



ROYAL  
HOLLOWAY  
UNIVERSITY  
OF LONDON

# SOCIAL NORMS AND APPLICATIONS WORKSHOP

March 17, 2026

# THE WISDOM OF DELIBERATING CROWDS

Two results and a half

Rosa Bauer

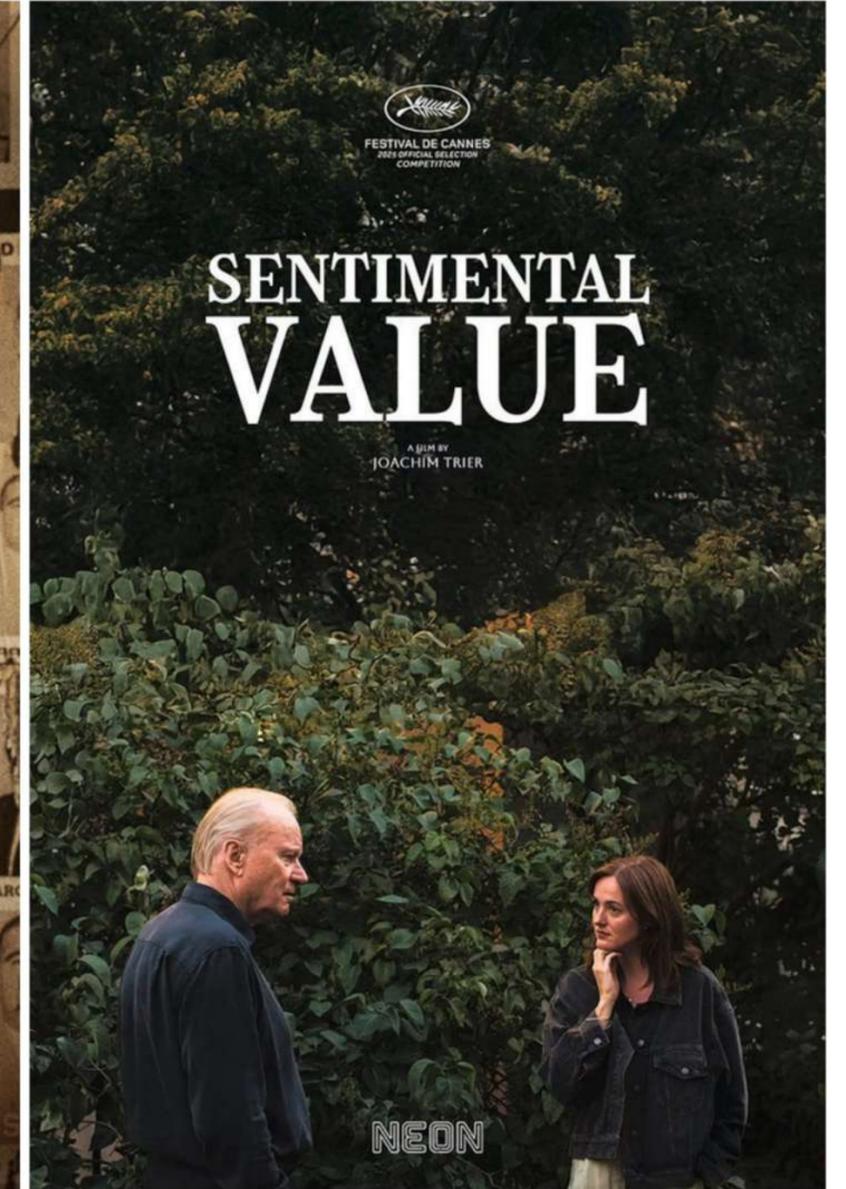
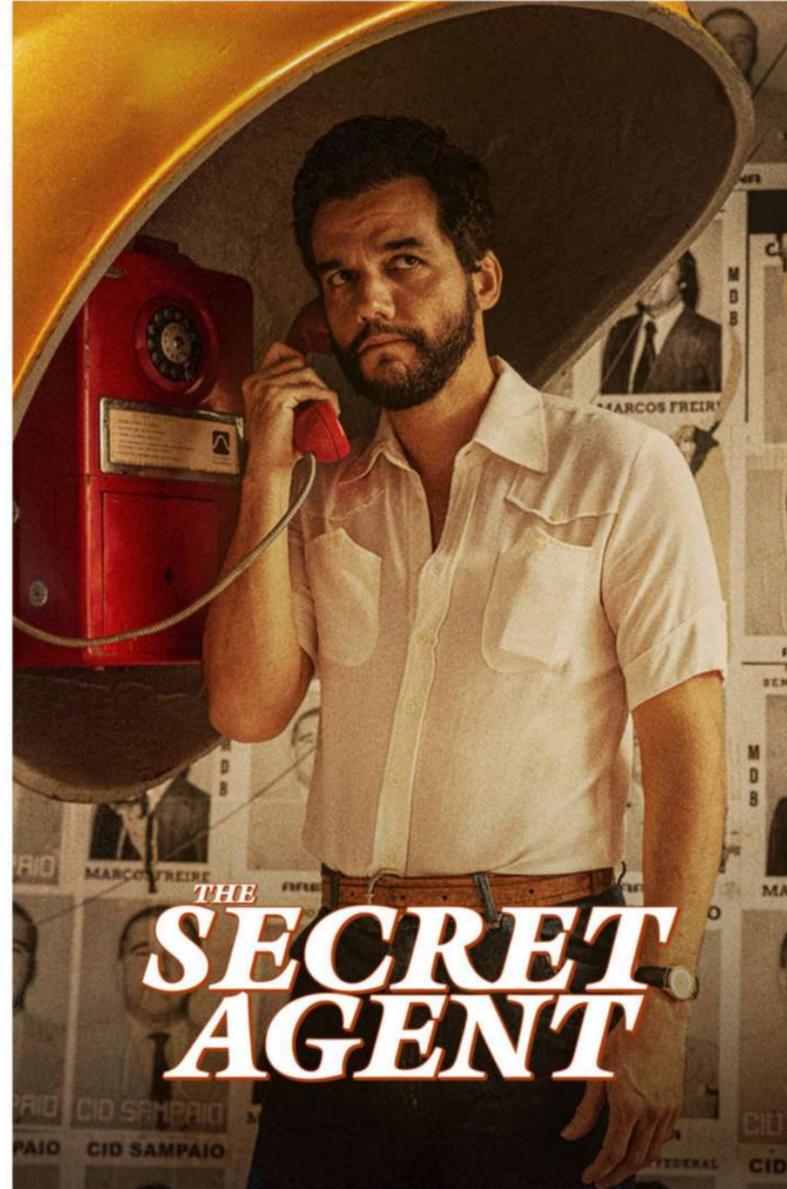
[rosa.bauer@icloud.com](mailto:rosa.bauer@icloud.com)

Adrian Haret

[adrian.haret@rhul.ac.uk](mailto:adrian.haret@rhul.ac.uk)

2026 Oscar for International Movie?

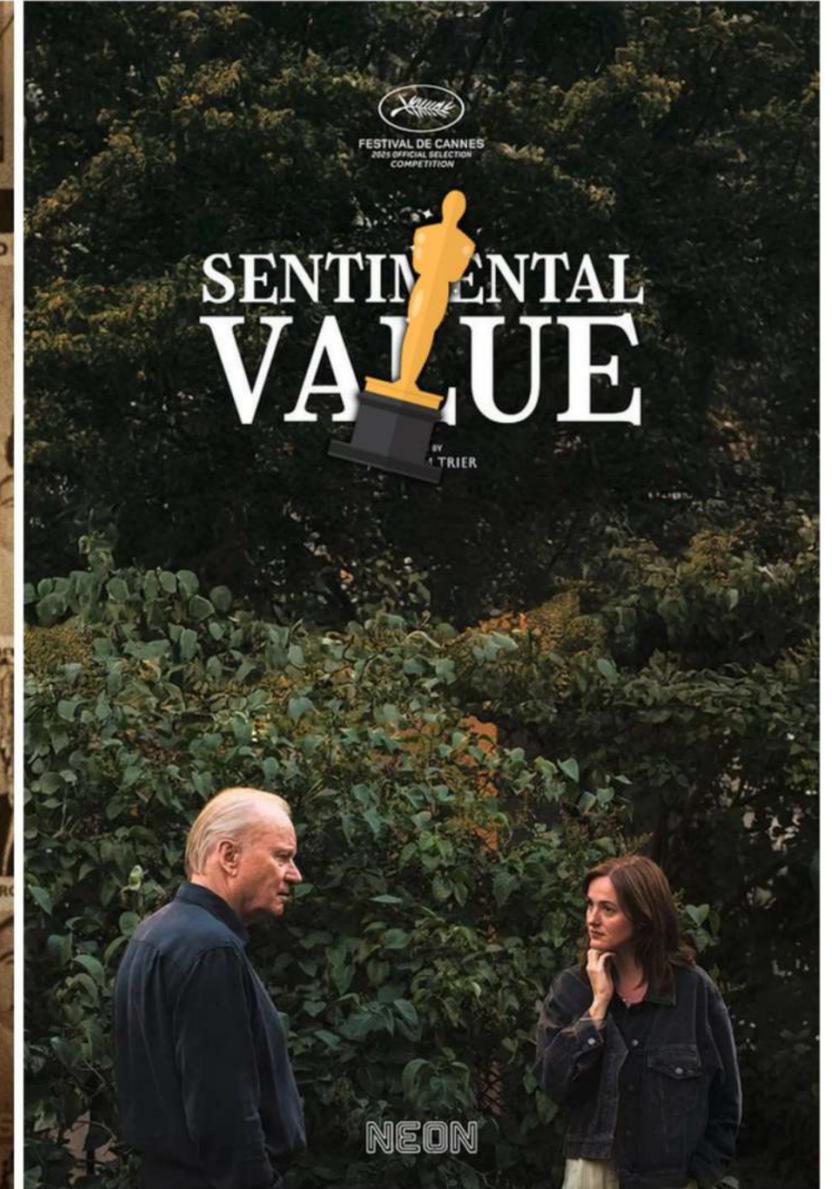
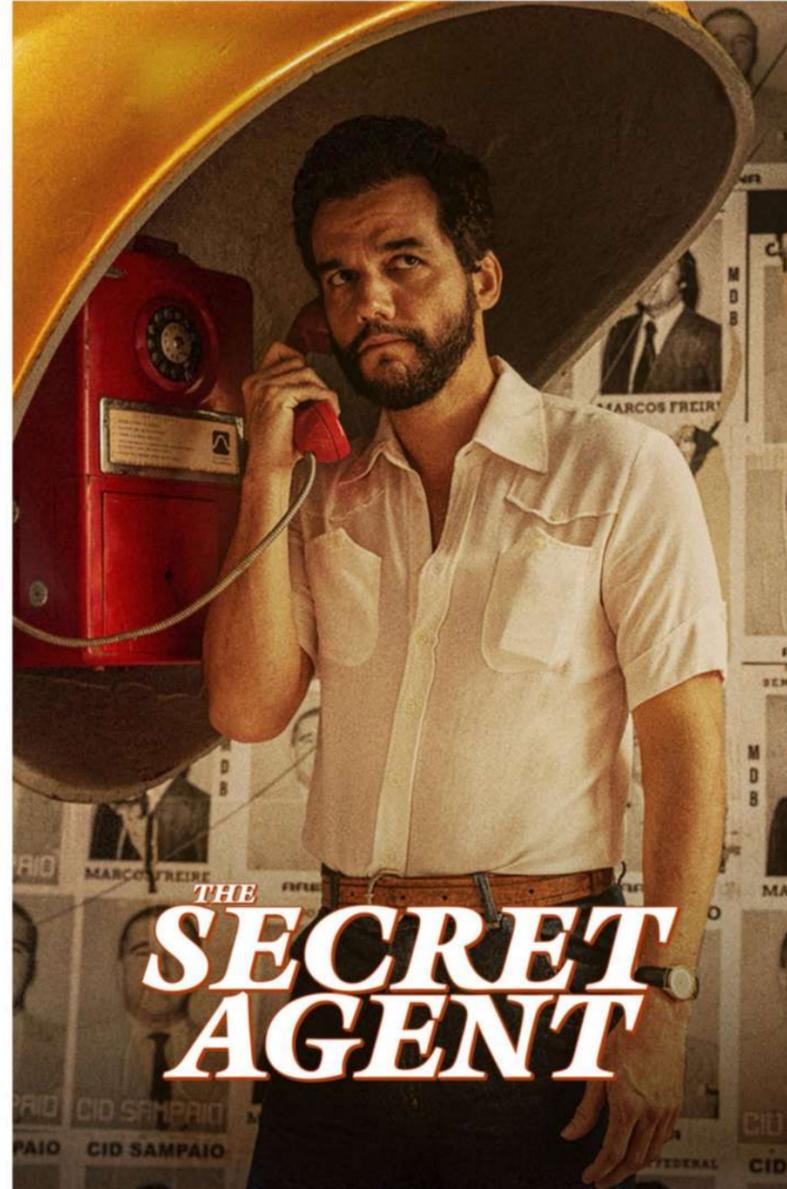
- The Secret Agent*
- Sentimental Value*



2026 Oscar for International Movie?

*The Secret Agent*

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Groups can sometimes be surprisingly accurate.



FRANCIS GALTON

I can personally attest to that...

I am talking, of course, about the ox.

17°·0 at Mosen, Basutoland, on August 23. The mean yearly value of the absolute maxima was 86°·9, and of the corresponding minima 41°·6. The mean temperature for the year was 0°·9 below the average. The stormiest month was October, and the calmest was April.

We have also received the official meteorological year-books for South Australia (1904) and Mysore (1905). Both of these works contain valuable means for previous years.

*Forty Years of Southern New Mexico Climate.*—Bulletin No. 59 of the New Mexico College of Agriculture contains the meteorological data recorded at the experimental station from 1892 to 1905 inclusive, together with results of temperature and rainfall observations at other stations in the Mesilla Valley for most of the years between 1851 and 1890, published some years ago by General Greeley in a "Report on the Climate of New Mexico." The station is situated in lat. 32° 15' N., long. 106° 45' W., and is 3868 feet above sea-level. The data have a general application to those portions of southern New Mexico with an altitude less than 4000 feet. The mean annual temperature for the whole period was 61°·6, mean maximum (fourteen years) 76°·8, mean minimum 41°·4, absolute maximum 106° (which occurred several times), absolute minimum 1° (December, 1895). The mean annual rainfall was 8·8 inches; the smallest yearly amount was 3·5 inches, in 1873, the largest 17·1 inches, in 1905. Most of the rain falls during July, August, and September. The relative humidity is low, the mean annual amount being about 51 per cent. The bulletin was prepared by J. D. Tinsley, vice-director of the station.

*Meteorological Observations in Germany.*—The results of the observations made under the system of the Deutsche Seewarte, Hamburg, for 1905, at ten stations of the second order, and at fifty-six storm-warning stations, have been received. This is the twenty-eighth yearly volume published by the Seewarte, and forms part of the series of German meteorological year-books. We have frequently referred to this excellent series, and the volume in question is similar in all respects to its predecessors; it contains most valuable data relating to the North Sea and Baltic coasts. We note that the sunshine at Hamburg was only 29 per cent. of the possible annual amount, and that there were 103 sunless days; the rainfall was 25·9 inches, the rainy days being 172 in number.

#### VOX POPULI.

IN these democratic days, any investigation into the trustworthiness and peculiarities of popular judgments is of interest. The material about to be discussed refers to a small matter, but is much to the point.

A weight-judging competition was carried on at the annual show of the West of England Fat Stock and Poultry Exhibition recently held at Plymouth. A fat ox having been selected, competitors bought stamped and numbered cards, for 6d. each, on which to inscribe their respective names, addresses, and estimates of what the ox would weigh after it had been slaughtered and "dressed." Those who guessed most successfully received prizes. About 800 tickets were issued, which were kindly lent me for examination after they had fulfilled their immediate purpose. These afforded excellent material. The judgments were unbiased by passion and uninfluenced by oratory and the like. The sixpenny fee deterred practical joking, and the hope of a prize and the joy of competition prompted each competitor to do his best. The competitors included butchers and farmers, some of whom were highly expert in judging the weight of cattle; others were probably guided by such information as they might pick up, and by their own fancies. The average competitor was probably as well fitted for making a just estimate of the dressed weight of the ox, as an average voter is of judging the merits of most political issues on which he votes, and the variety among the voters to judge justly was probably much the same in either case.

After weeding thirteen cards out of the collection, as being defective or illegible, there remained 787 for discussion. I arrayed them in order of the magnitudes of the estimates, and converted the *wt.*, *quarters*, and *lbs.* in which they were made, into *lbs.*, under which form they will be treated.

[NO. 1949, VOL. 75]

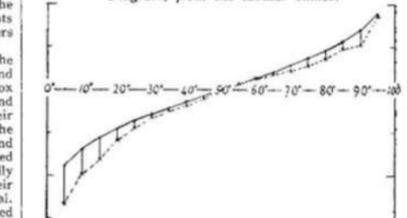
*Distribution of the estimates of the dressed weight of a particular living ox, made by 787 different persons.*

Degrees of the length of Array 0°-100°	Estimates in lbs.	Centiles		Excess of Observed over Normal
		Observed deviates from 1207 lbs.	Normal p.e. = 37	
5	1074	-133	-90	+43
10	1109	-98	-70	+28
15	1126	-81	-57	+24
20	1148	-59	-46	+13
25	1162	-45	-37	+8
30	1174	-33	-29	+4
35	1181	-26	-21	+5
40	1188	-19	-14	+5
45	1197	-10	-7	+3
50	1207	0	0	0
55	1214	+7	+7	0
60	1219	+12	+14	-2
65	1225	+18	+21	-3
70	1230	+23	+29	-6
75	1236	+29	+37	-8
80	1243	+36	+46	-10
85	1254	+47	+57	-10
90	1267	+52	+70	-18
95	1293	+86	+90	-4

75, 75, the first and third quartiles, stand at 25° and 75° respectively. 50, the median or middlemost value, stands at 50°. The dressed weight proved to be 1197 lbs.

According to the democratic principle of "one vote one value," the middlemost estimate expresses the *vox populi*, every other estimate being condemned as too low or too high by a majority of the voters (for fuller explanation see "One Vote, One Value," NATURE, February 28, p. 414). Now the middlemost estimate is 1207 lbs., and the weight of the dressed ox proved to be 1198 lbs.; so the *vox populi* was in this case 9 lbs., or 0·8 per cent. of the whole weight too high. The distribution of the estimates about their middlemost value was of the usual type, so far that they clustered closely in its neighbourhood and became rapidly more sparse as the distance from it increased.

Diagram, from the tabular values.



The continuous line is the normal curve with p.e. = 37. The broken line is drawn from the observations. The lines connecting them show the differences between the observed and the normal.

But they were not scattered symmetrically. One quarter of them deviated more than 45 lbs. above the middlemost (3·7 per cent.), and another quarter deviated more than 29 lbs. below it (2·4 per cent.), therefore the range of the two middle quarters, that is, of the middlemost half, lay within those limits. It would be an equal chance that the estimate written on any card picked at random out of the collection lay within or without those limits. In other words, the "probable error" of a single observation may be reckoned as  $\frac{1}{2}(45+29)$ , or 37 lbs. (3·1 per cent.). Taking this for the p.e. of the normal curve that is best adapted for comparison with the observed values, the results are obtained which appear in above table, and graphically in the diagram.



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I can personally attest to that...

I am talking, of course, about the ox.

JAMES SUROWIECKI

Plus many other examples!



A NEW YORK TIMES BUSINESS BESTSELLER

"As entertaining and thought-provoking as *The Tipping Point* by Malcolm Gladwell. . . . *The Wisdom of Crowds* ranges far and wide."

—*The Boston Globe*

# THE WISDOM OF CROWDS

JAMES  
SUROWIECKI

WITH A NEW AFTERWORD BY THE AUTHOR



Surowiecki, J. (2005). *The Wisdom of Crowds*. Anchor.



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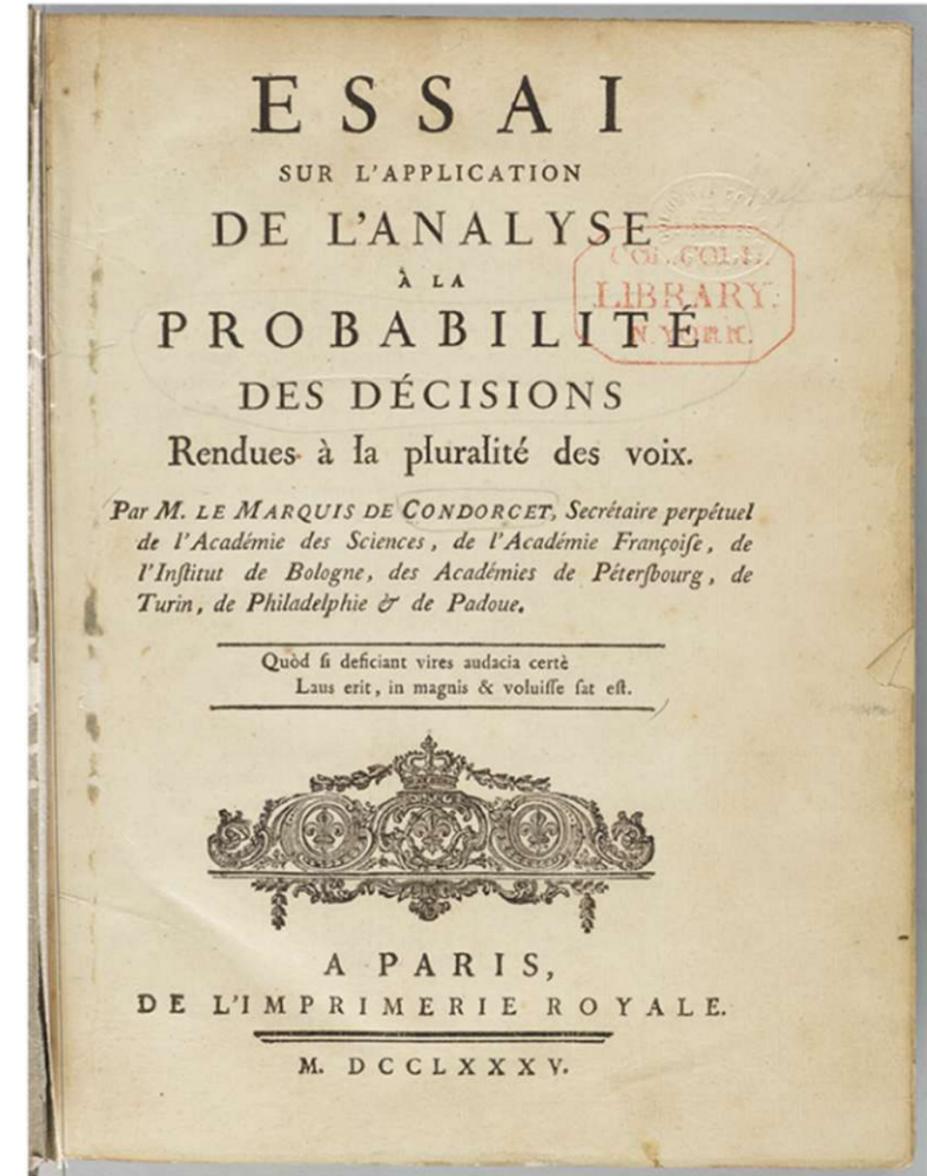
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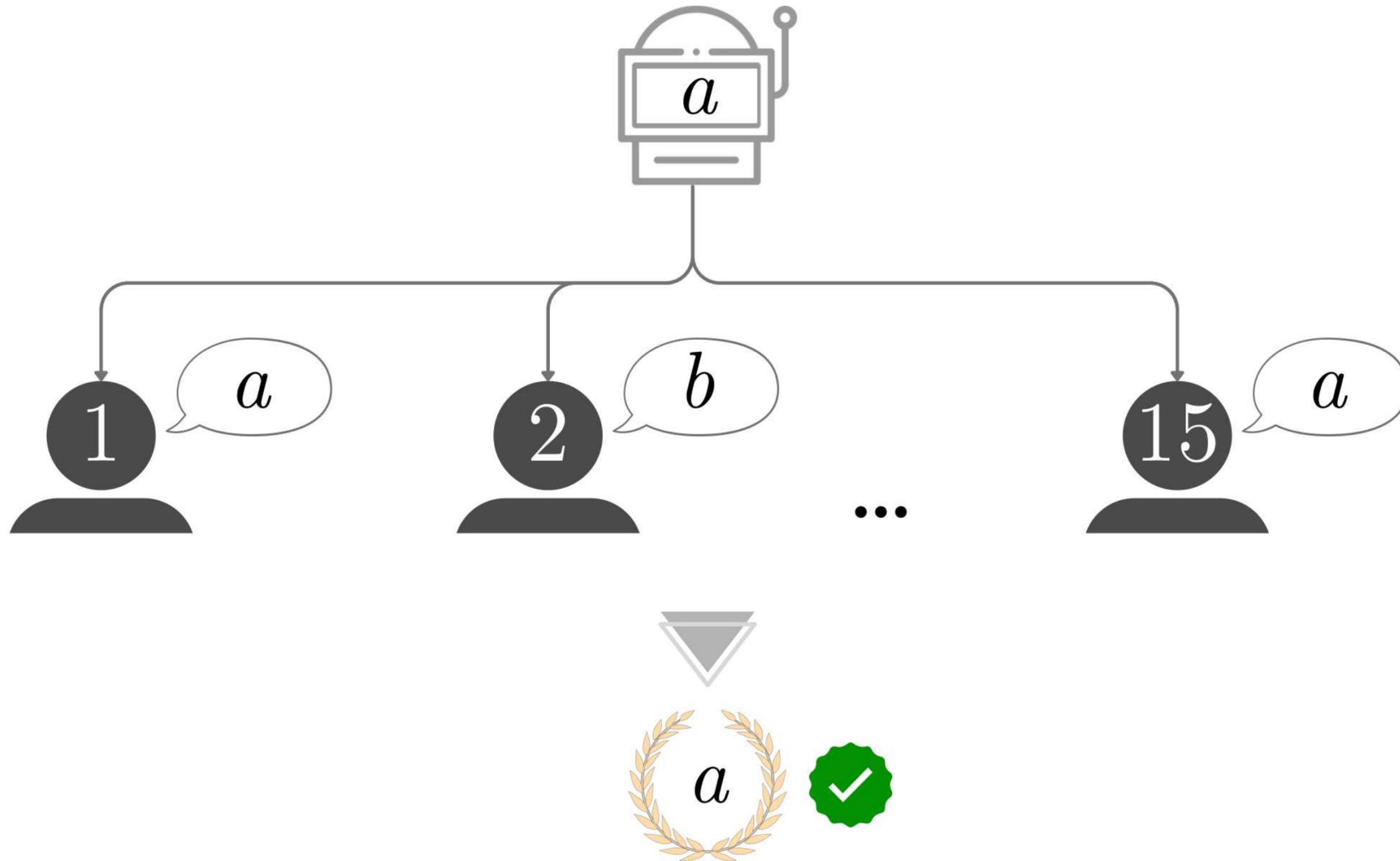
CONDORCET

I think I can prove why...



