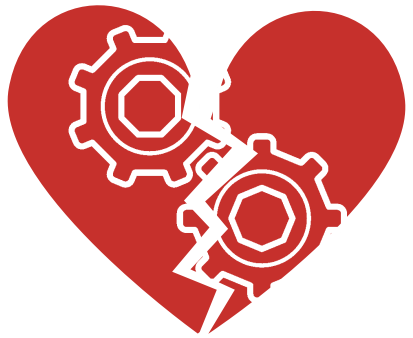
Roth, A. E. (1982). The economics of matching: Stability and incentives. *Mathematics of Operations Research* 7(4):617– 628.

DAVID GALE

Too bad.

LLOYD SHAPLEY

But maybe we can find some other mechanism that is stable and strategyproof for both sides.



ALVIN E. ROTH

Well, actually, we can't.

ULLE ENDRISS

And these days we can prove this sort of thing with the help of computers.

Roth, A. E. (1982). The economics of matching: Stability and incentives. *Mathematics of Operations Research* 7(4):617– 628.

Endriss, U. (2020). Analysis of One-to-One Matching Mechanisms via SAT Solving: Impossibilities for Universal Axioms. *AAAI 2020*: 1918-1925.

Stability, as mentioned earlier, is important: without it participants will not want to participate, and the market unravels.

Like the early versions of the residency matching matching programs.

Stability, as mentioned earlier, is important: without it participants will not want to participate, and the market unravels.

Like the early versions of the residency matching matching programs.

Which, by the way… whatever happened to those?

ALVIN E. ROTH

In 1951, a centralized procedure for matching residents to hospital was introduced.

It replaced a chaotic, non-centralized market.

By implementing a hospital-proposing Deferred Acceptance mechanism.

Extended to accommodate many-to-one matches: many residents, one hospital.

Roth, A. E. (1984). The Evolution of the Labor Market for Medical Interns and Residents: A Case Study in Game Theory. *The Journal of Political Economy*, 92(6), 991–1016.

ALVIN E. ROTH

In 1951, a centralized procedure for matching residents to hospital was introduced.

It replaced a chaotic, non-centralized market.

By implementing a hospital-proposing Deferred Acceptance mechanism.

Extended to accommodate many-to-one matches: many residents, one hospital.

It was a success and lives on to this day as *The National Resident Matching Program*, or *The Match*.

Tweaked to accommodate other constraints, e.g., preference of couples.

THE MATCH
NATIONAL RESIDENT MATCHING PROGRAM®

Roth, A. E. (1984). The Evolution of the Labor Market for Medical Interns and Residents: A Case Study in Game Theory. *The Journal of Political Economy*, 92(6), 991-1016.

In 2012, Lloyd Shapley and Al Roth were awarded the Nobel Prize in Economics.

For "the theory of stable allocations and the practice of market design."