Marie Jean Antoine Nicolas de Caritat, Marquis of Condorcet (1785). *Essai sur l'application de l'analyse à la probabilité des décisions rendues à la pluralité des voix.*

# CONDORCET JURY THEOREM: INTUITION



# MODEL

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alternatives  $a, b$

correct alternative  $a$

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collective decision over $n$ voters	$\text{Vote}(n) \in \{a, b\}$ , the alternative with the most votes // we assume $n$ is odd to avoid ties
voter $i$ 's accuracy	$p_i = \Pr[v_i = a]$ // probability of $i$ voting correctly

# ASSUMPTIONS

## CONDORCET JURY THEOREM

**(Equal accuracies)**  $p_i = p_j = p$ , for all  $i, j \in N$ .

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**(Independence)**  $\Pr[v_i = x, v_j = y] = \Pr[v_i = x] \cdot \Pr[v_j = y]$ ,  
for all distinct  $i, j \in N$ .

// agents vote independently of each other

# TODAY'S FIRST RESULT

## THEOREM (THE CONDORCET JURY THEOREM, OR CJT)

For odd  $n$  voters, each with accuracy  $p > 1/2$  and voting independently of each other, it holds that:

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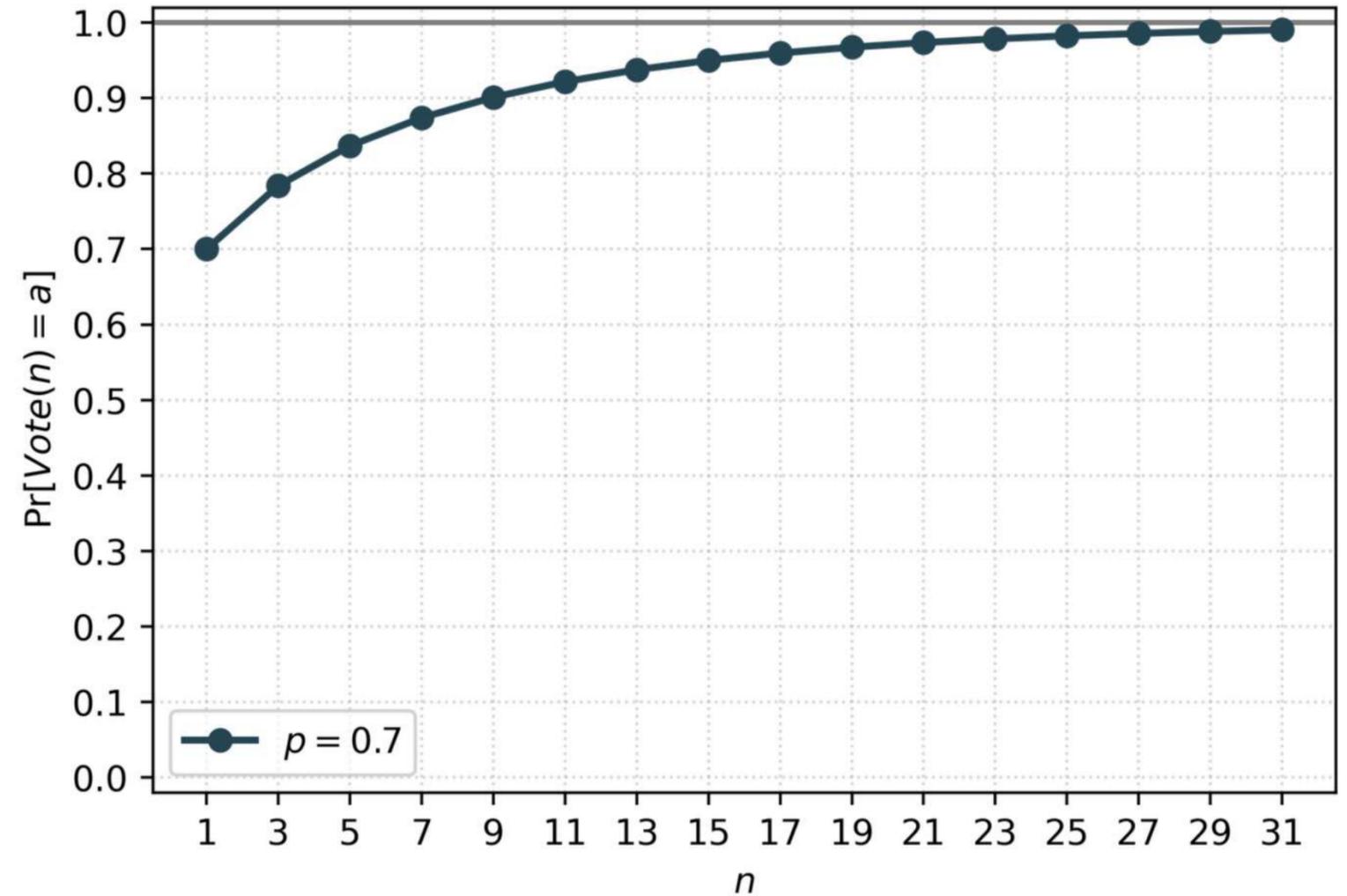
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**(Asymptotic behavior)**  $\lim_{n \rightarrow \infty} \Pr[\text{Vote}(n) = a] = 1$ .

// group accuracy approaches 1 asymptotically

# CJT IN A NUTSHELL

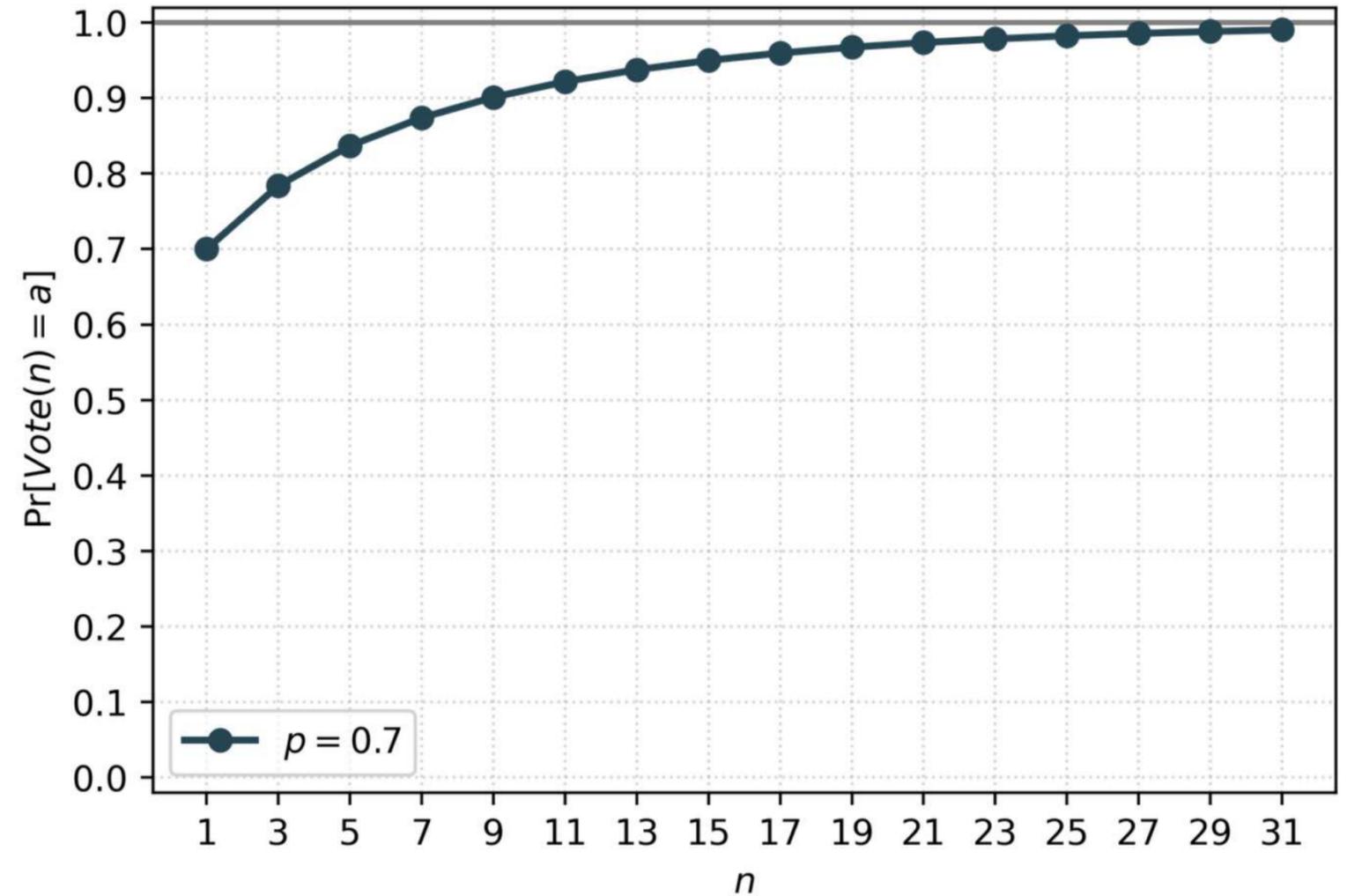
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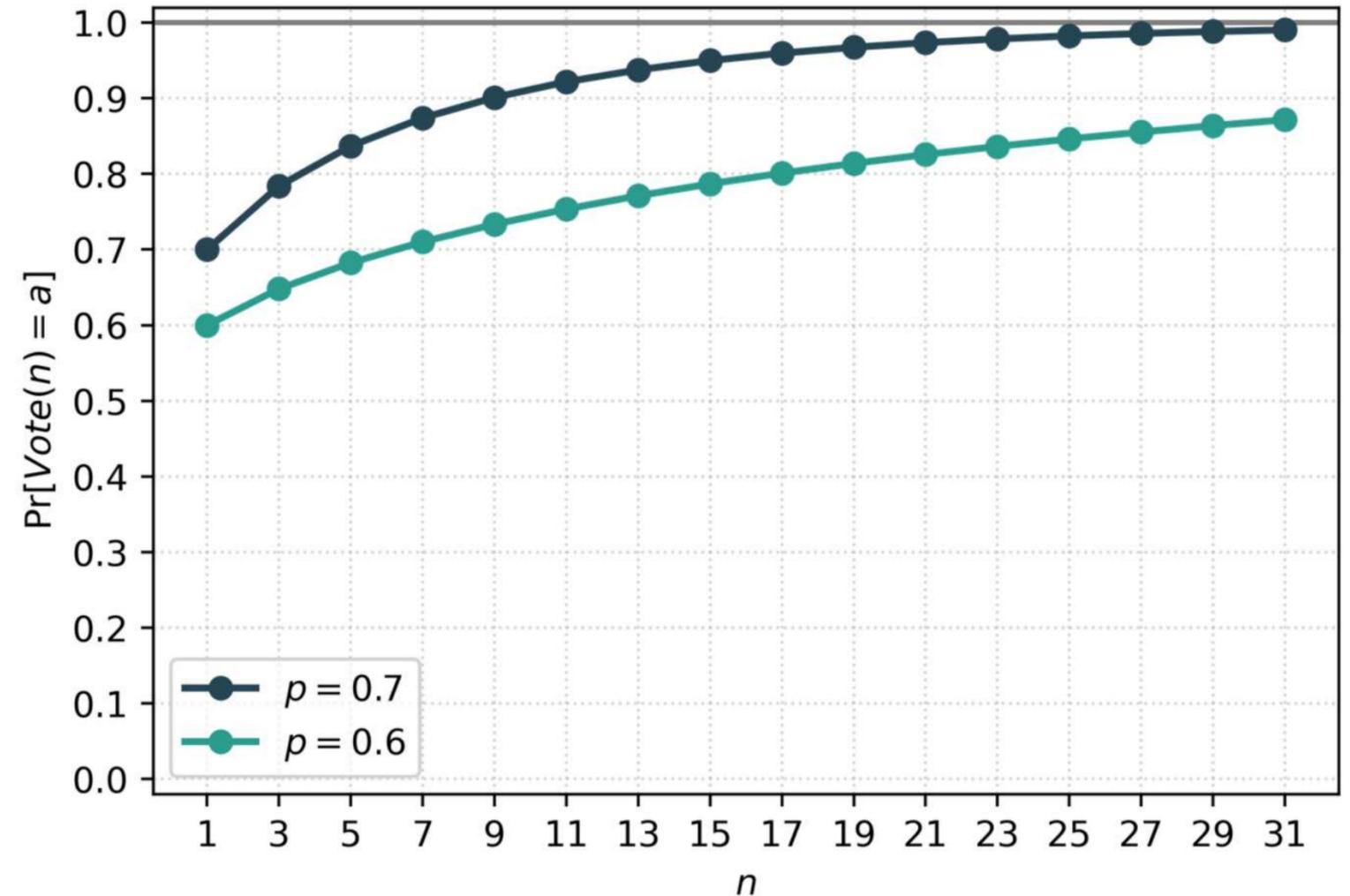


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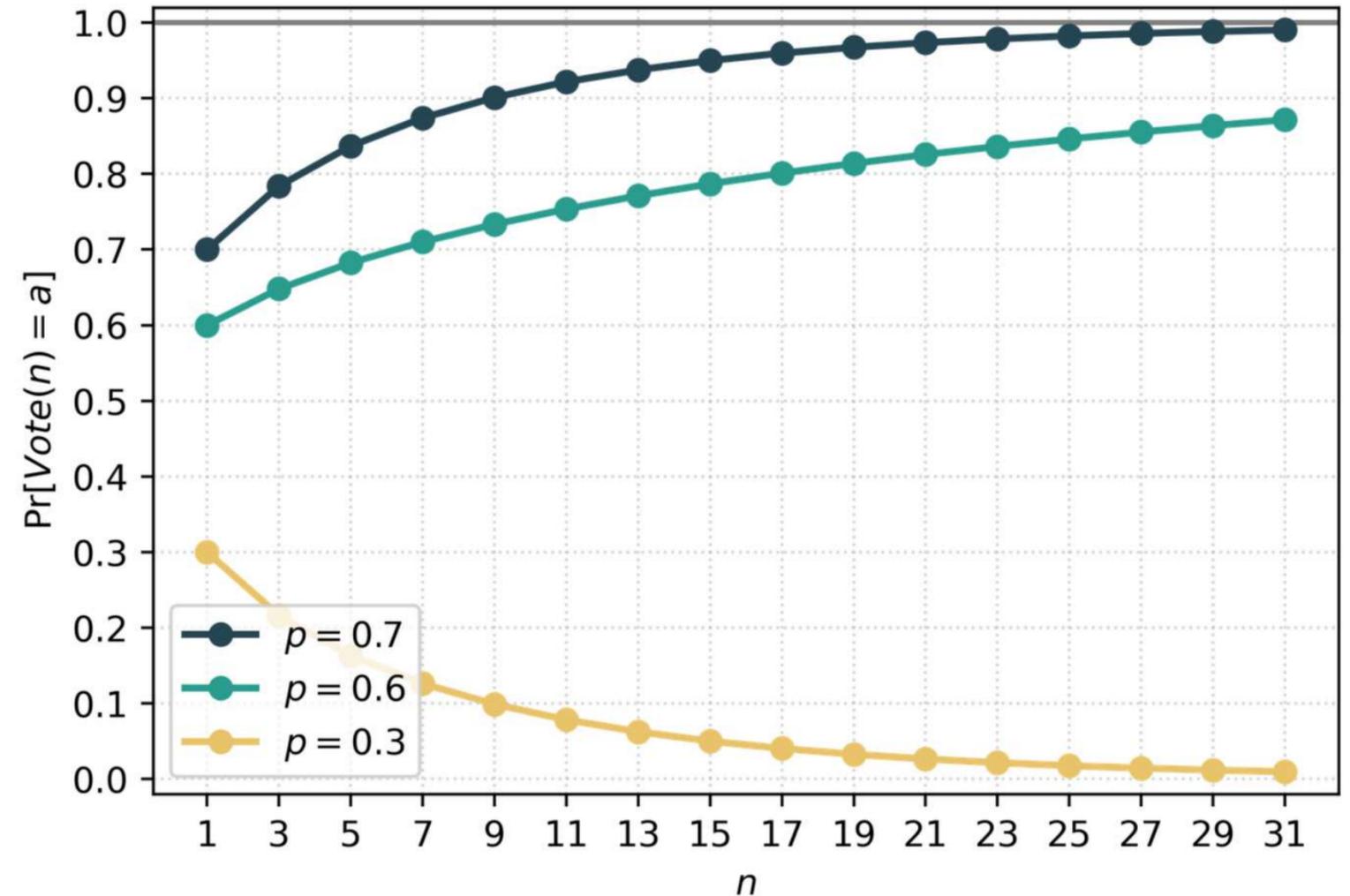
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The effect is more pronounced the higher  $p$  is...

...as long as  $p > 1/2$ .



This also leads to a defense of democracy:  
not just because it's fair, but also because it  
leads to better decisions.



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We can get good decisions with large, inclusive  
deliberating body of citizens.

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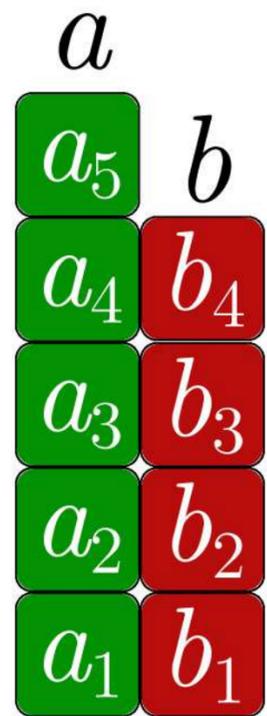
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ADRIAN

Well, let's model *that*!

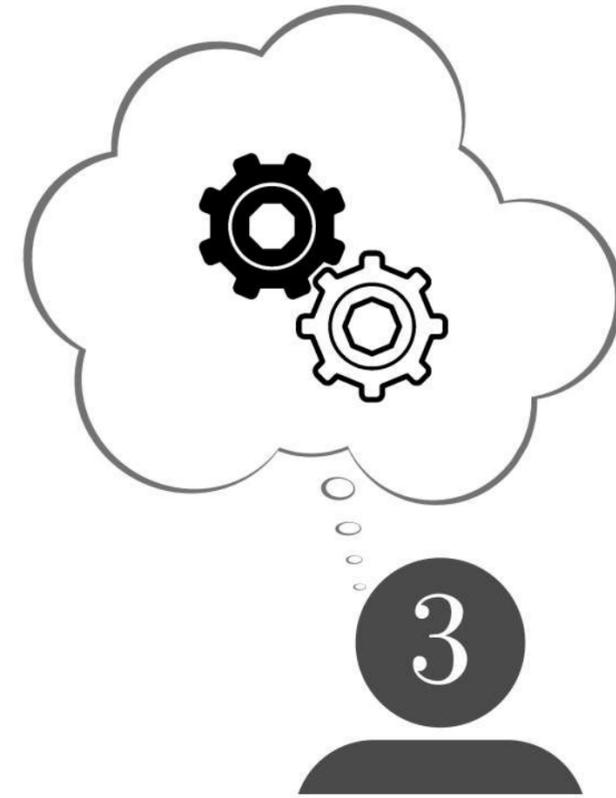
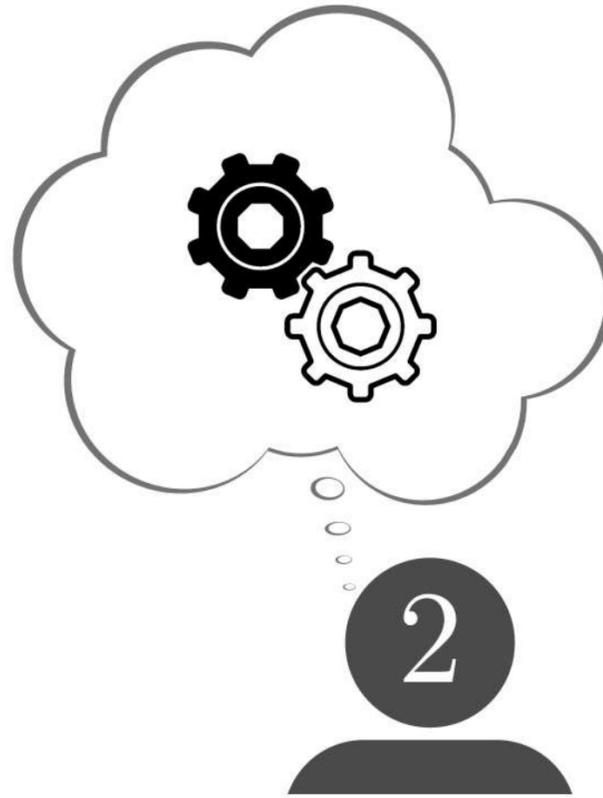
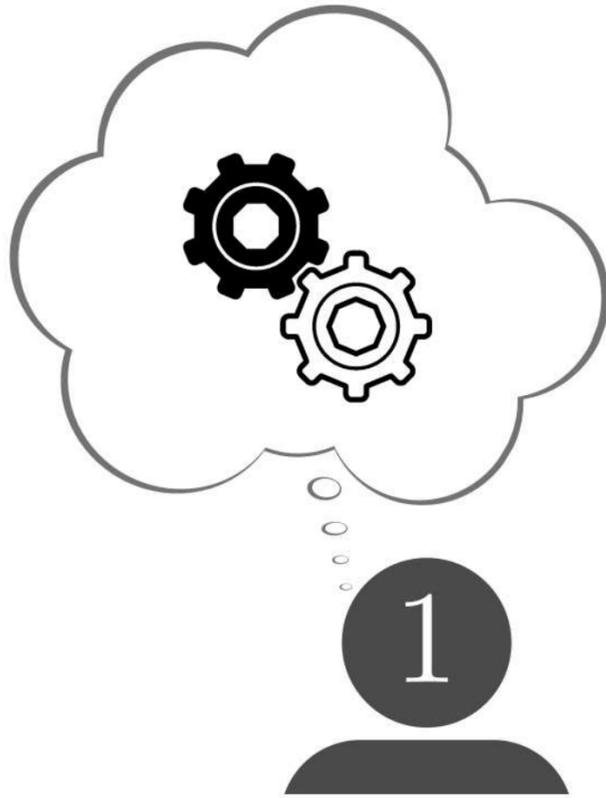
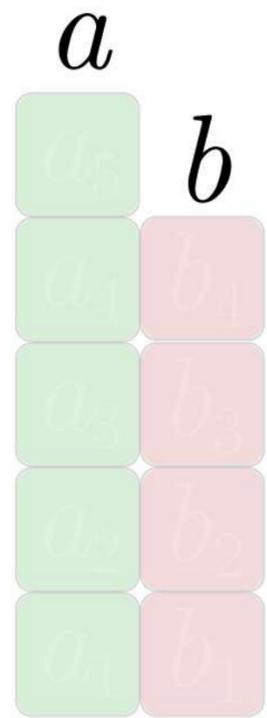


# DELIBERATION EXAMPLE



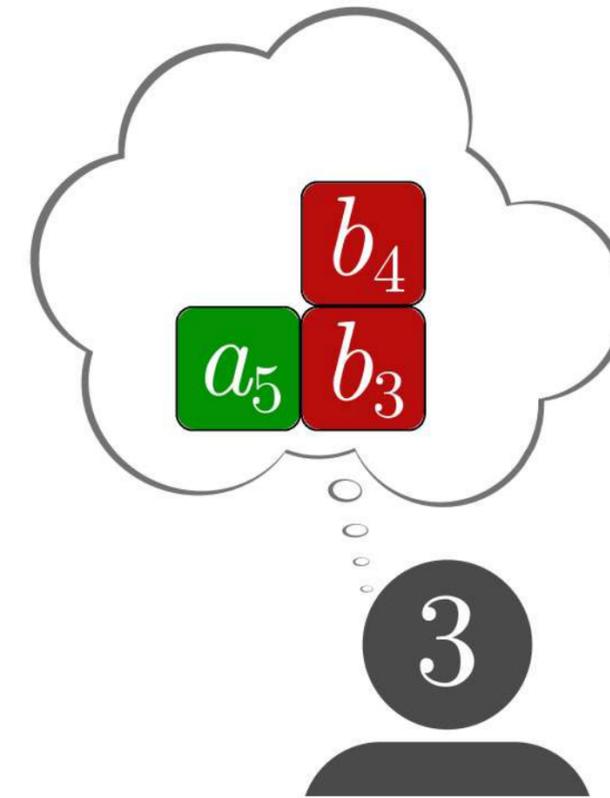
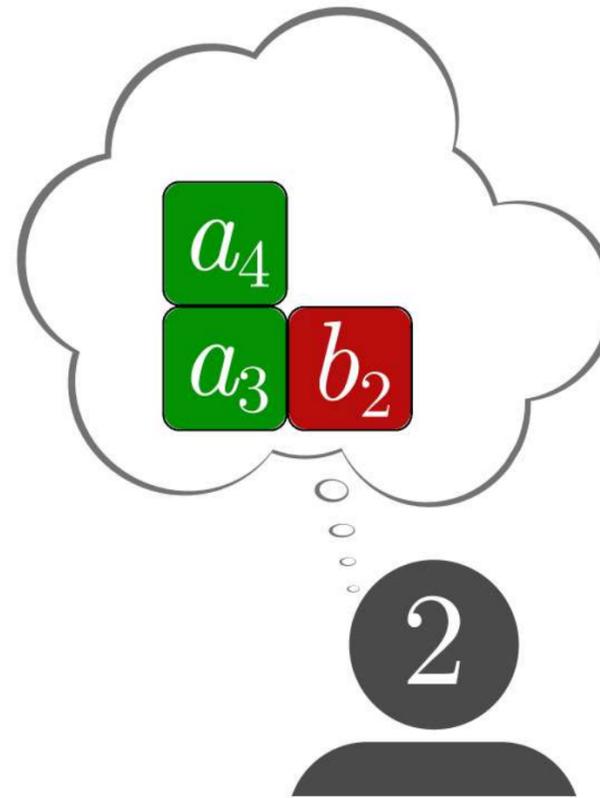
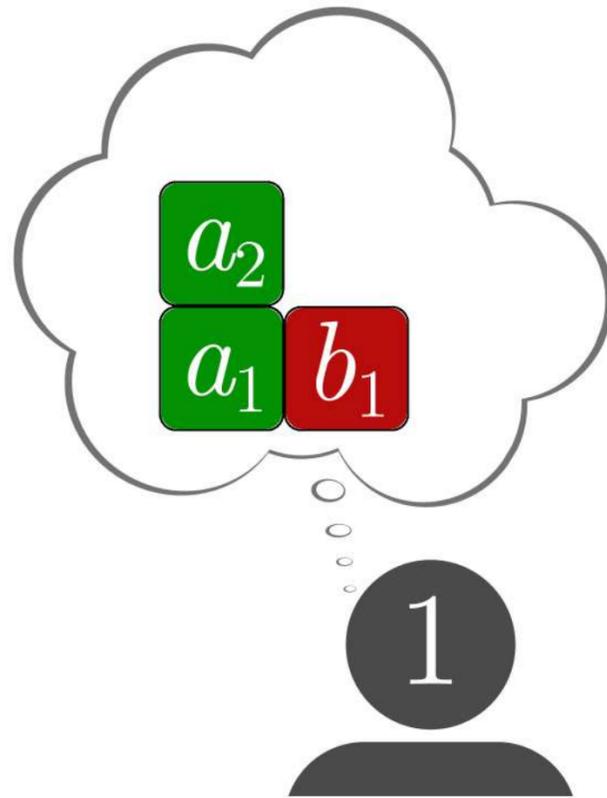
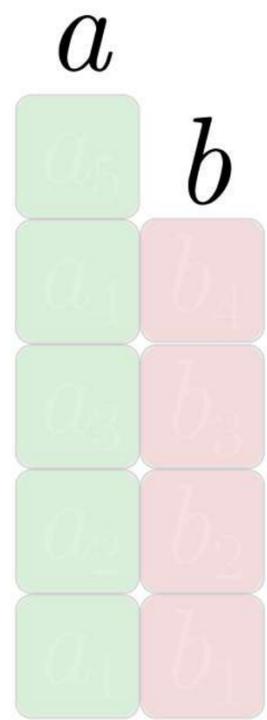
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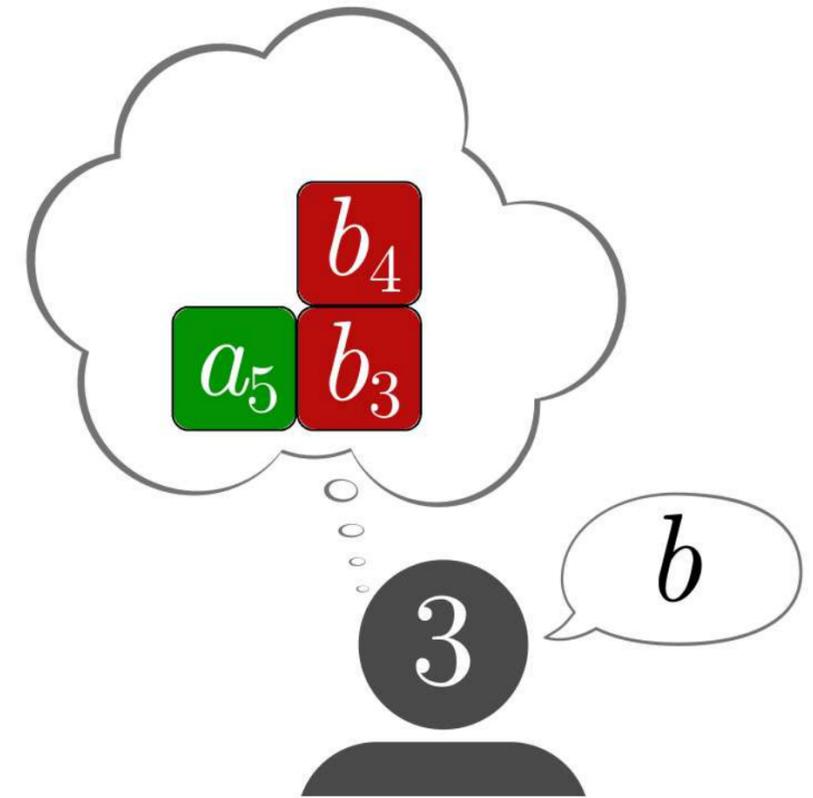
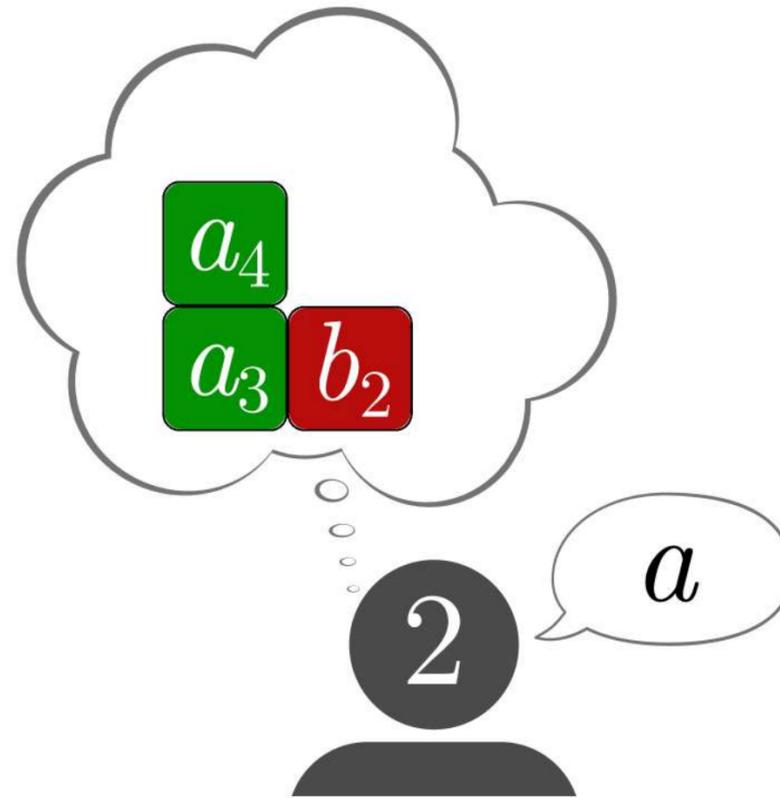
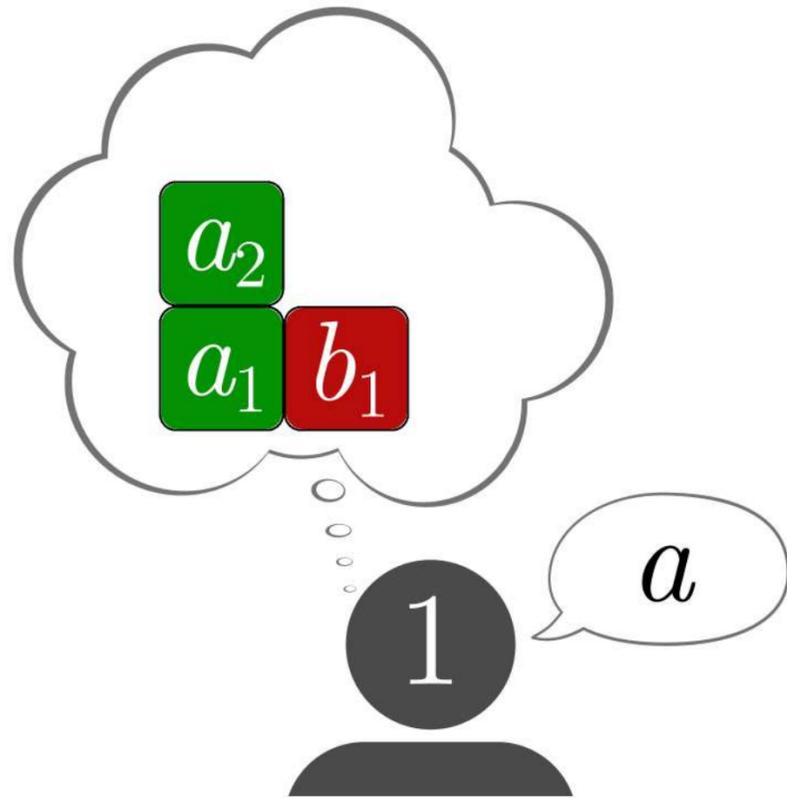
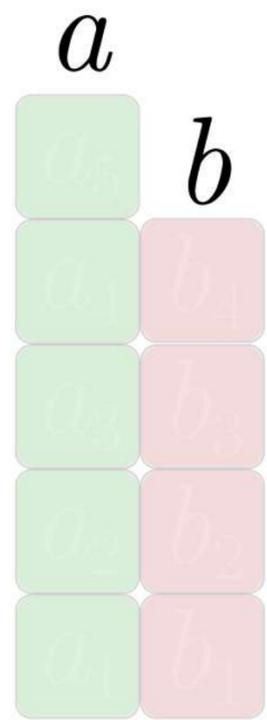
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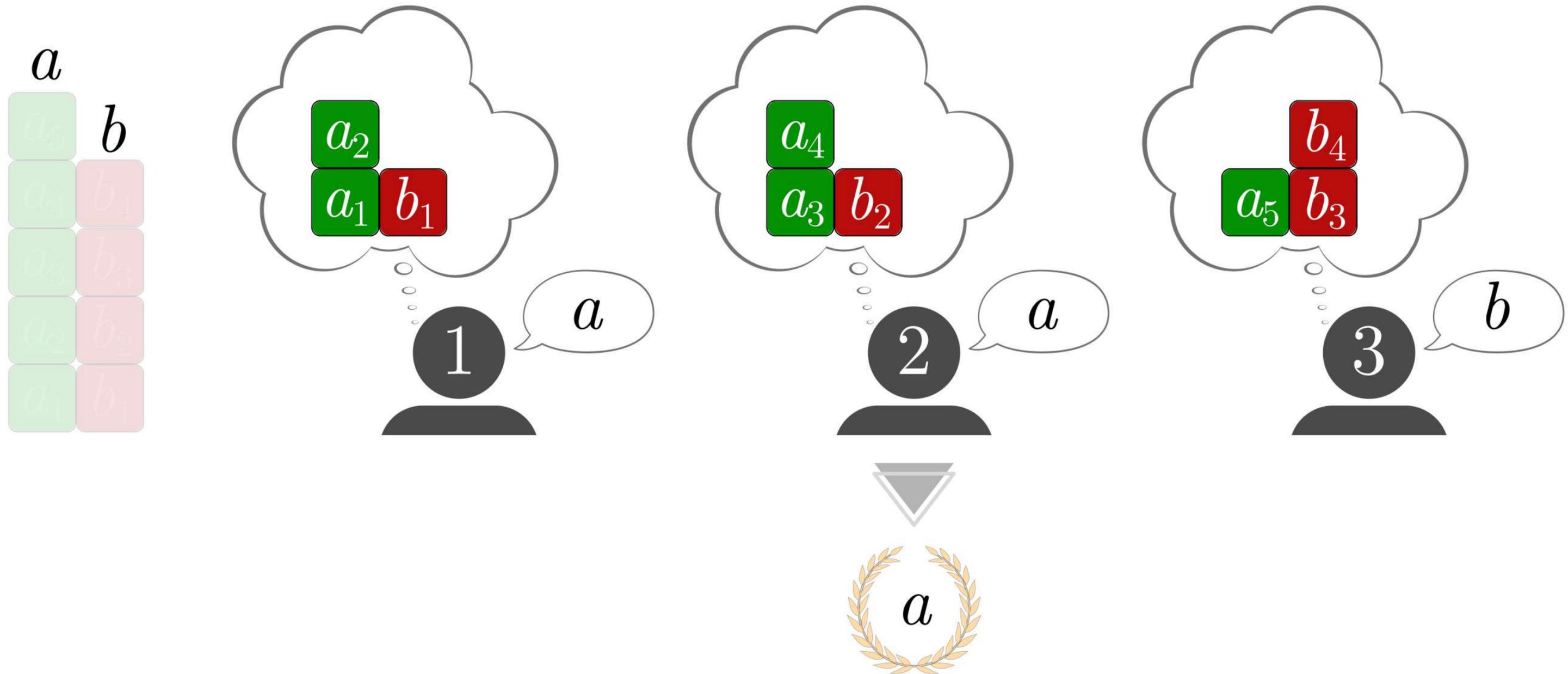
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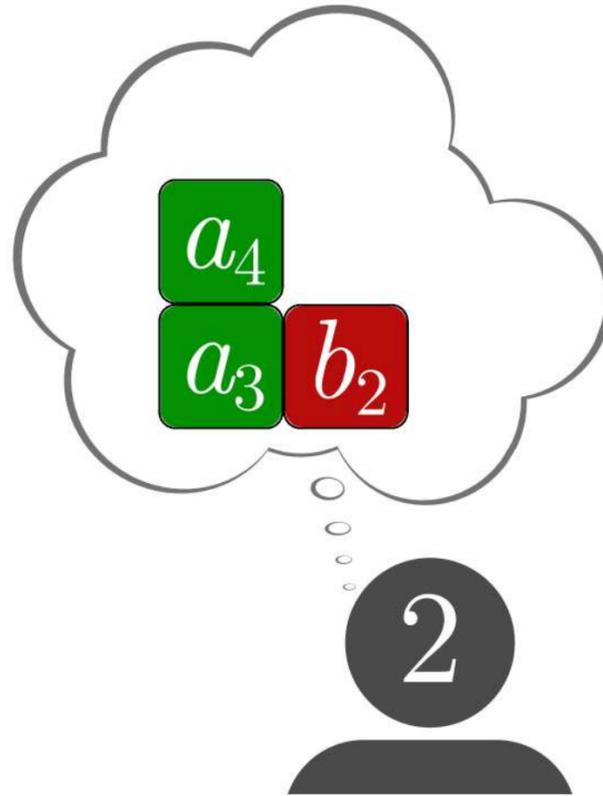
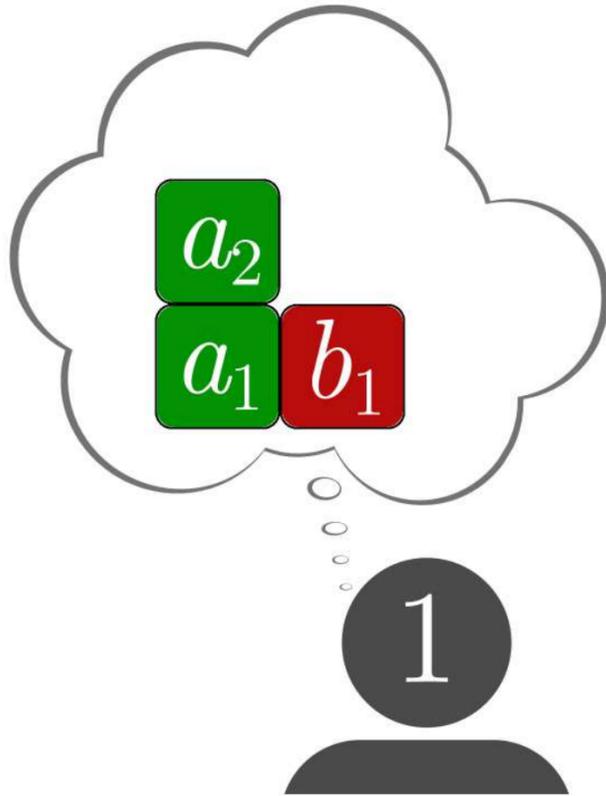
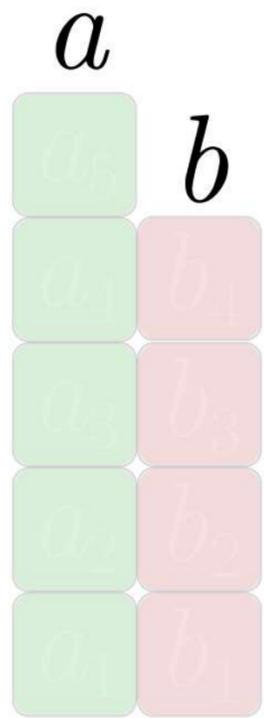
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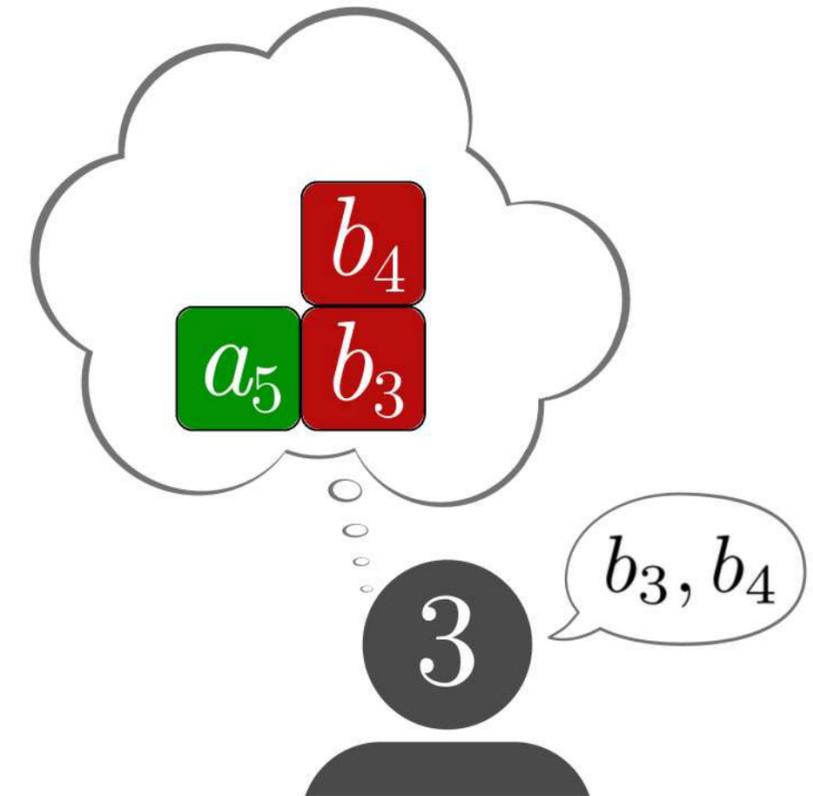
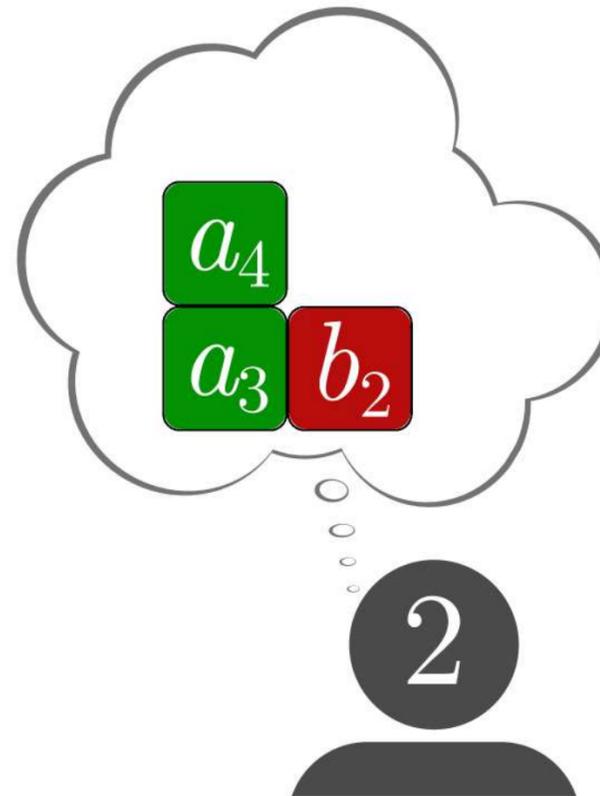
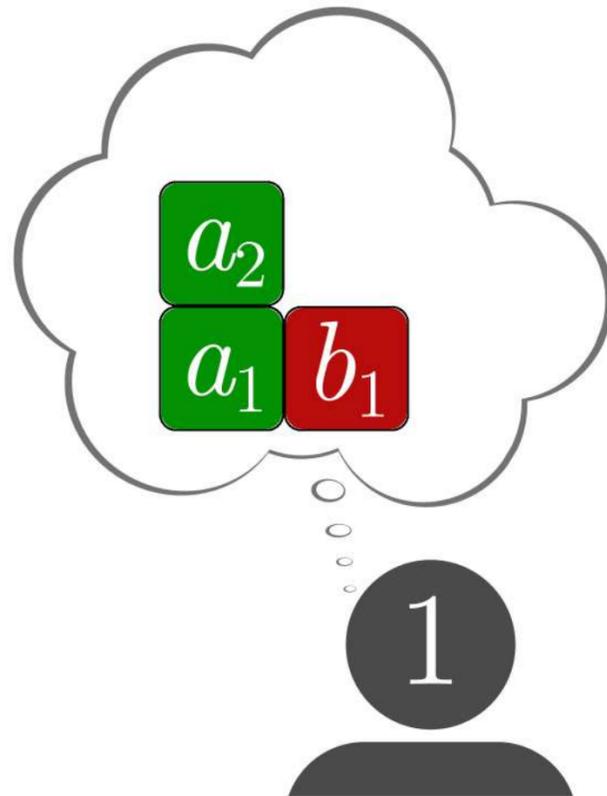
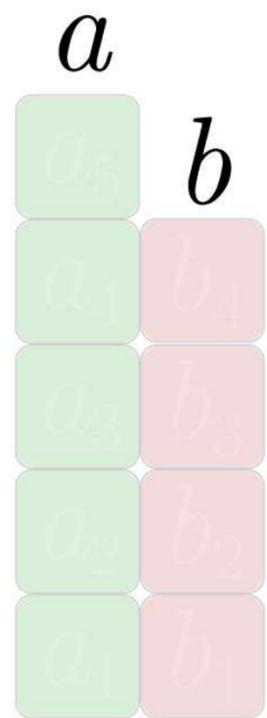
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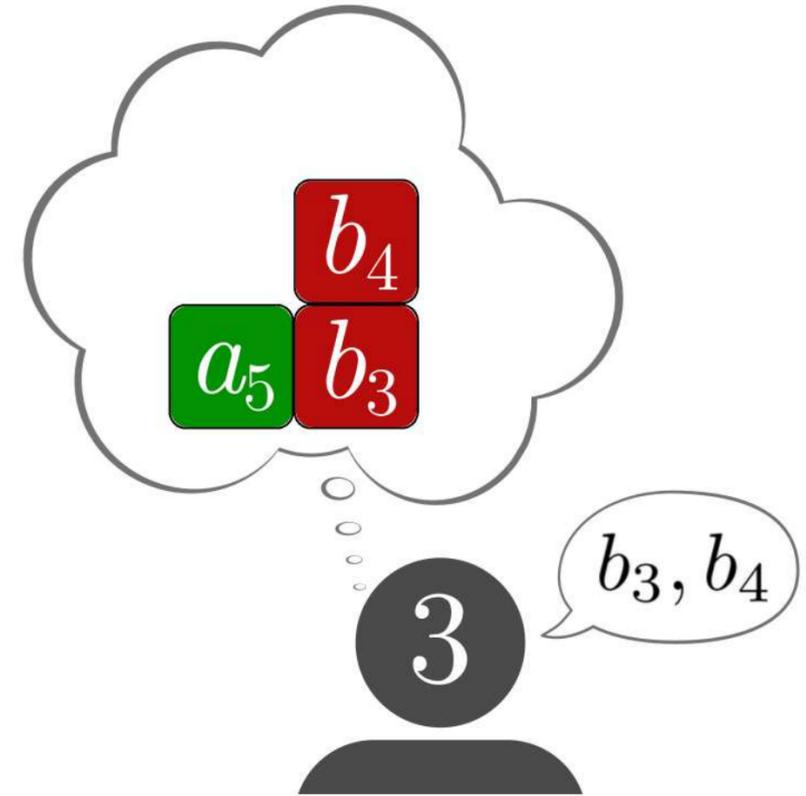
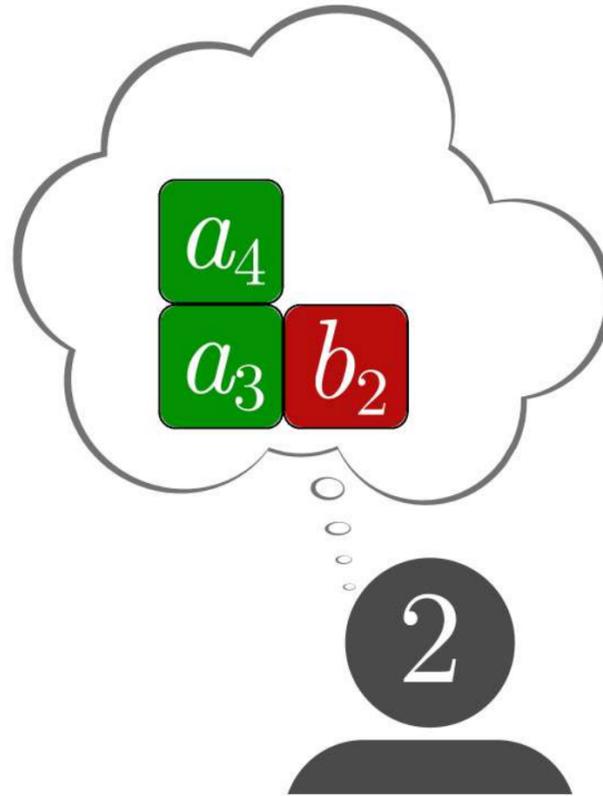
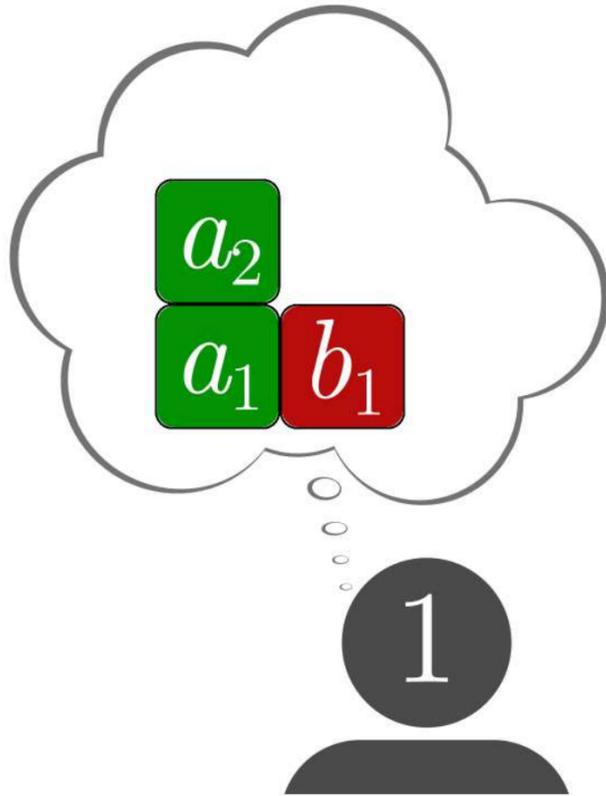
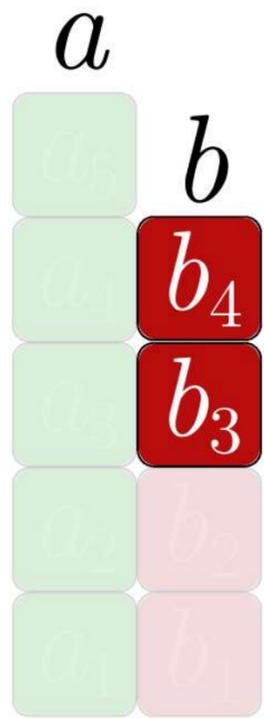
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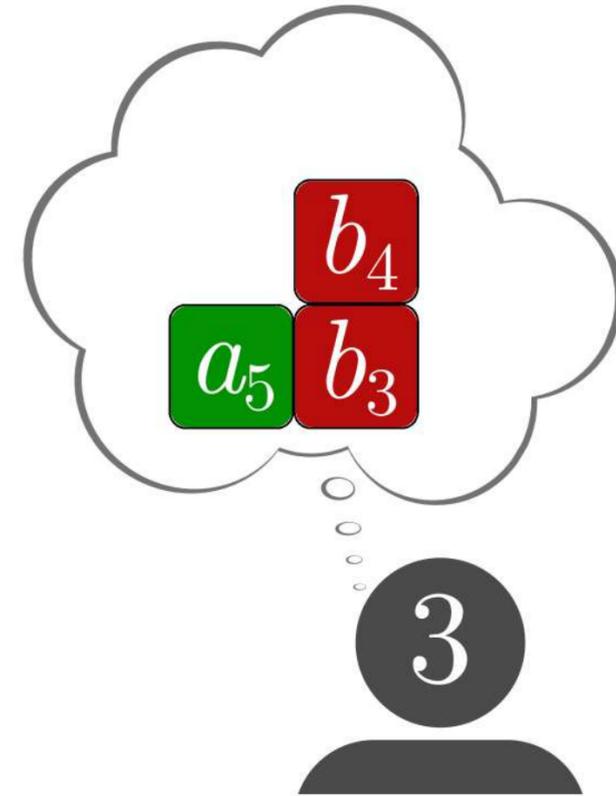
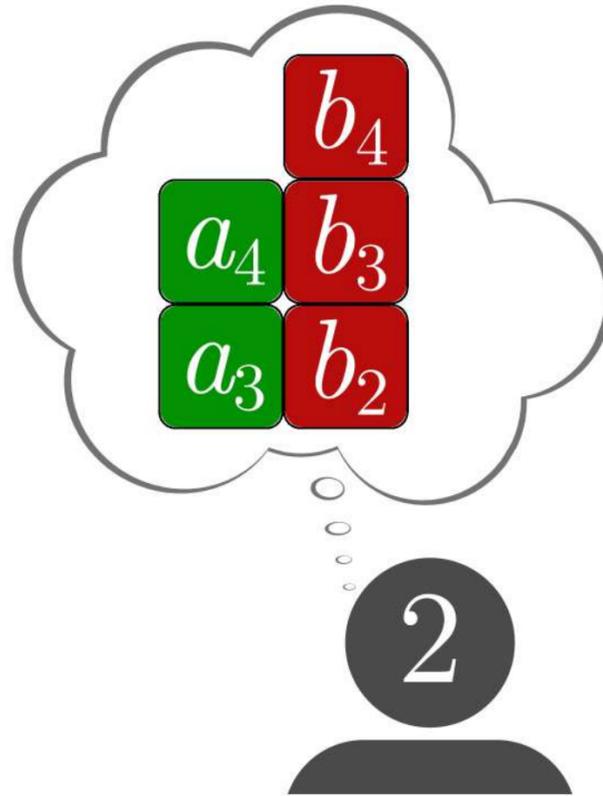
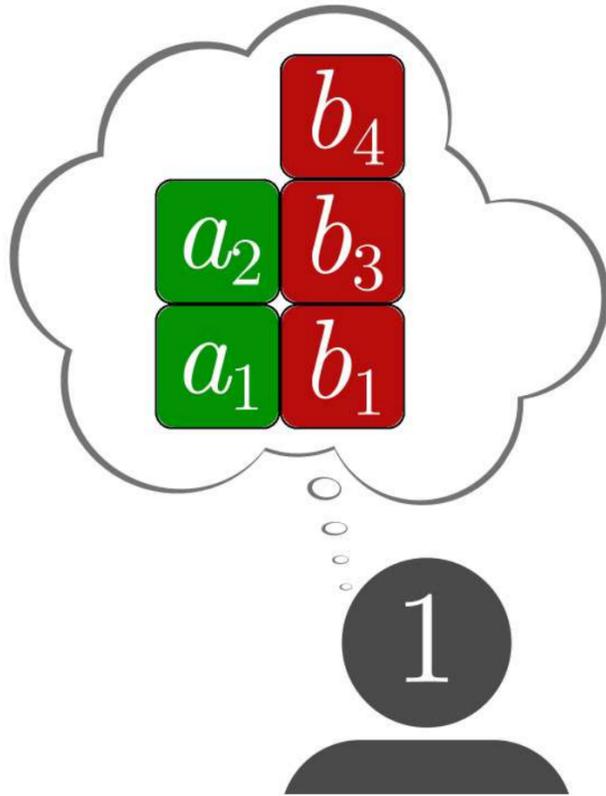
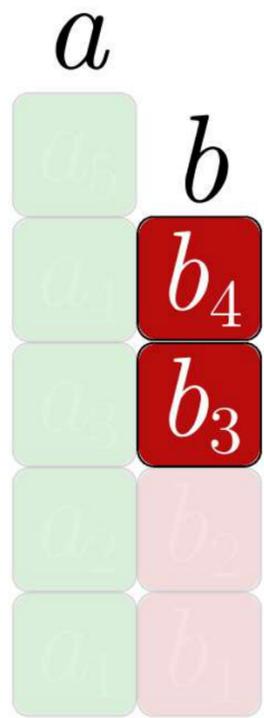
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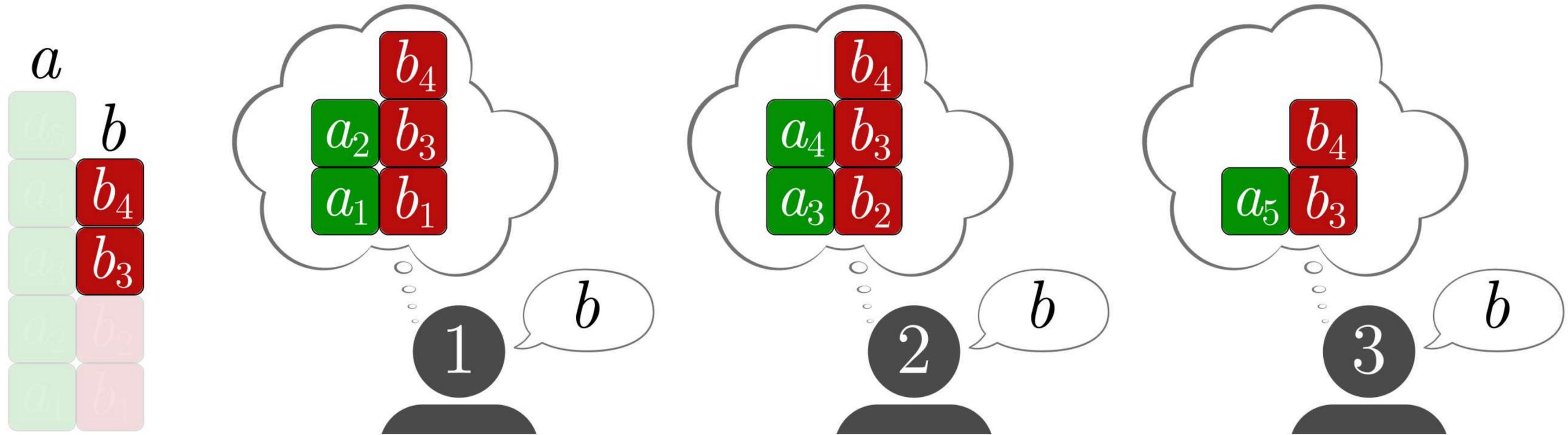
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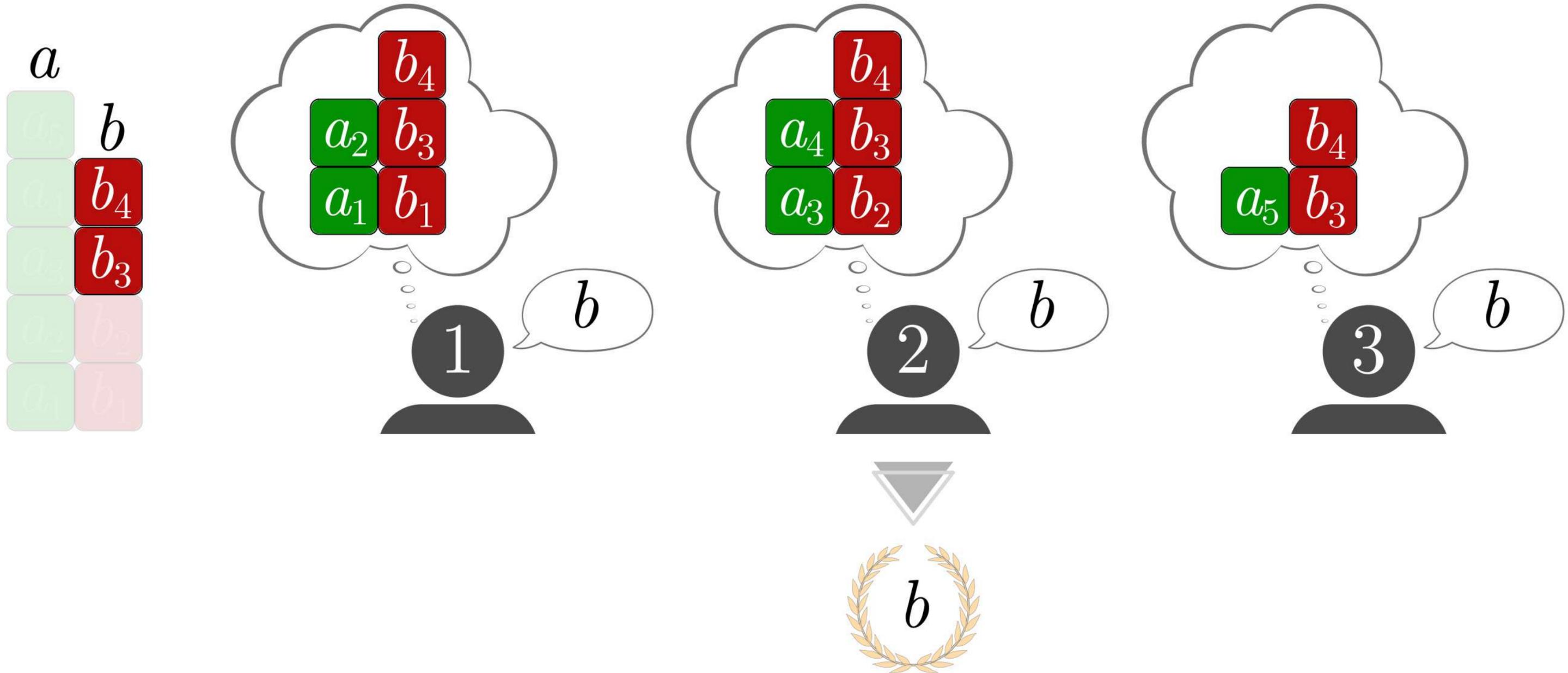
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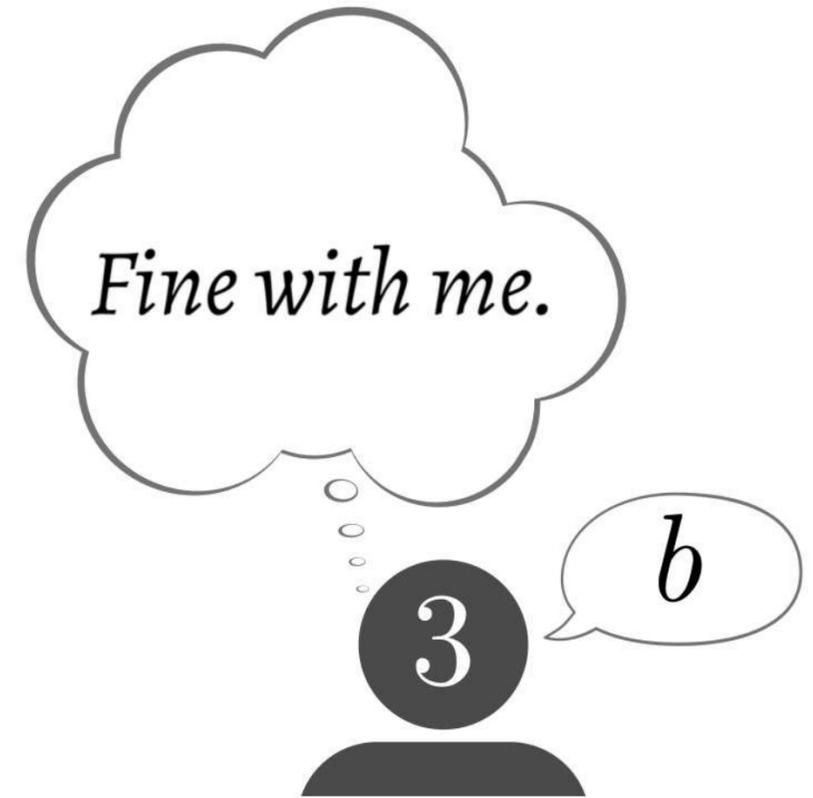
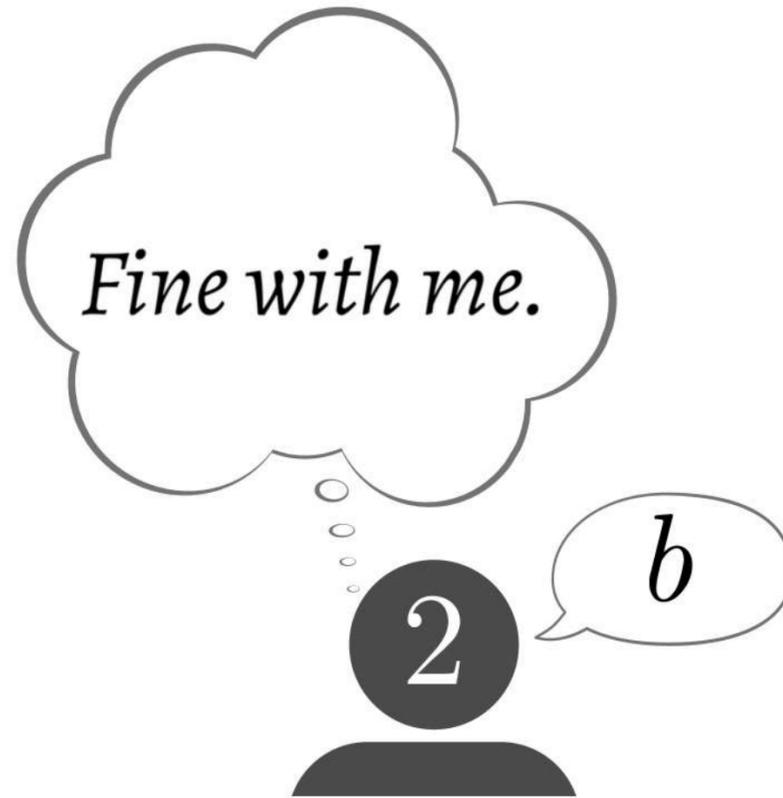
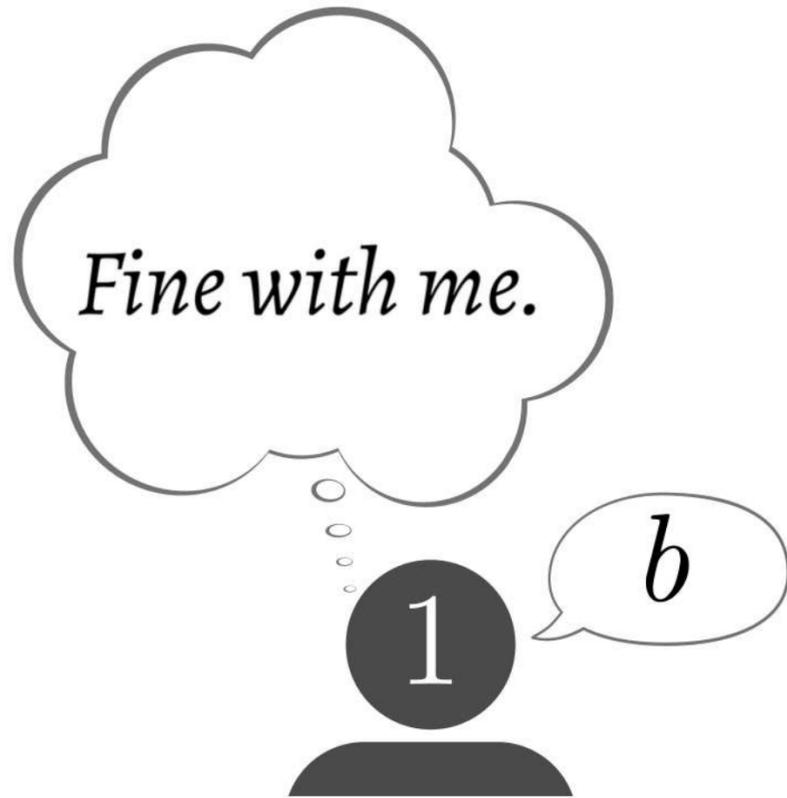
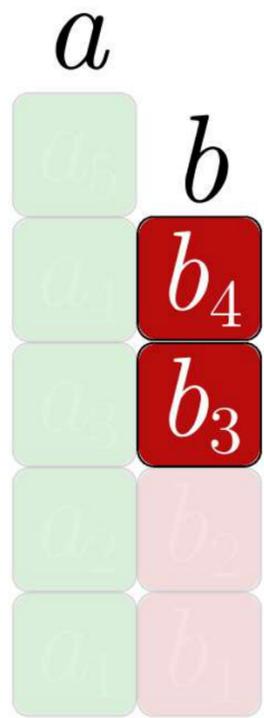
# DELIBERATION EXAMPLE

$t = 1$



# DELIBERATION EXAMPLE

$t = 2$



In this example deliberation leads to the wrong answer...

In this example deliberation leads to the wrong answer... but how likely is that to happen?

# MODEL

DELIBERATION

alternatives  $a, b$

best alternative  $a$

voters  $N = \{1, \dots, n\}$

voter  $i$ 's vote

collective decision over  $n$  voters

# MODEL

## DELIBERATION

alternatives	$a, b$
evidence for $a$	$A = \{a_1, \dots, a_\ell\}$
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deliberation rounds	$t \in \{0, 1, \dots\}$
$i$ 's private evidence at $t$	$A_i^t \subseteq A, B_i^t \subseteq B$

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$i$ 's private evidence at $t$	$A_i^t \subseteq A, B_i^t \subseteq B$	
voter $i$ 's vote at $t$	$v_i^t = \begin{cases} a & \text{if }  A_i^t  >  B_i^t  \\ b & \text{if }  A_i^t  <  B_i^t  \\ \emptyset & \text{if }  A_i^t  =  B_i^t  \end{cases}$	
	// vote for alternative with more evidence, abstain	
		// in case of tie

collective decision over  $n$  voters

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		// vote for alternative with more evidence, abstain // in case of tie
collective decision over $n$ voters at $t$	$\text{Vote}^t(n) \in \{a, b, \emptyset\}$	
		// alternative with most votes, or $\emptyset$ in case of tie

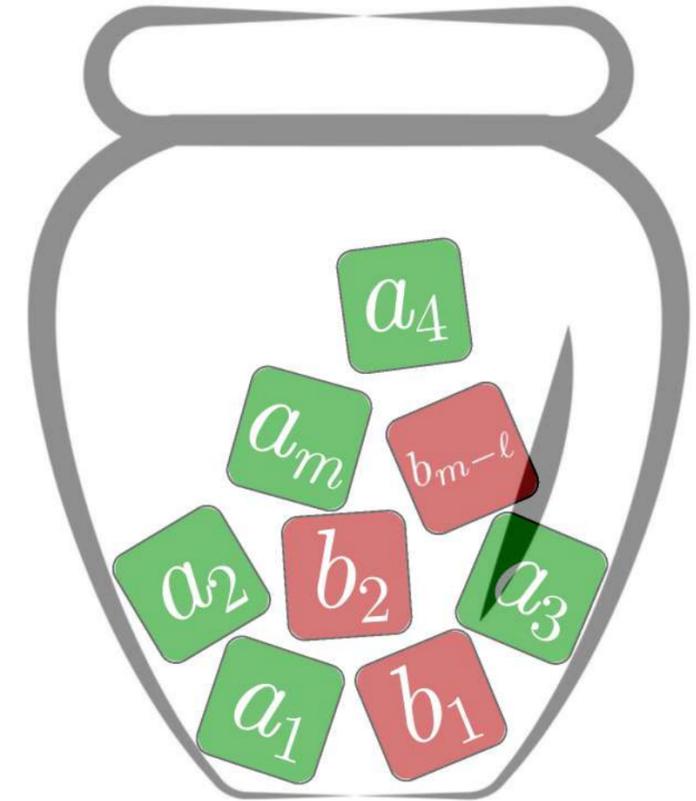
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## DELIBERATION

Fix a  $k \in \{1, \dots, m\}$ .

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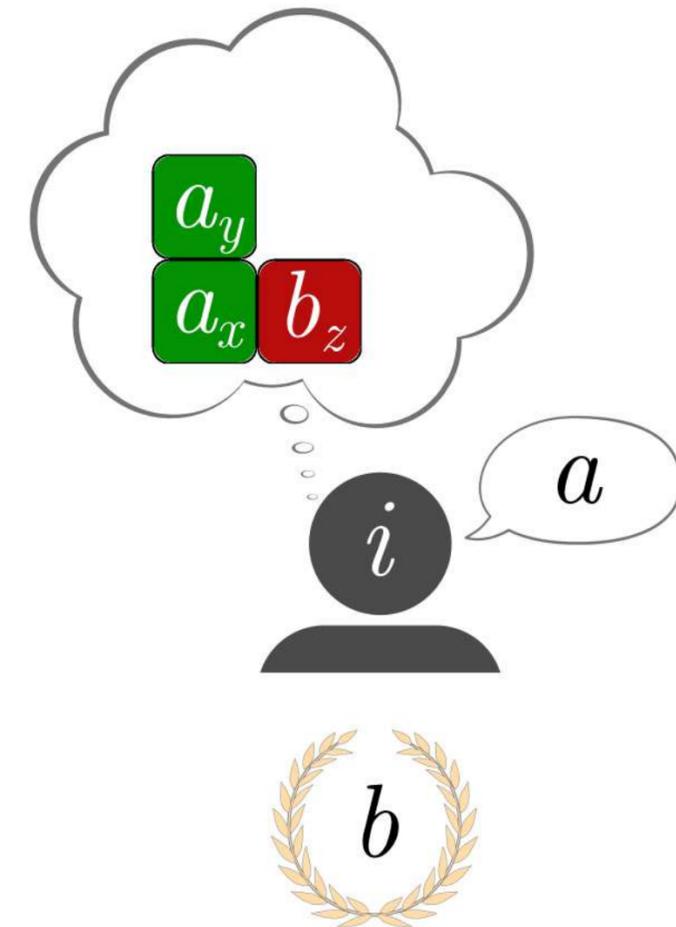
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round  $t$



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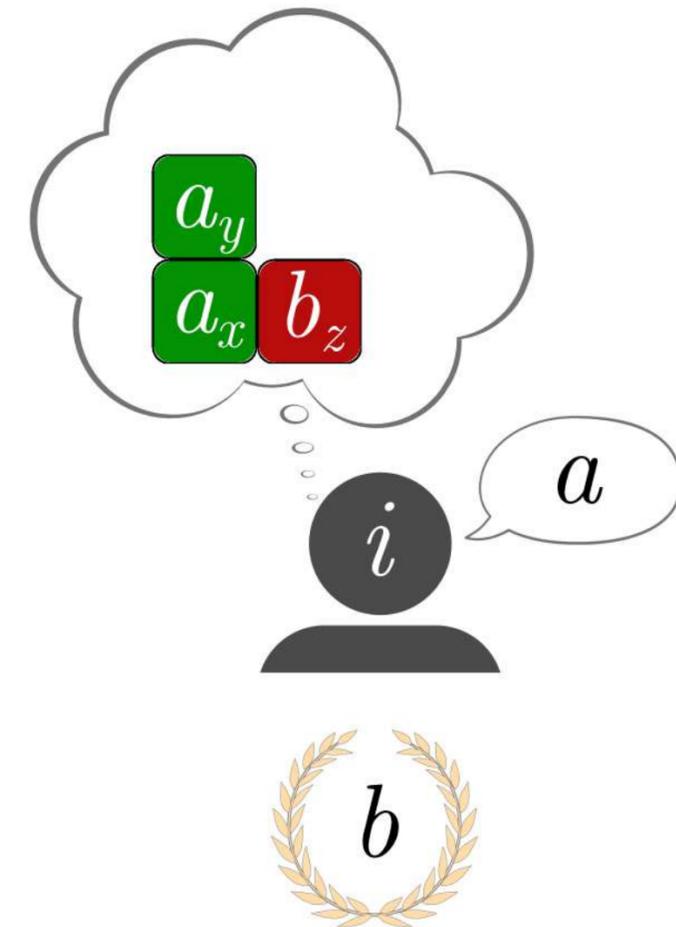
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//  $v_i^{t-1} = a$  and  $\text{Vote}^{t-1}(n) = b$ , or vice versa

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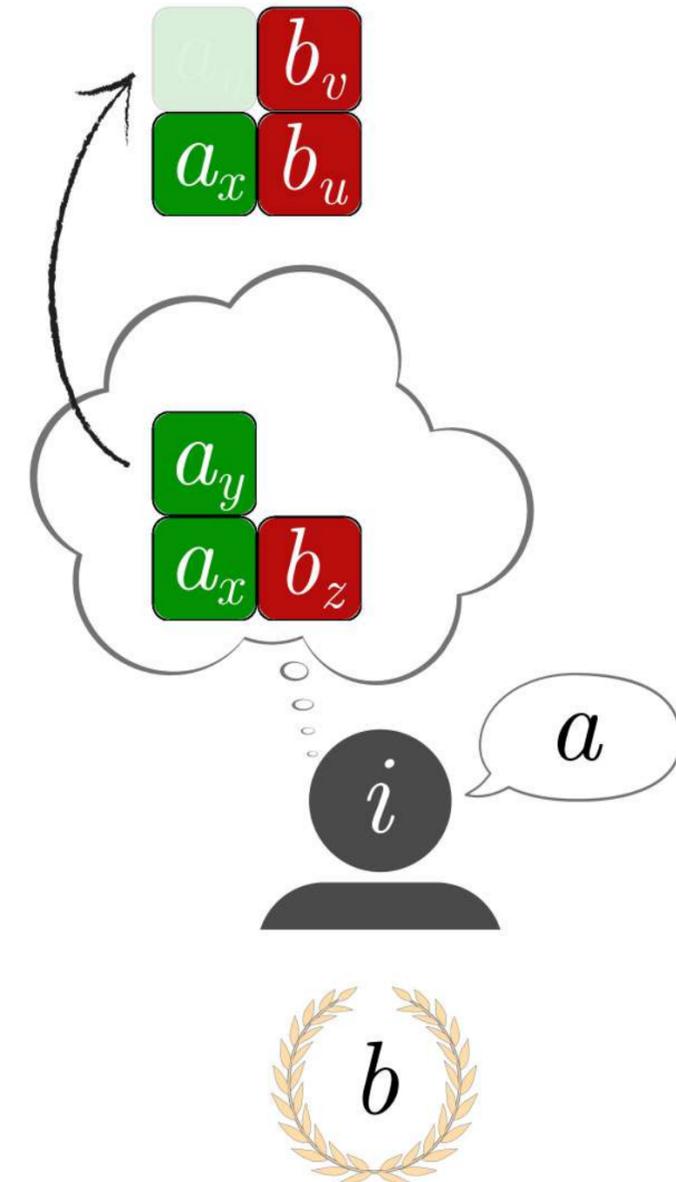
- ▷ each agent  $i$  compares opinion at  $t-1$  with collective decision at  $t-1$ : if  $i$  disagrees with the group, they disclose all evidence for their preferred alternative that hasn't been already disclosed;

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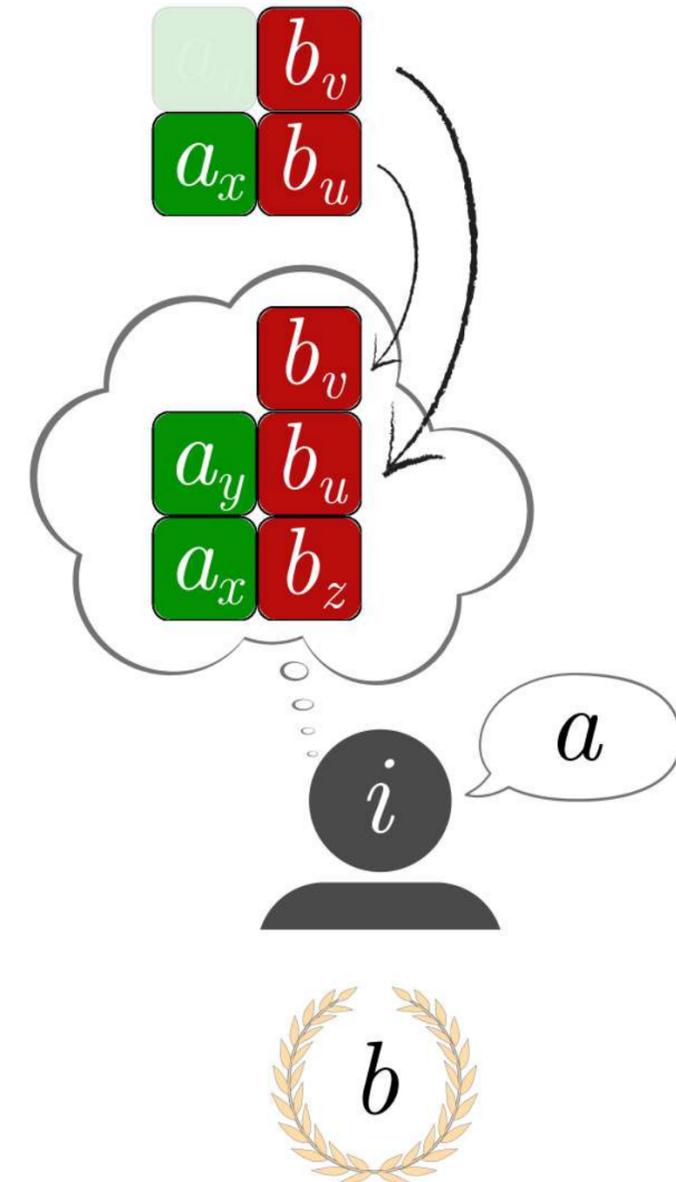
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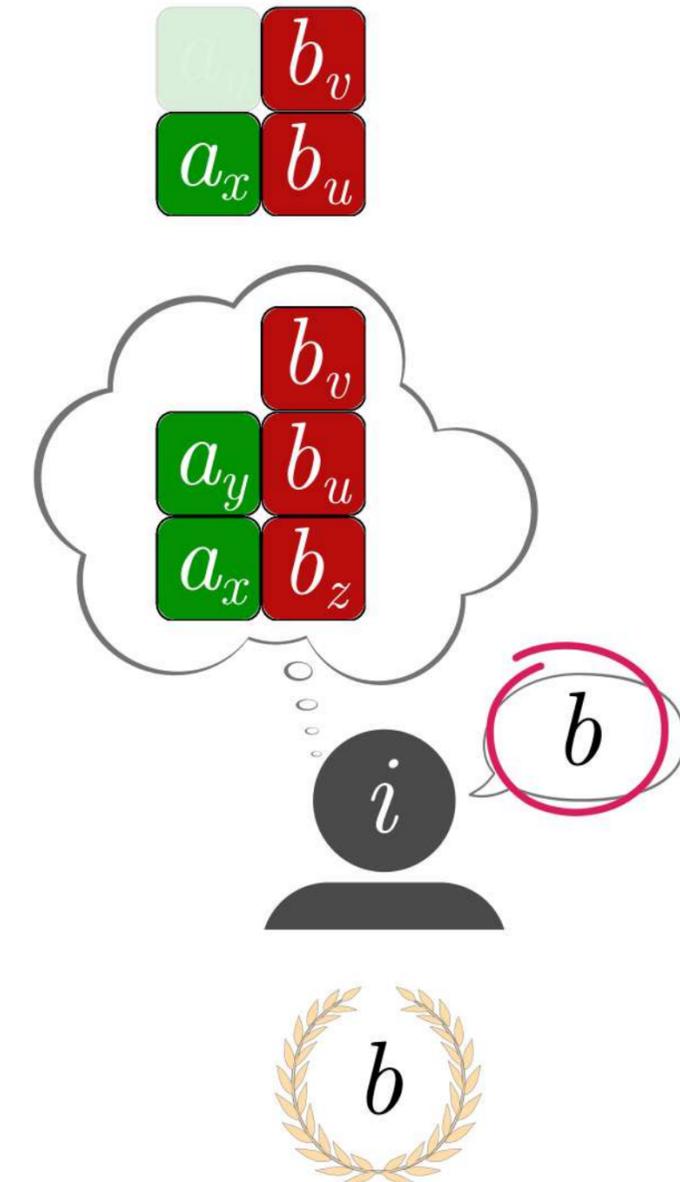
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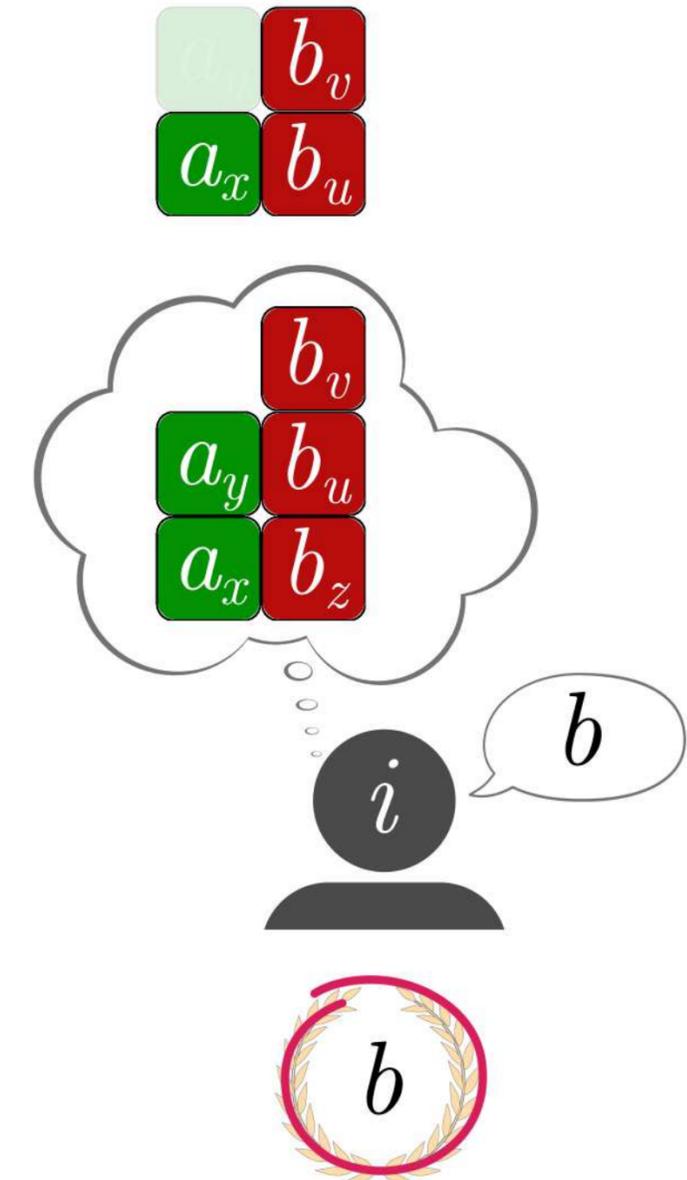
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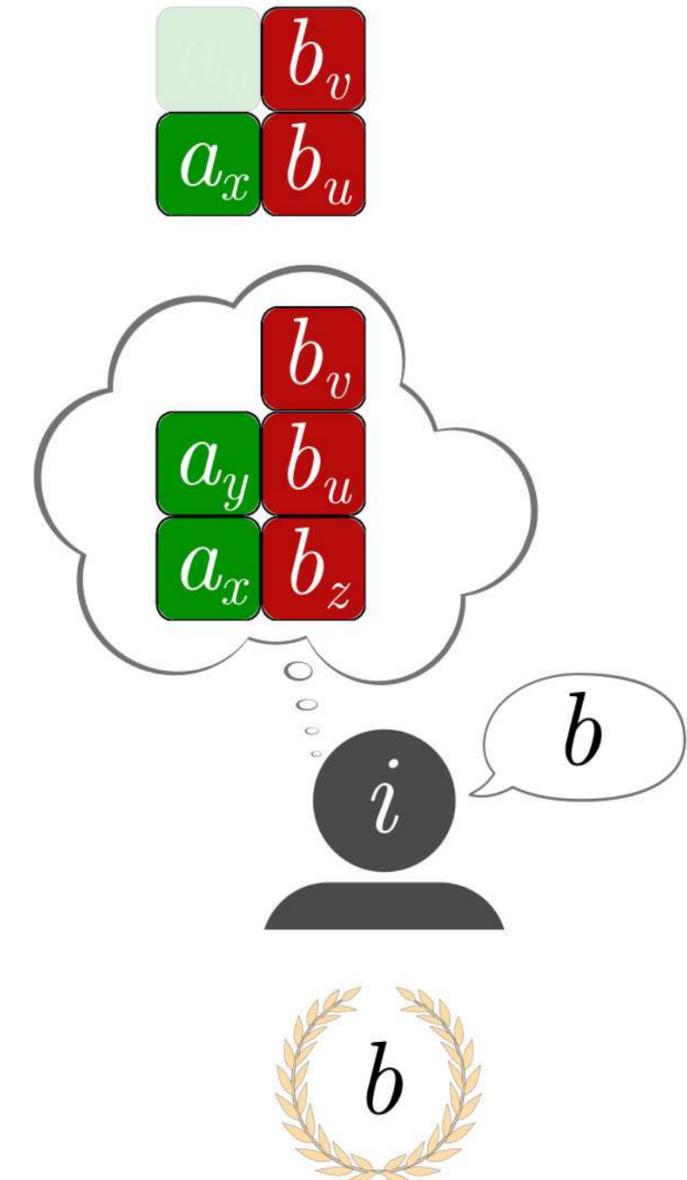
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Keep going until no new evidence is disclosed.

round  $t$



Note that deliberation terminates at some point. Call *the vote at termination*  $\text{Vote}^*(n)$ .

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What can we say about  $\text{Vote}^*(n)$ ? Especially as compared to  $\text{Vote}^0(n)$ .



ROSA

The assumption here is that agents are eager to correct a (perceived) mistake... is that realistic?



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In our experiments people tend to speak up in favor of the plurality option...



Table 1  
Some Possible Pregroup Distributions of Seven Pro-A ( $a_i$ ) and Four Pro-B ( $b_i$ ) Items of Information Over Three Group Members

Item position	Group member		
	X	Y	Z
Case 1: All information shared			
Pro-A	$a_1, a_2, a_3, a_4, a_5, a_6, a_7$	$a_1, a_2, a_3, a_4, a_5, a_6, a_7$	$a_1, a_2, a_3, a_4, a_5, a_6, a_7$
Pro-B	$b_1, b_2, b_3, b_4$	$b_1, b_2, b_3, b_4$	$b_1, b_2, b_3, b_4$
Case 2: Unbiased distribution			
Pro-A Shared	$a_1$	$a_1$	$a_1$
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Case 3: Mildly biased distribution			
Pro-A Shared	$a_1, a_2, a_3, a_4$	$a_1, a_2, a_3, a_4$	$a_1, a_2, a_3, a_4$
Pro-A Unshared	$a_5$	$a_6$	$a_7$
Pro-B*	$b_1, b_2, b_3, b_4$	$b_1, b_2, b_3, b_4$	$b_1, b_2, b_3, b_4$
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\* All shared.

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ADRIAN

So with our truth-seeking agents we should the group pooling information efficiently...

...right?

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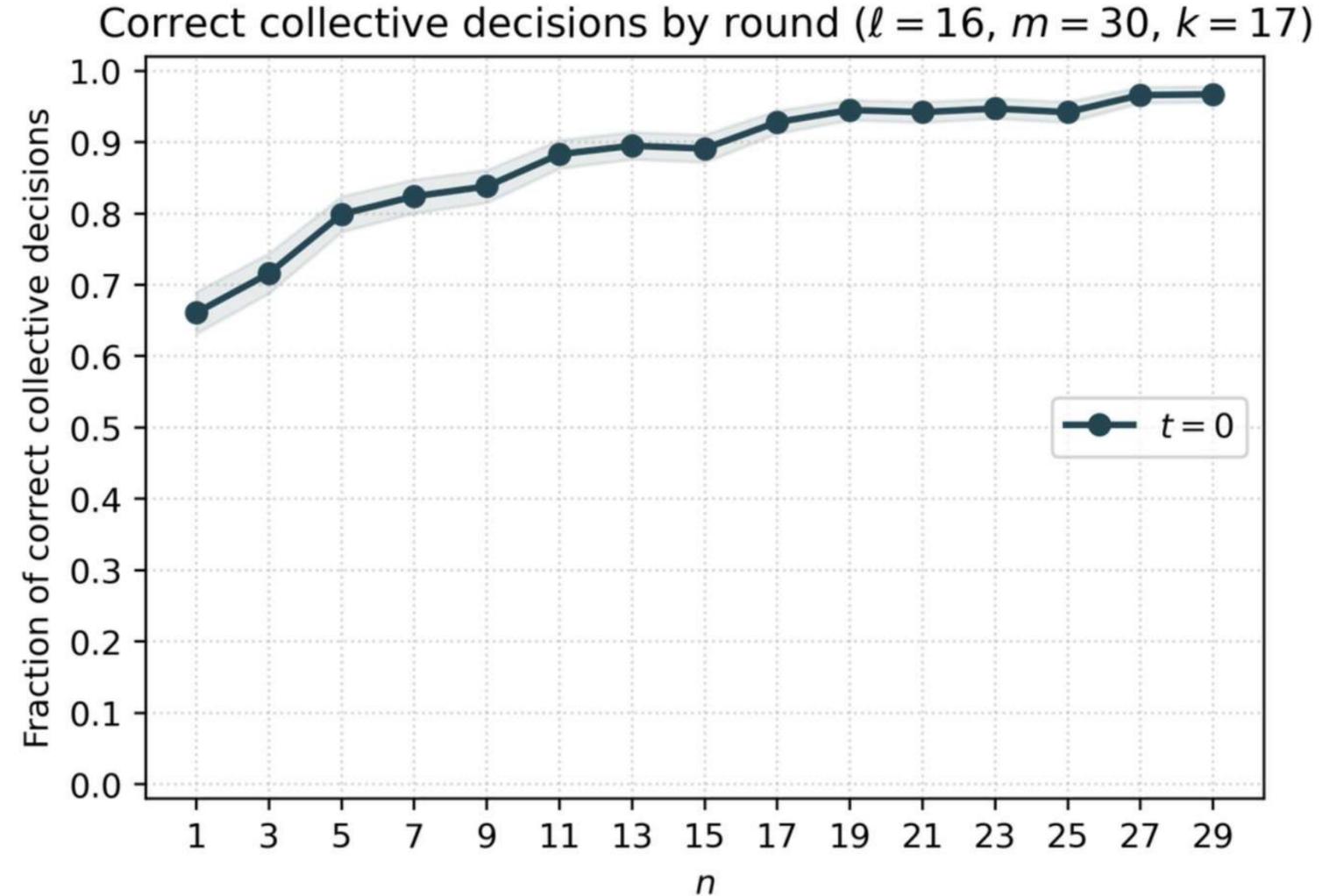
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# LET'S SIMULATE

Generate 1000 deliberation instances, show statistics per round.

Round 0 shows increasing group accuracy.

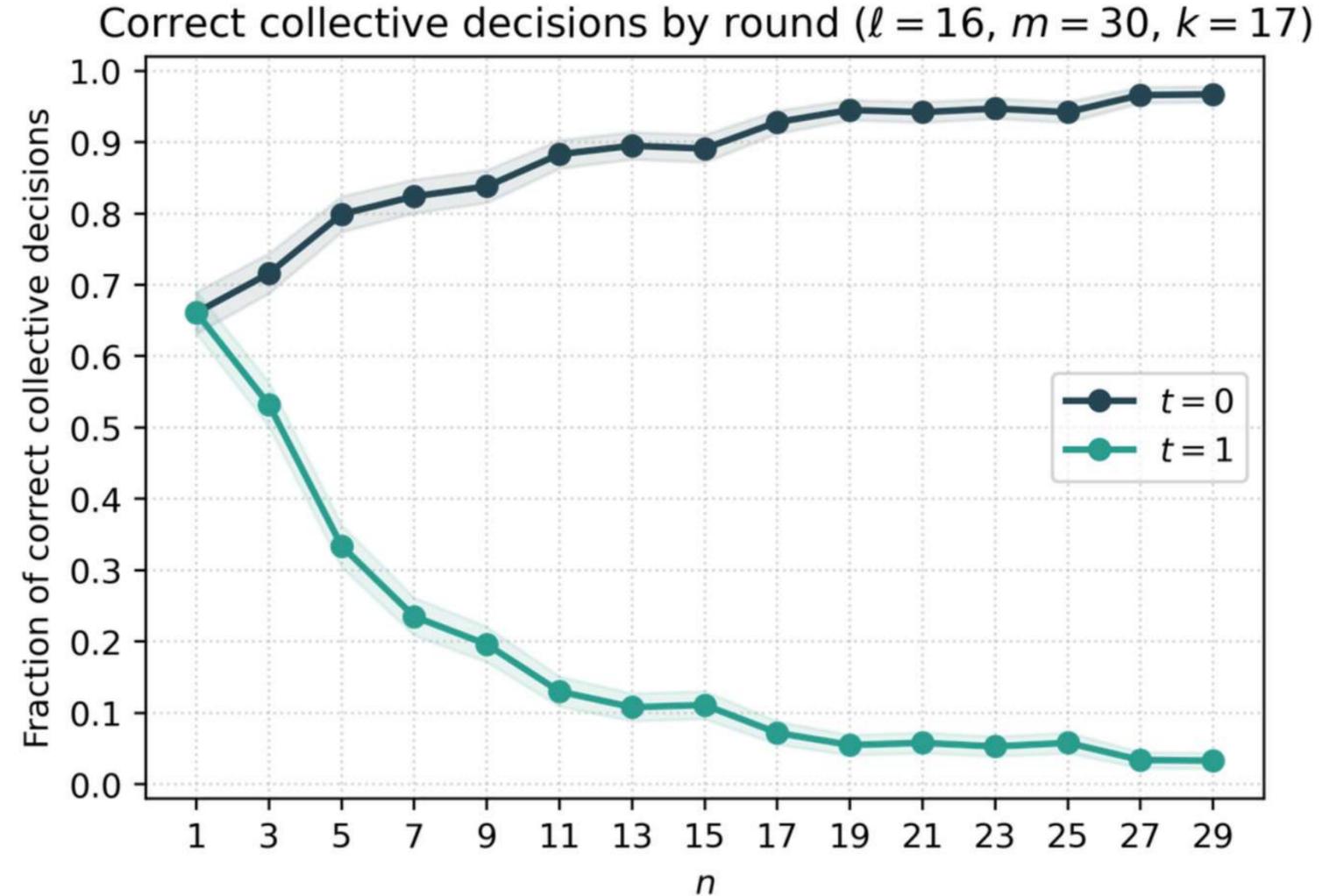


# LET'S SIMULATE: ARE WE COOKED?

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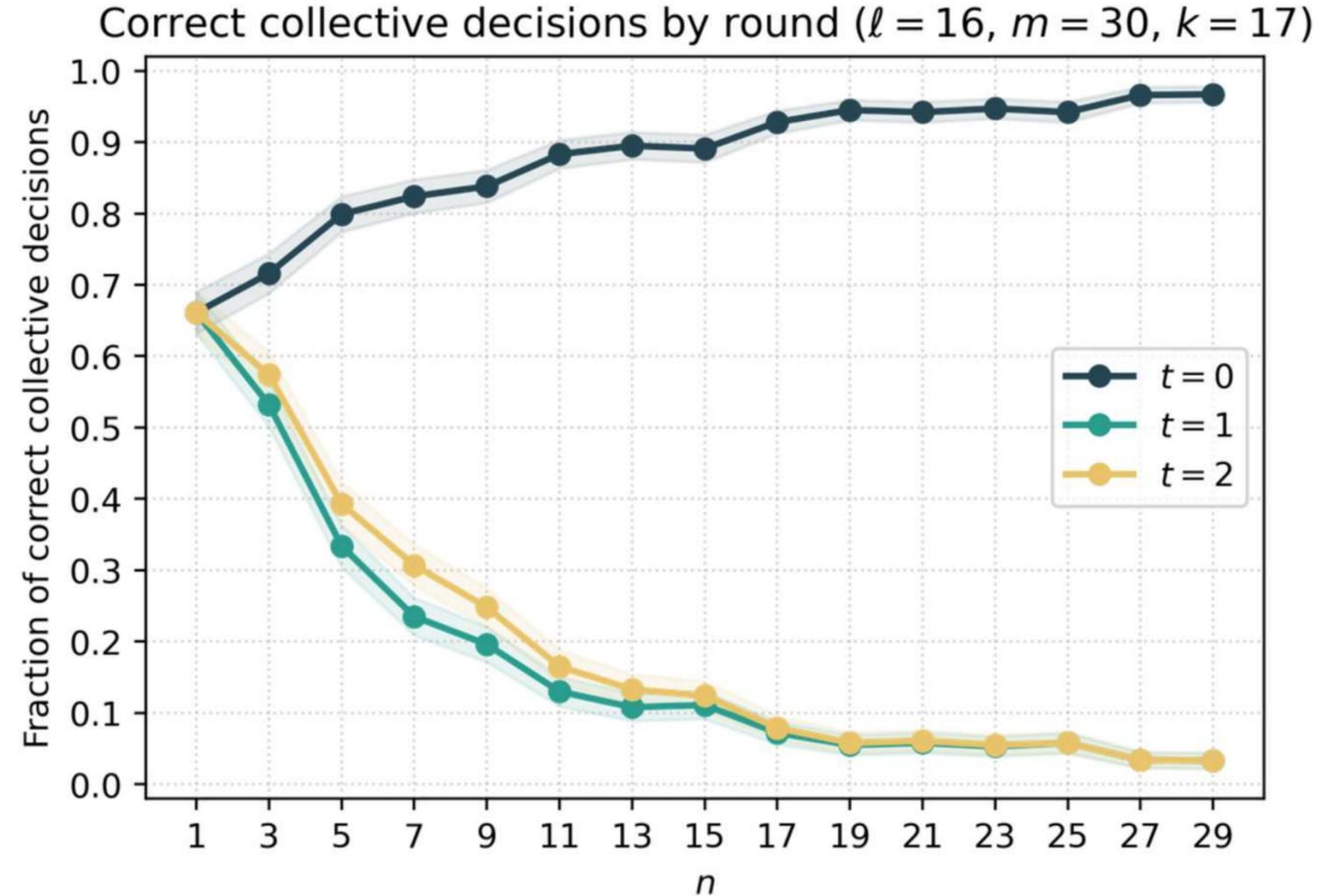
# LET'S SIMULATE: ARE WE COOKED?

Generate 1000 deliberation instances, show statistics per round.

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Round 2: still bad.



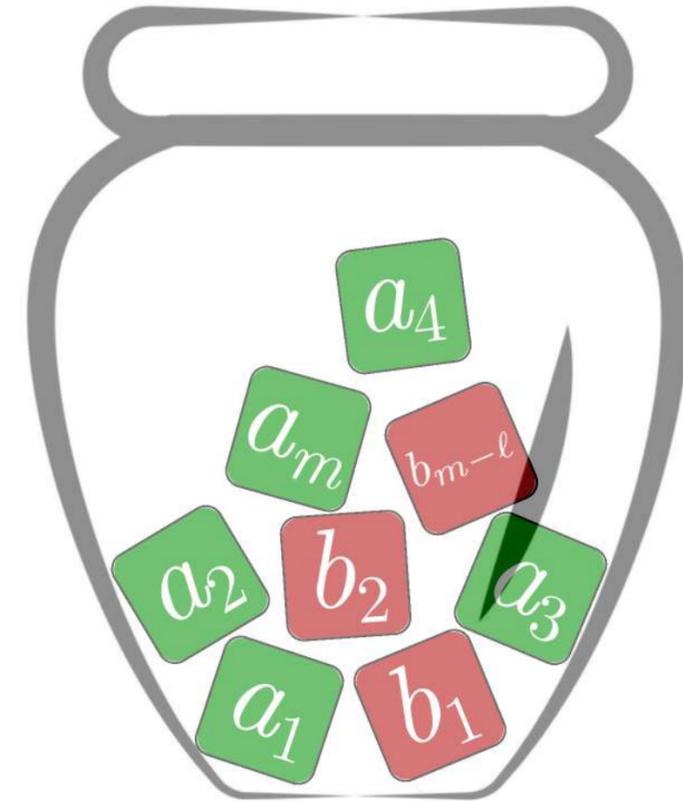
Is deliberation doomed?

Is deliberation doomed? To see what's going on, let's take snapshots of the deliberation process at various rounds.

# ROUND 0: SETUP

$$|A_i^0| \sim \text{Hypergeometric}(m, \ell, k)$$

// population size  $m$ ,  $\ell$  successes,  $k$  draws



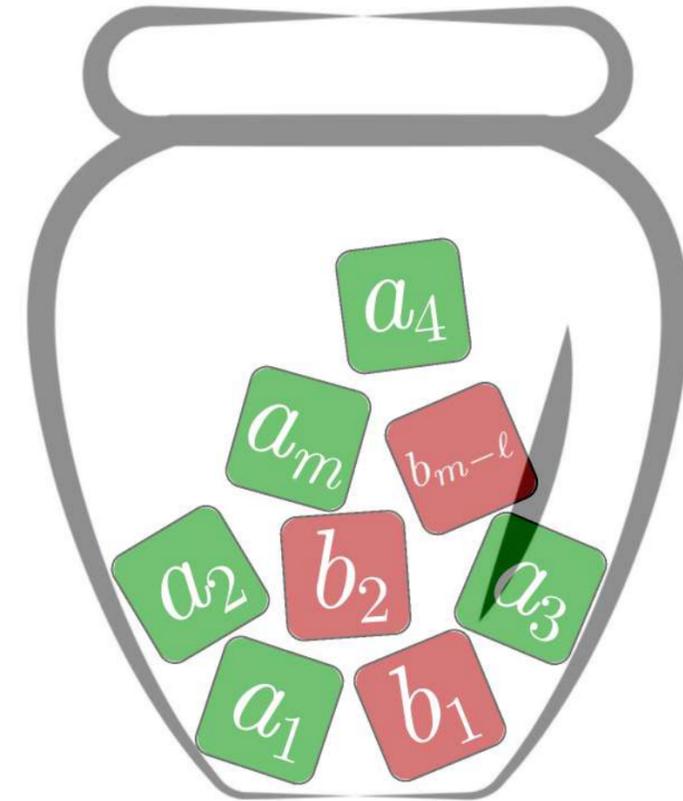
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⇓

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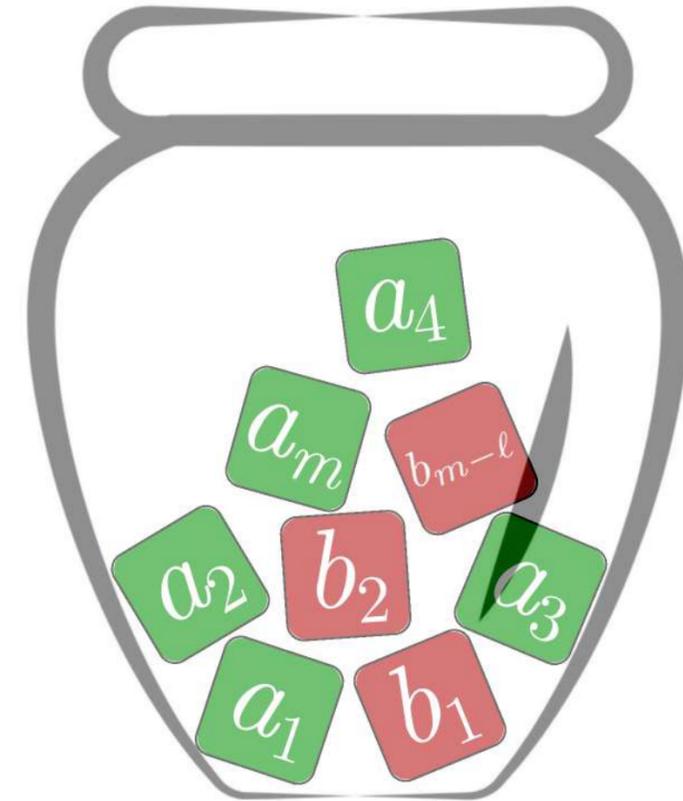
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$$\Pr\left[v_i^0 = a\right] = \Pr\left[|A_i^0| > \frac{k}{2}\right]$$



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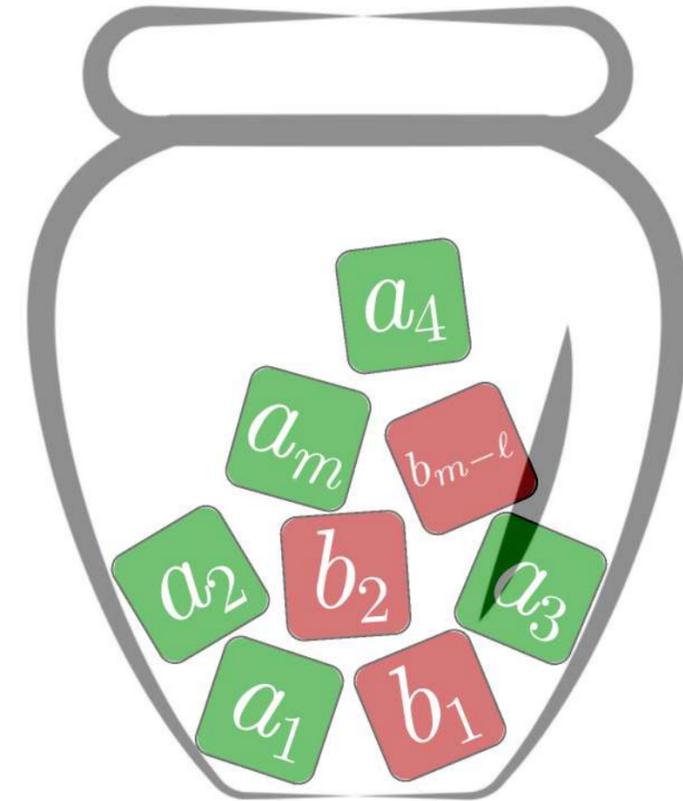
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⇓

$$\begin{aligned} \Pr\left[v_i^0 = a\right] &= \Pr\left[|A_i^0| > \frac{k}{2}\right] \\ &= \text{big sum} \end{aligned}$$



Does the group vote correctly initially?

# ROUND 0: GENERAL RESULTS

For any agent  $i$  it holds that:

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If  $k$  is odd then:

$$\Pr\left[|A_i^0| > \frac{k}{2}\right] + \Pr\left[|A_i^0| < \frac{k}{2}\right] = 1, \text{ and} \quad (1)$$

$$\Pr\left[|A_i^0| = j\right] > \Pr\left[|A_i^0| = k - j\right], \text{ for } j > \frac{k}{2}. \quad (2)$$

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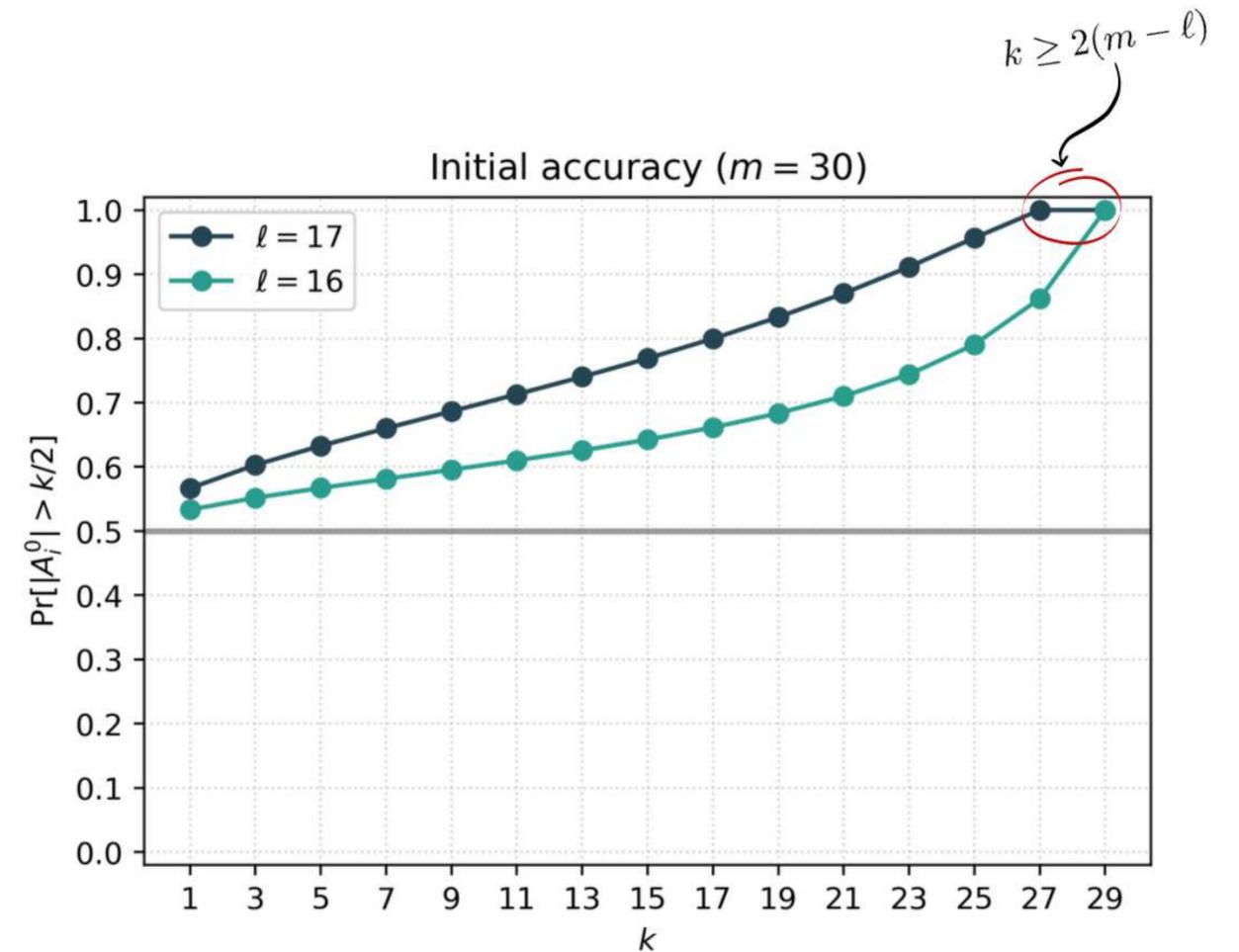
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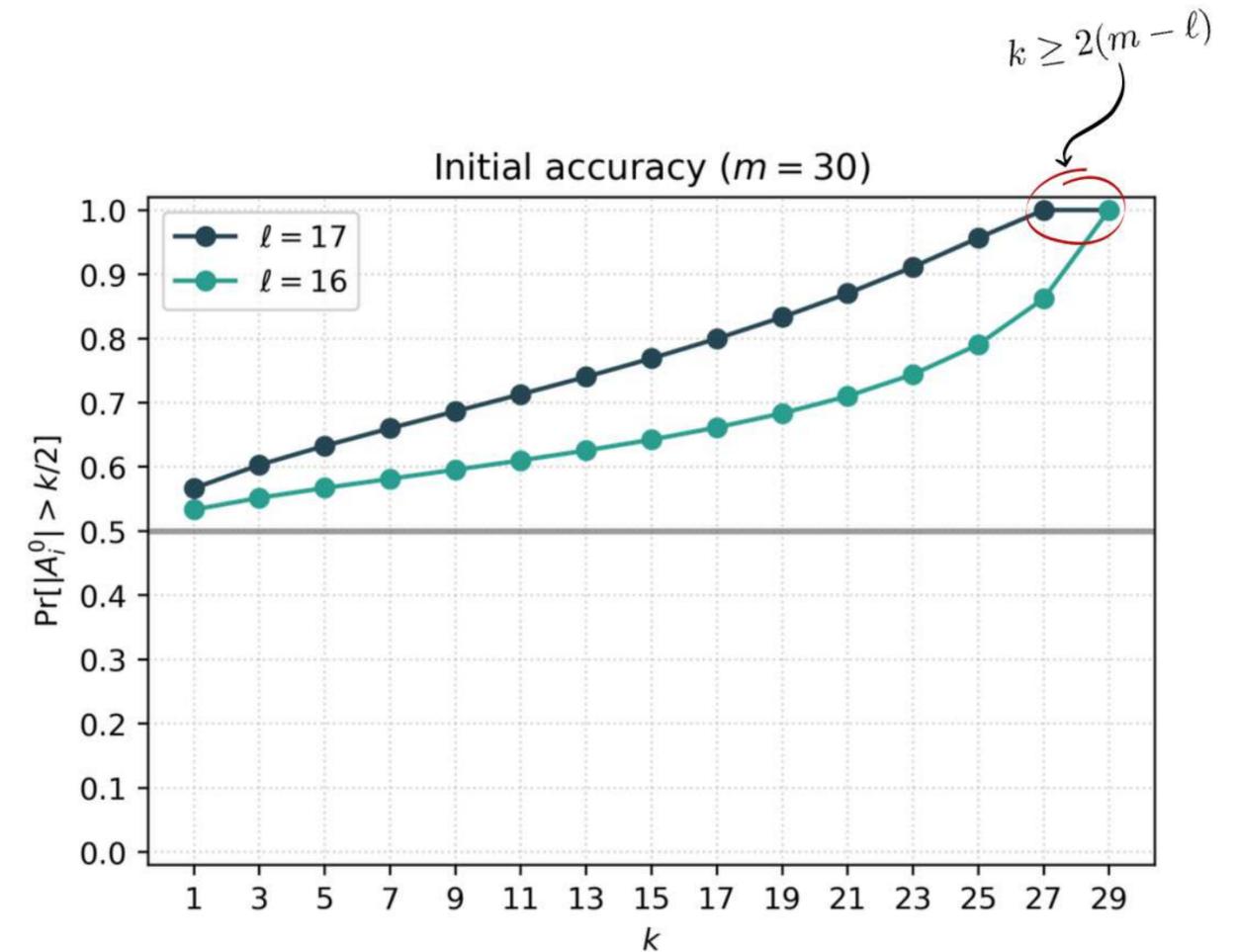
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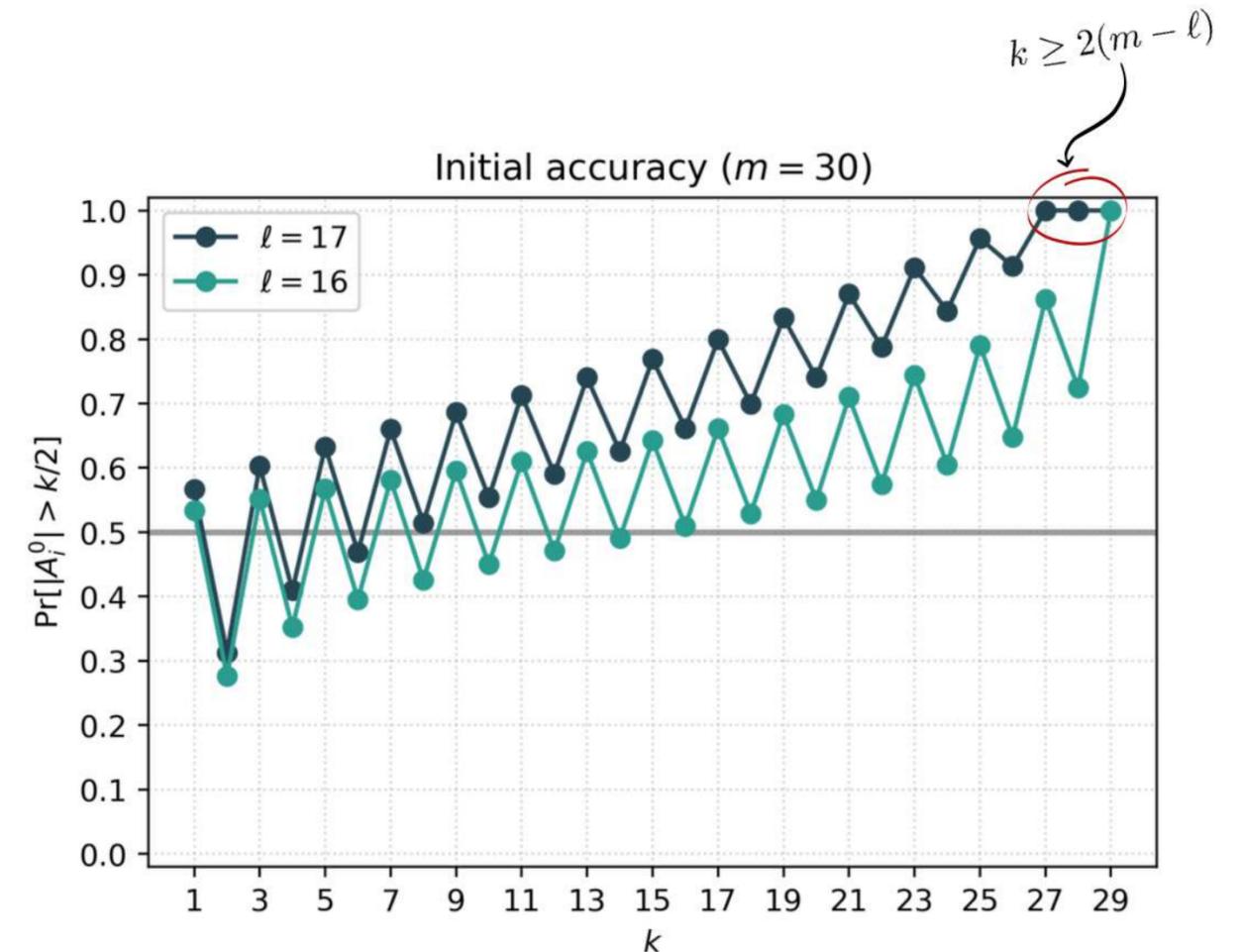
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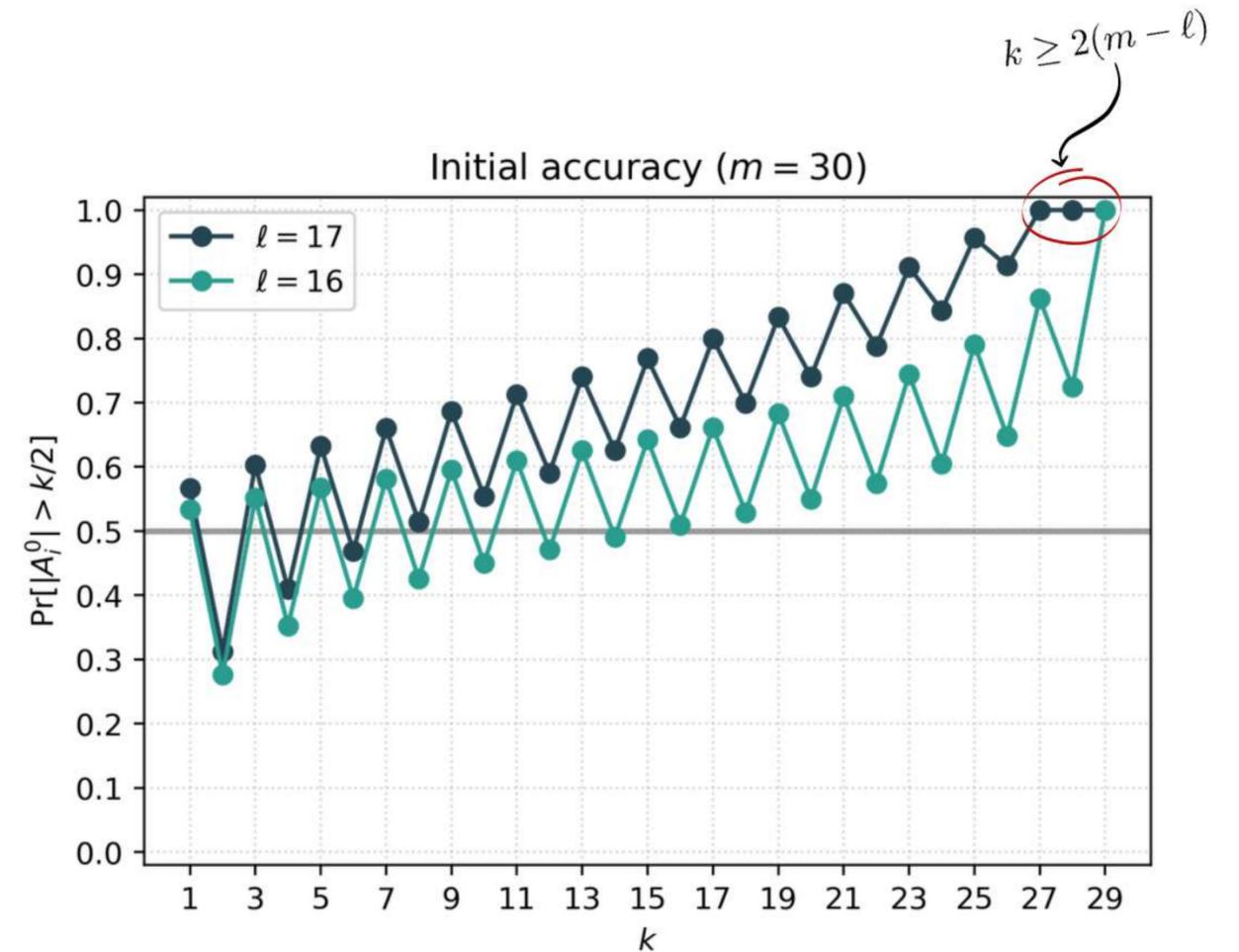
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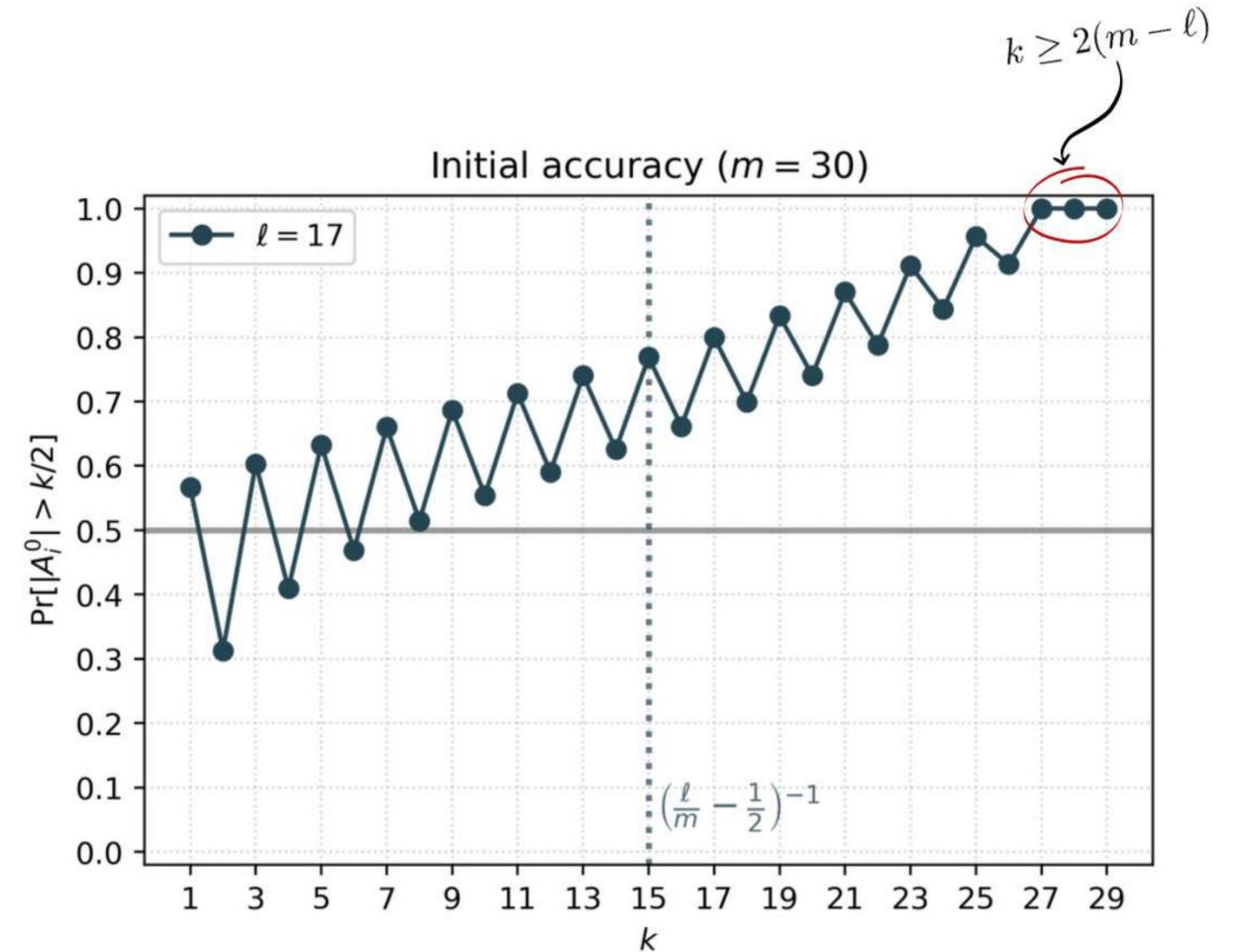
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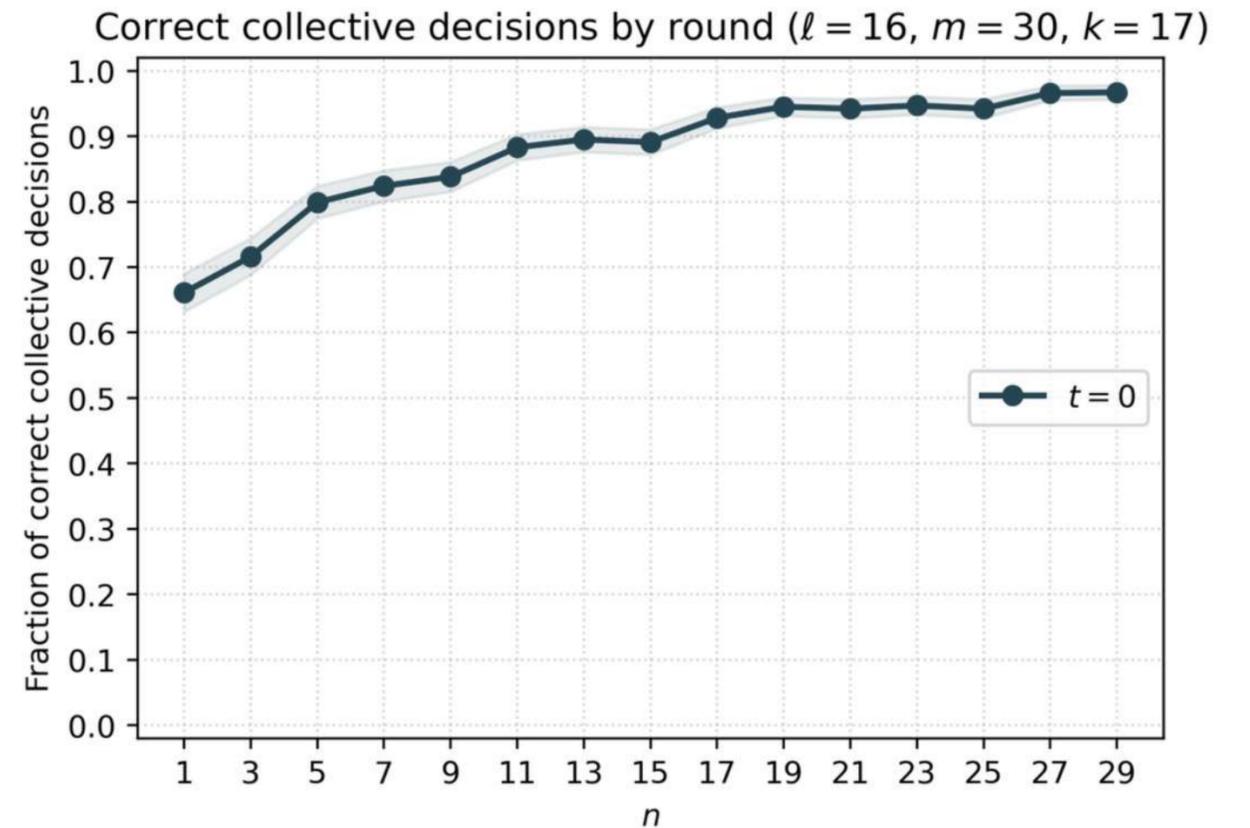
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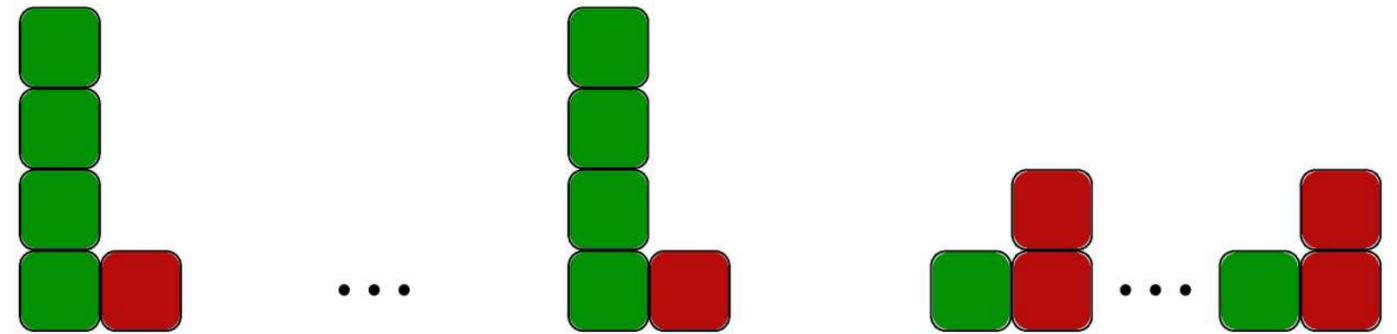


What happens at the next round?

# ROUND 1: SETUP

Round 1 starts with a majority for  $a...$

$t = 1$



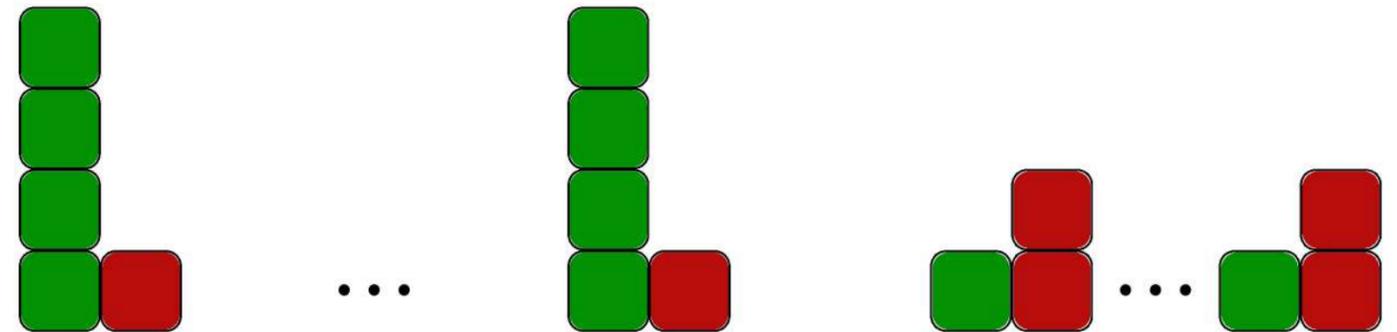
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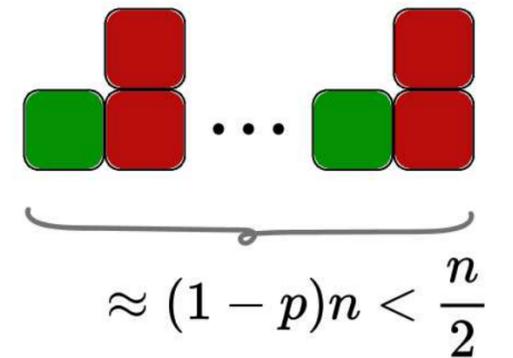
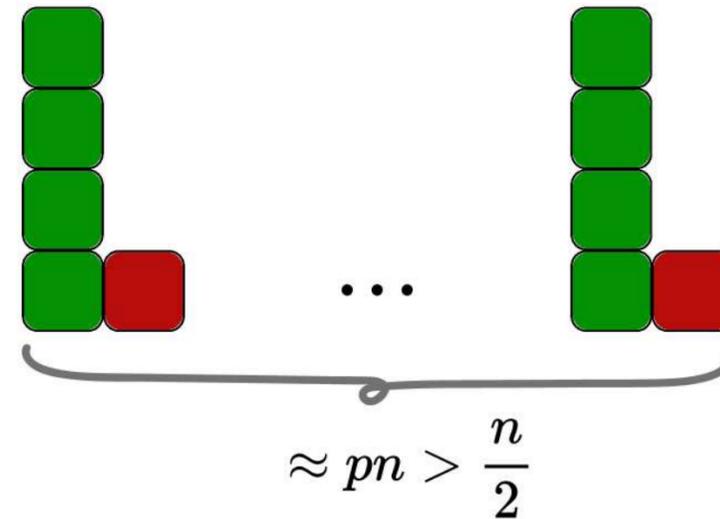
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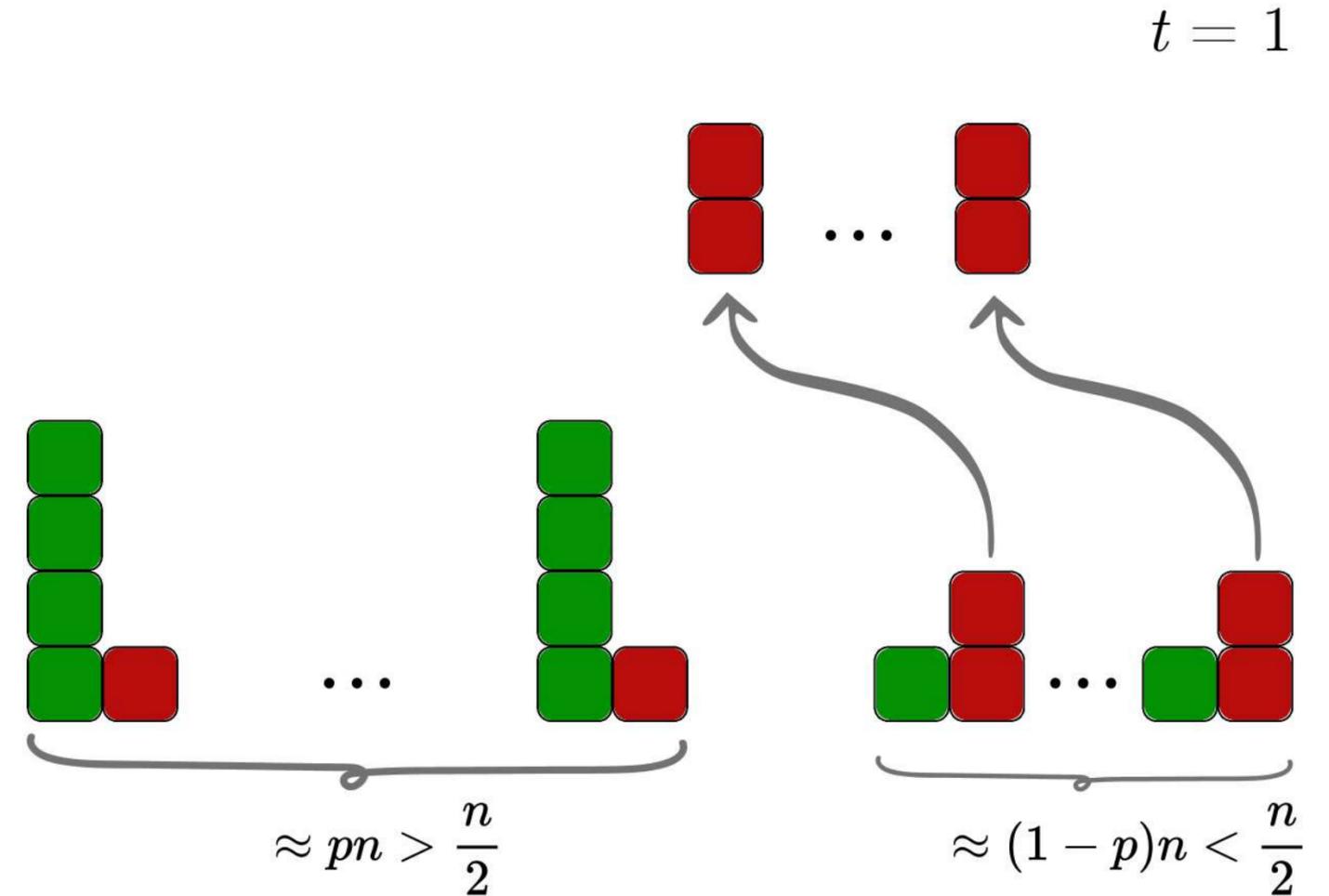
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$b$  voters disclose private evidence for  $b$

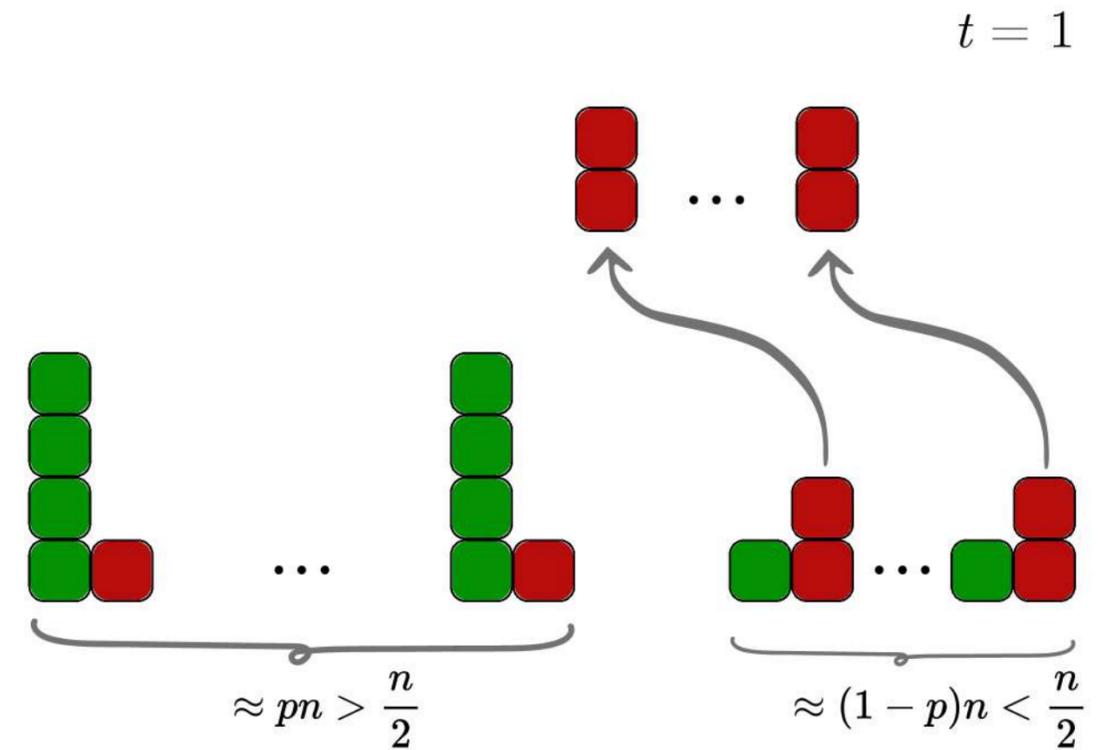


What happens with the collective decision at round 1?

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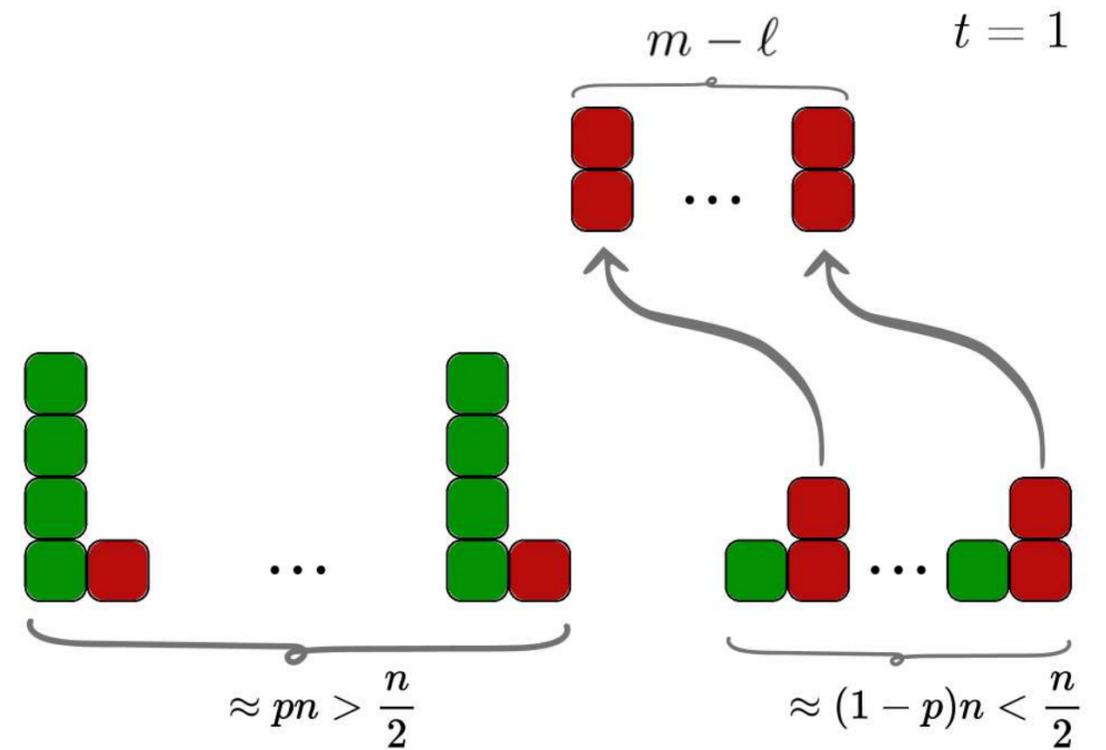


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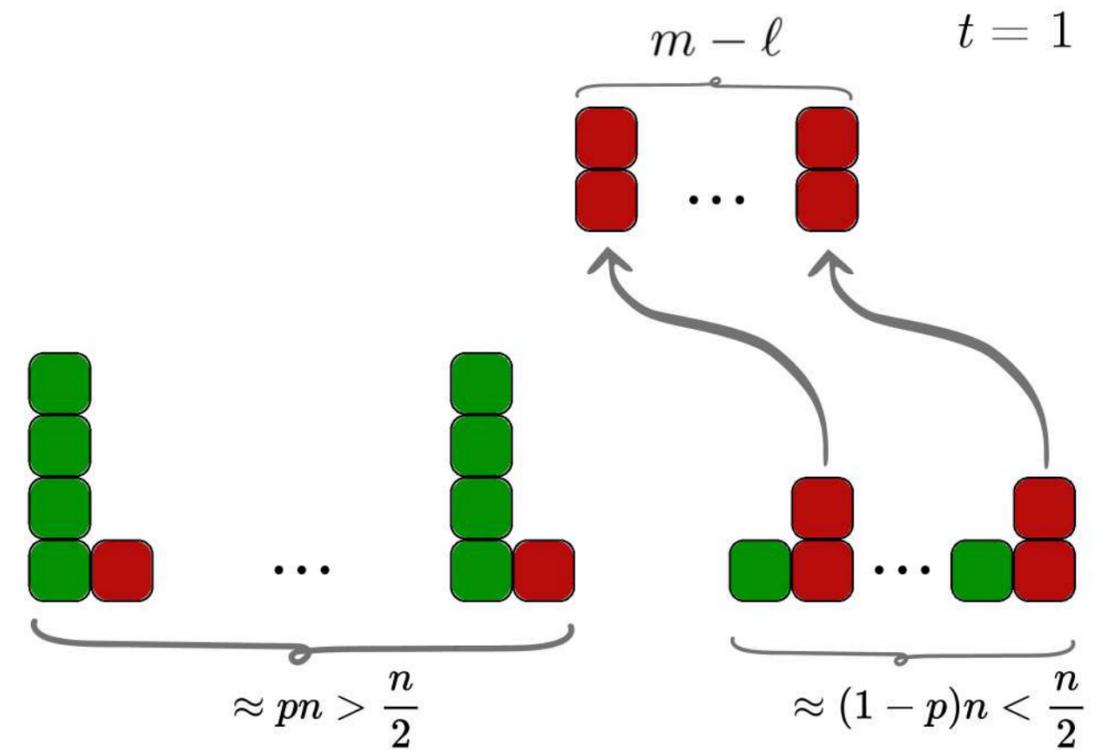


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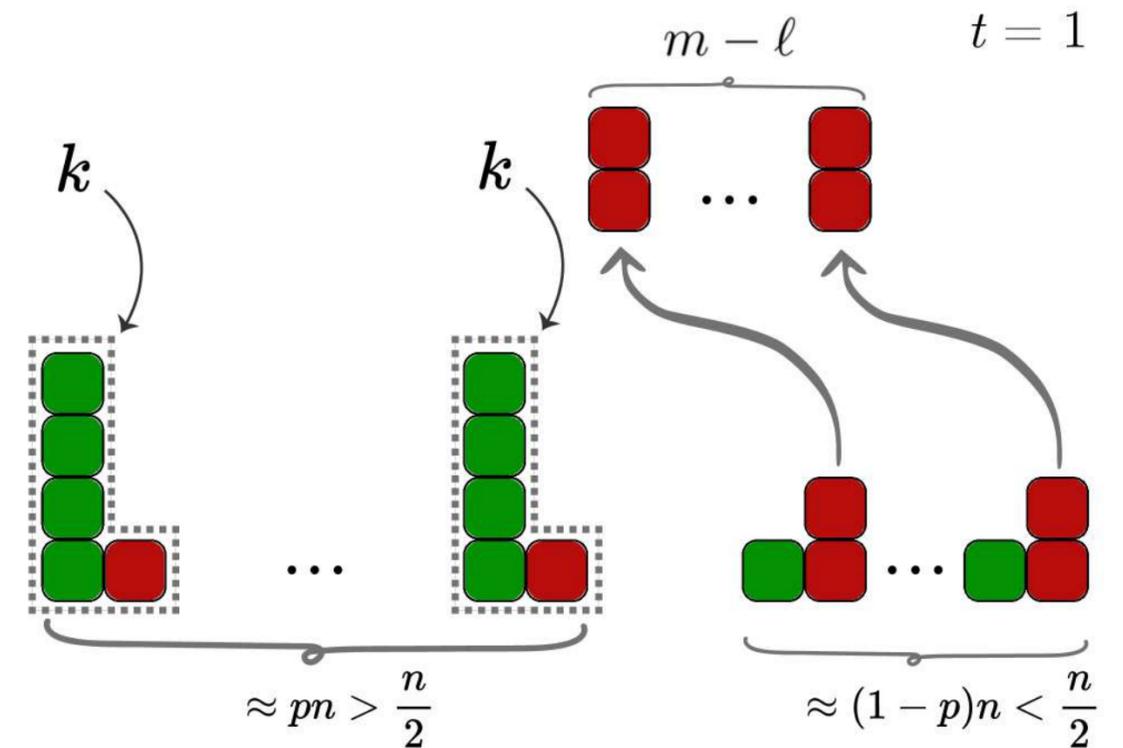
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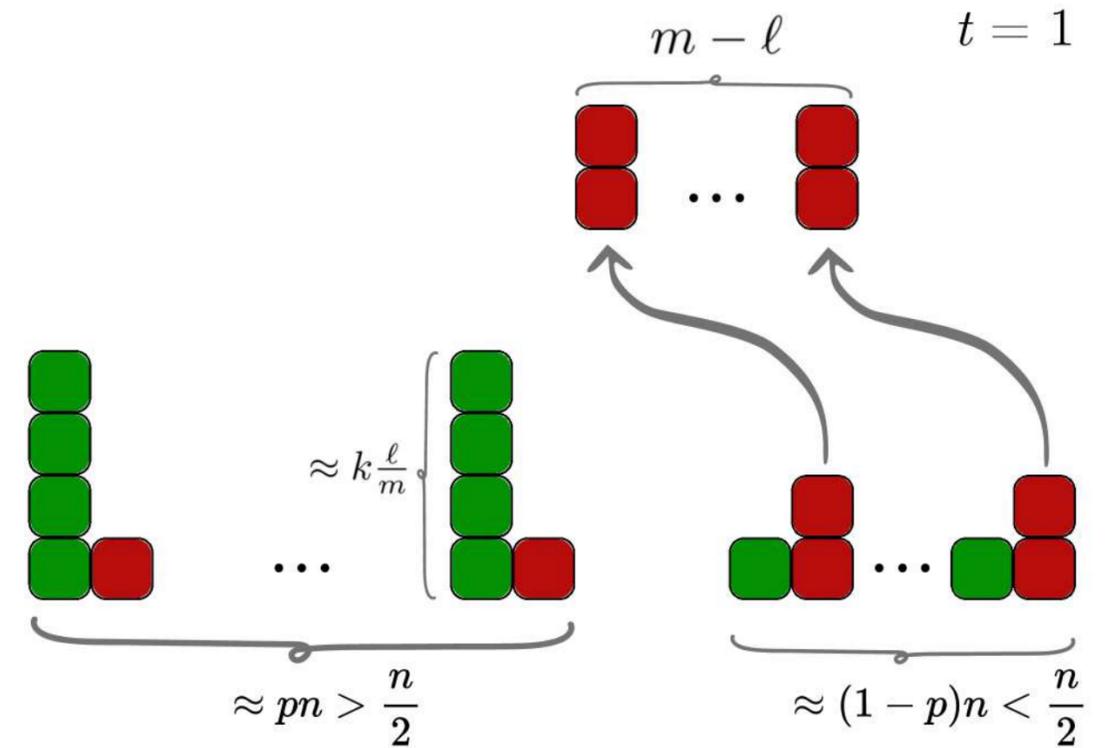
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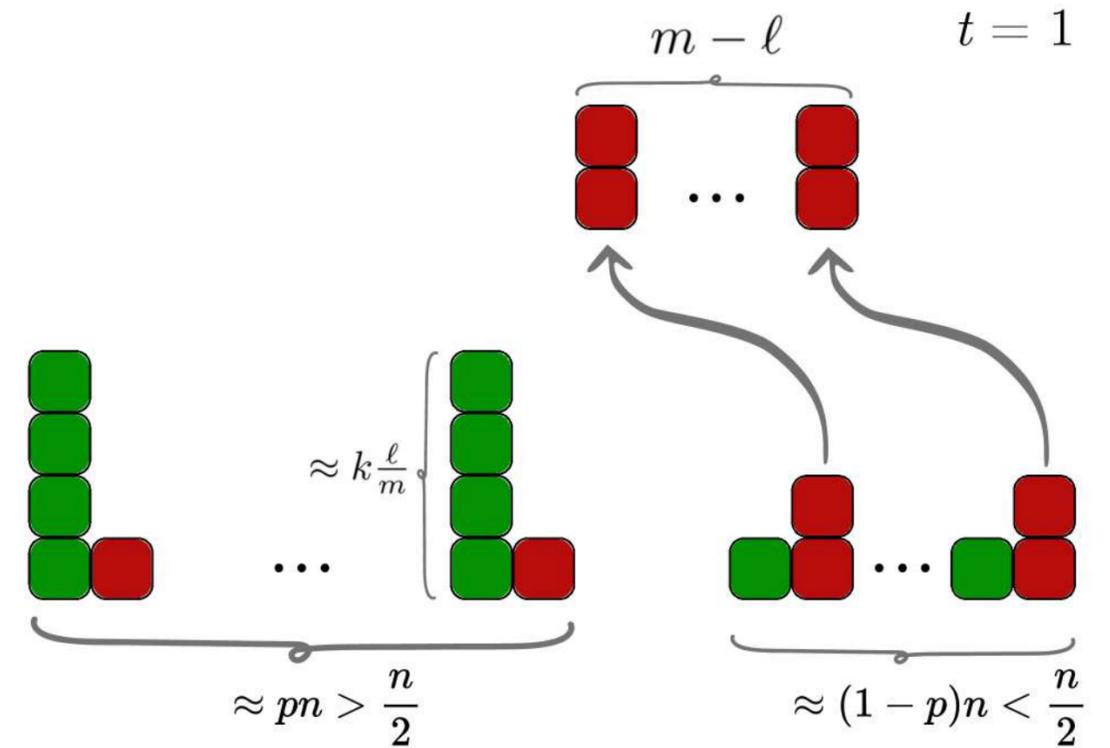
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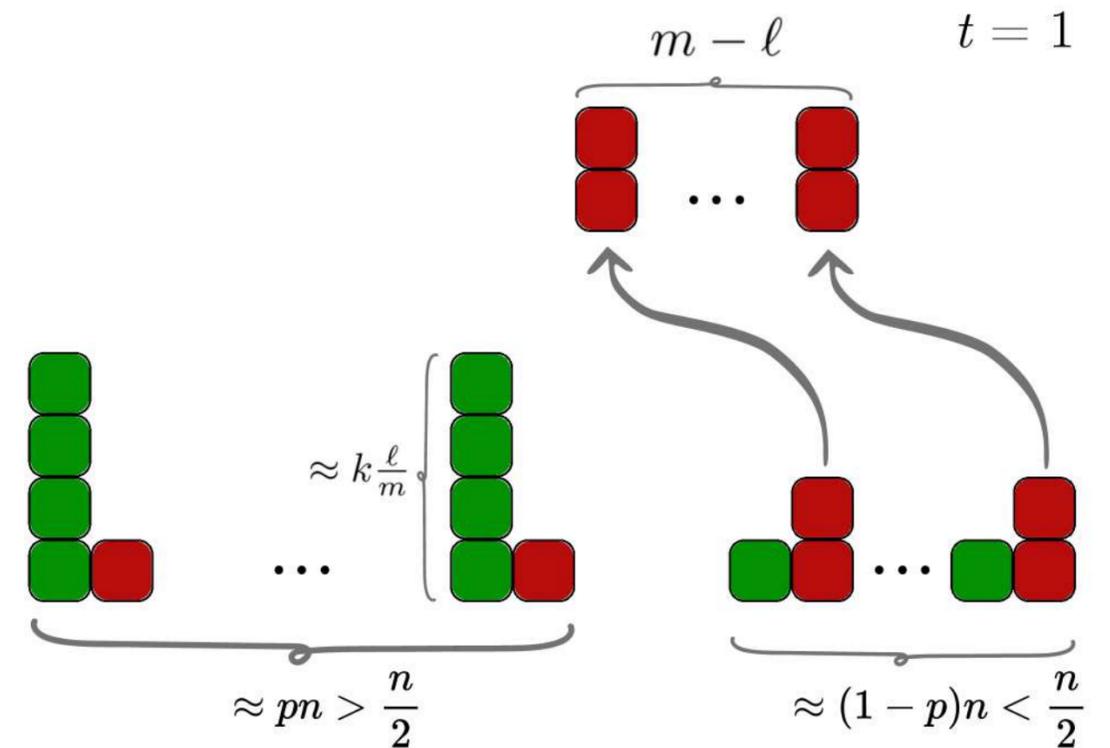
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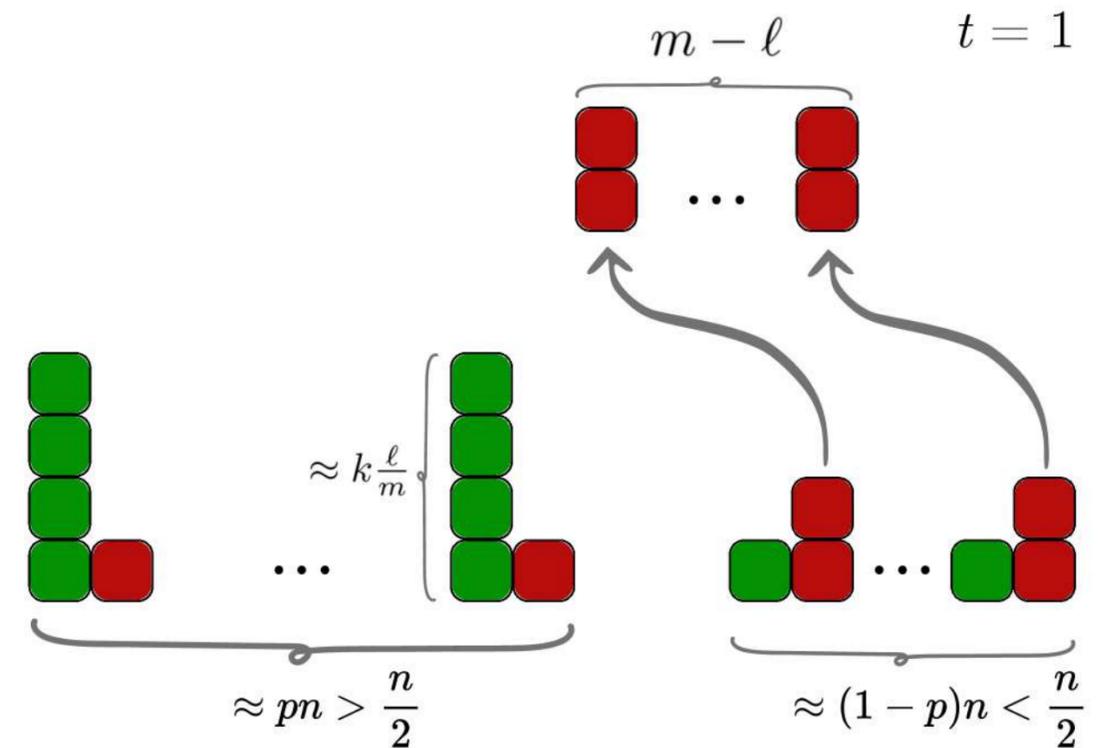
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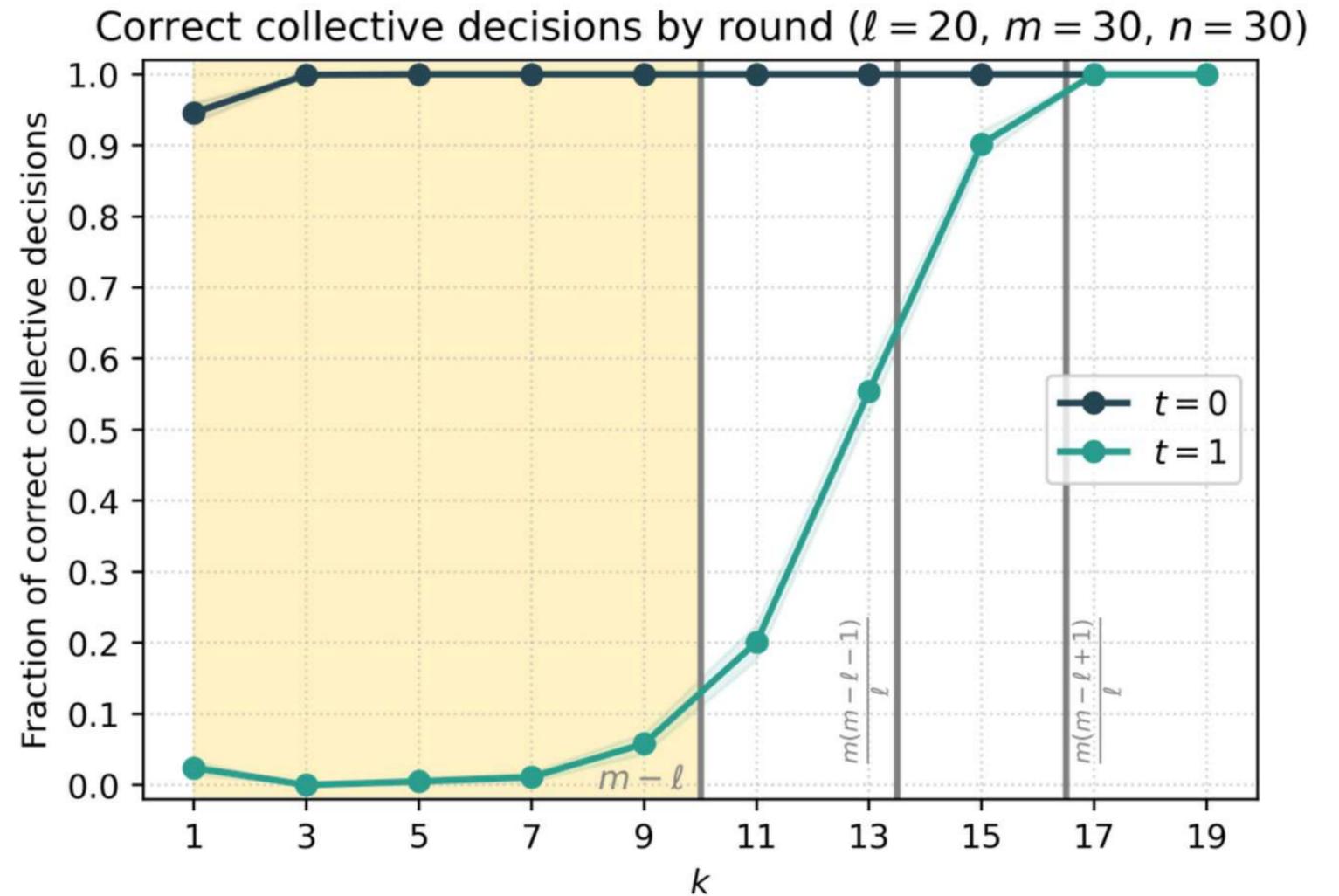
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(i) if  $k \leq m - \ell$ , then:

$$\lim_{n \rightarrow \infty} \Pr[\text{Vote}^1(n) = b] = 1,$$

and deliberation terminates with  $b$  as winner;

// total flip to  $b$



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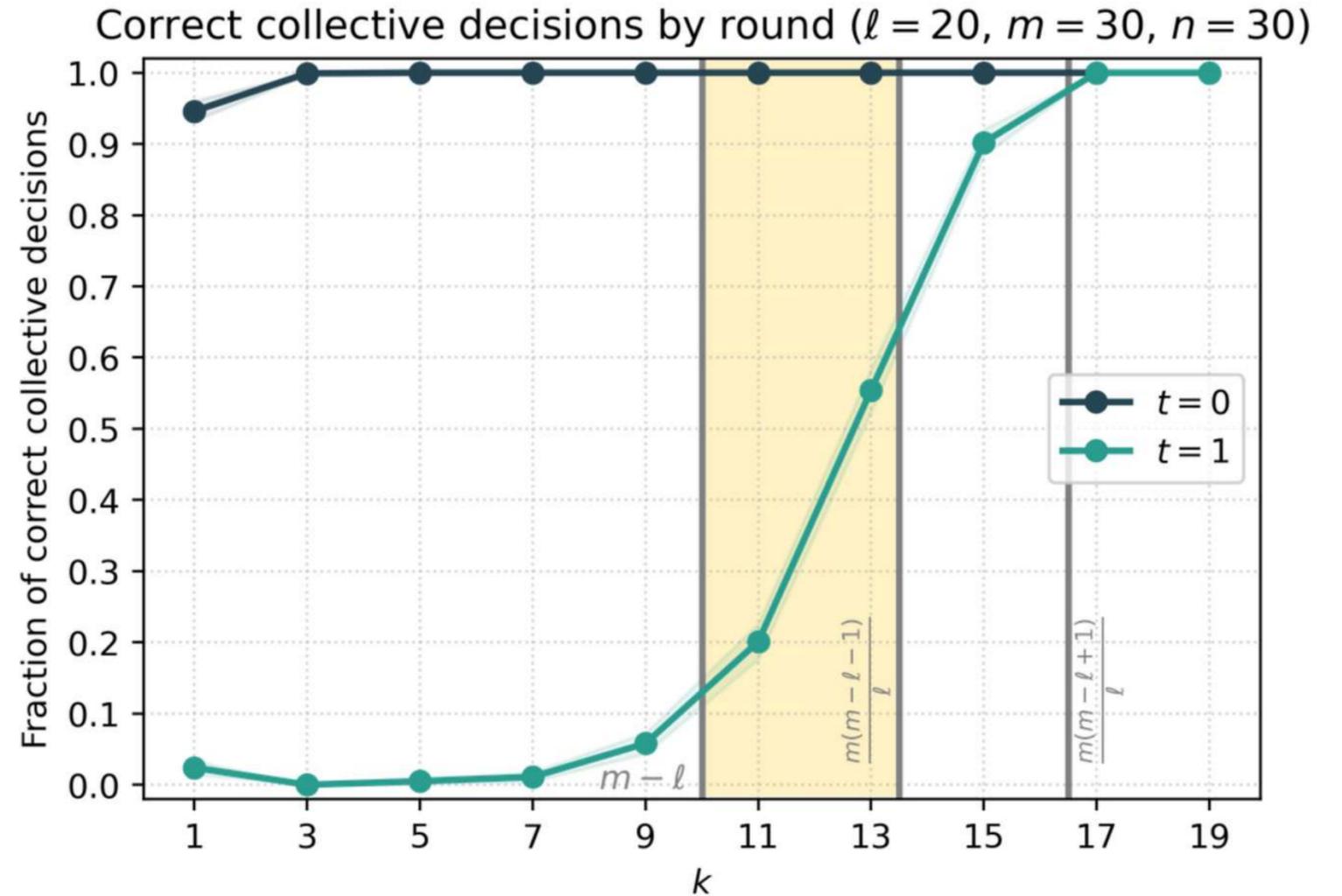
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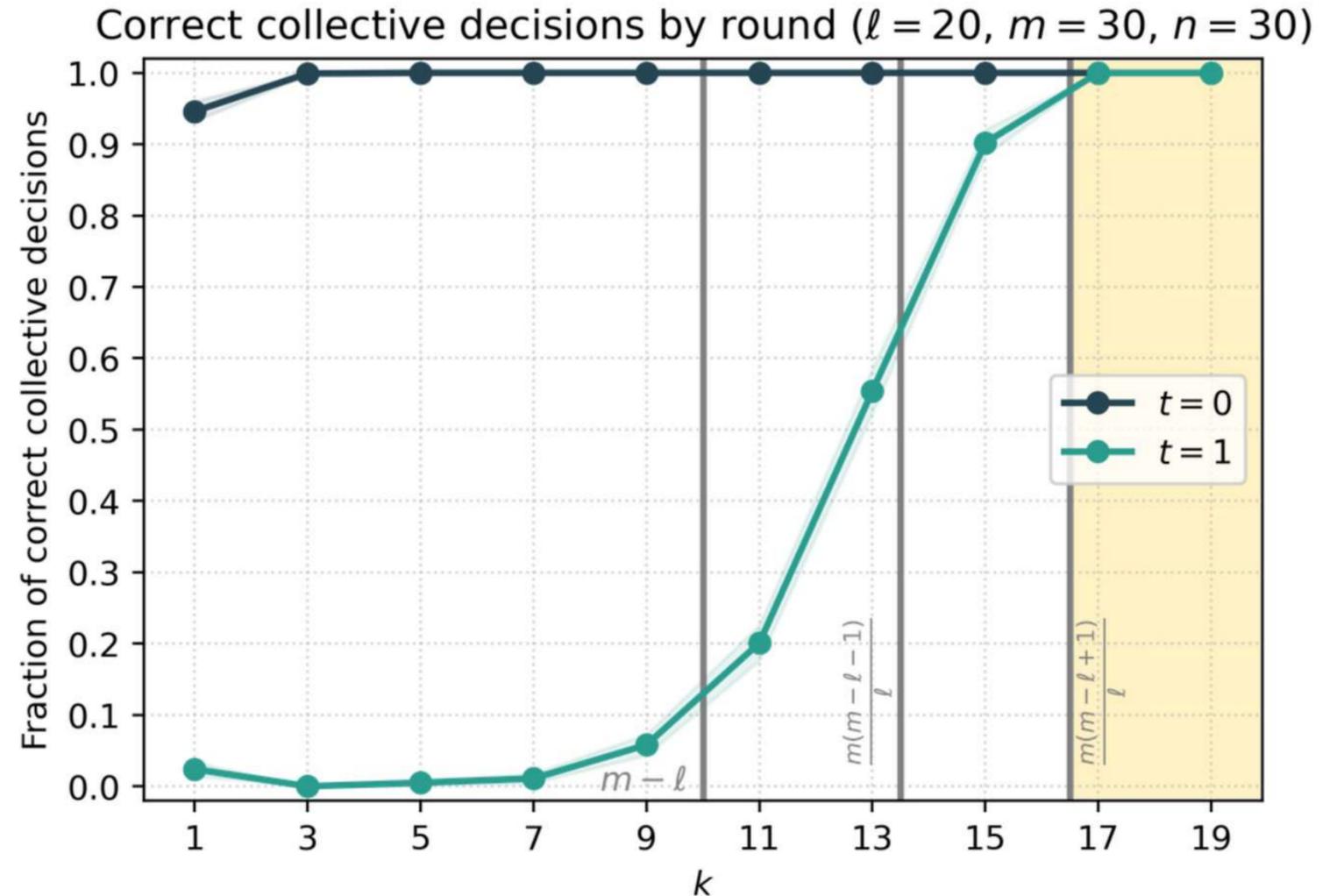
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(iii) if  $\frac{m(m-\ell+1)}{\ell} < k < 2(m - \ell)$ , then:

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and deliberation terminates with  $a$  as winner.  
// majority for  $a$  holds

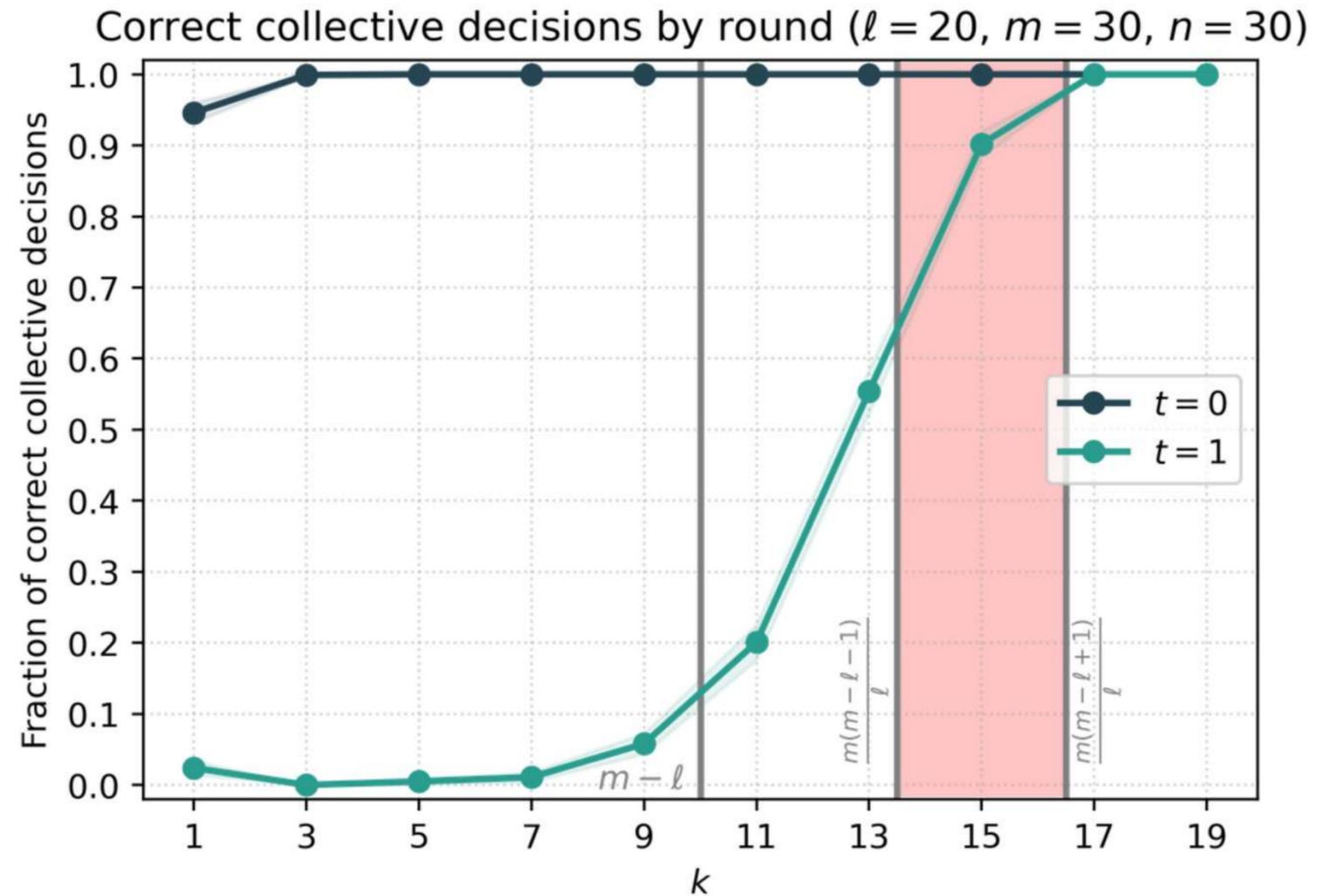


# ROUND 1: OUTCOME

For the intermediate range

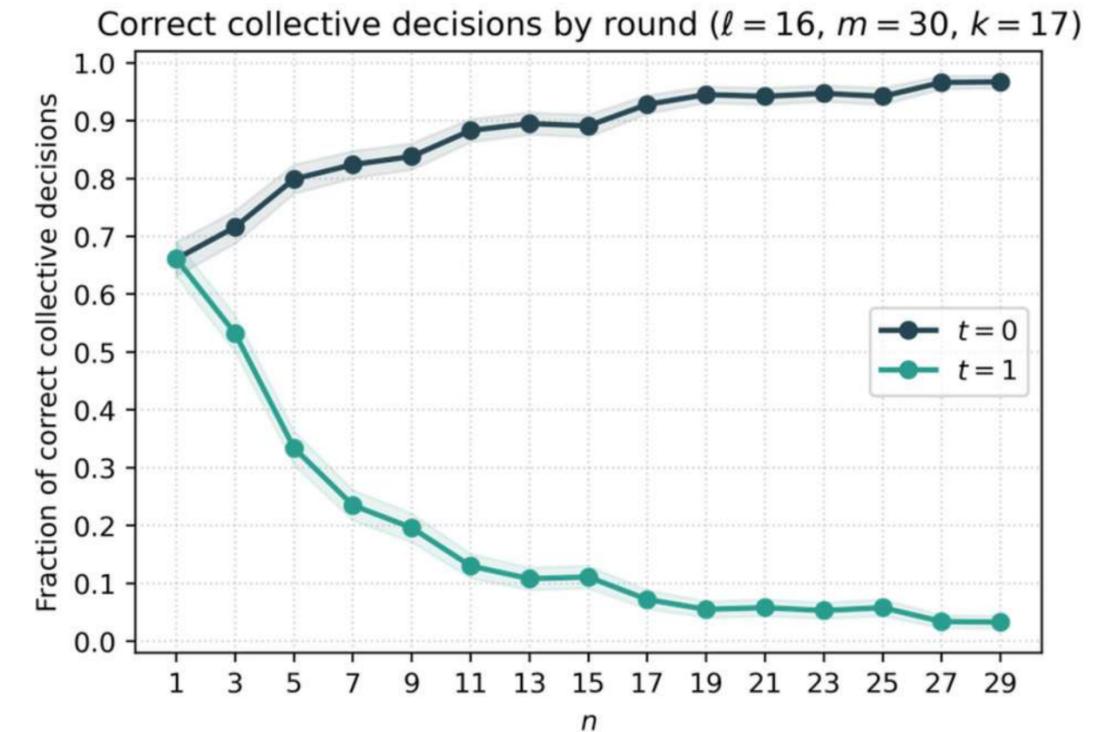
$$\frac{m(m-l-1)}{l} < k < \frac{m(m-l+1)}{l}$$

the outcome is less clear.



So unless  $k$  is large enough, the  $b$ -minority likely flips the vote to  $b$  at  $t = 1$ .

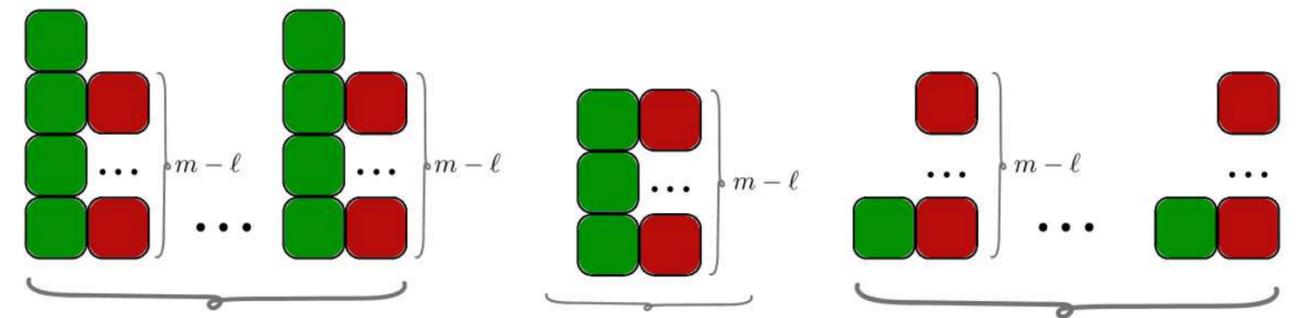
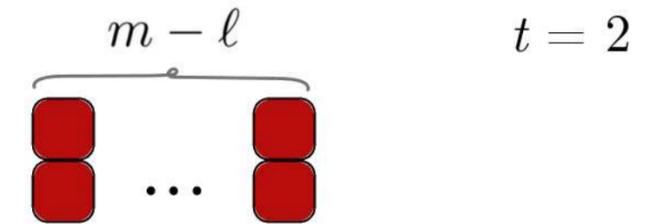
So unless  $k$  is large enough, the  $b$ -minority likely flips the vote to  $b$  at  $t = 1$ . Which explains what we saw in the earlier simulation.



# ROUND 2: SETUP

If  $m - \ell < k < \frac{m(m-\ell-1)}{\ell}$  we enter round 2 with a decision for  $b$ :

$$\begin{aligned}
 p_b &:= \Pr[v_i^1 = b] \\
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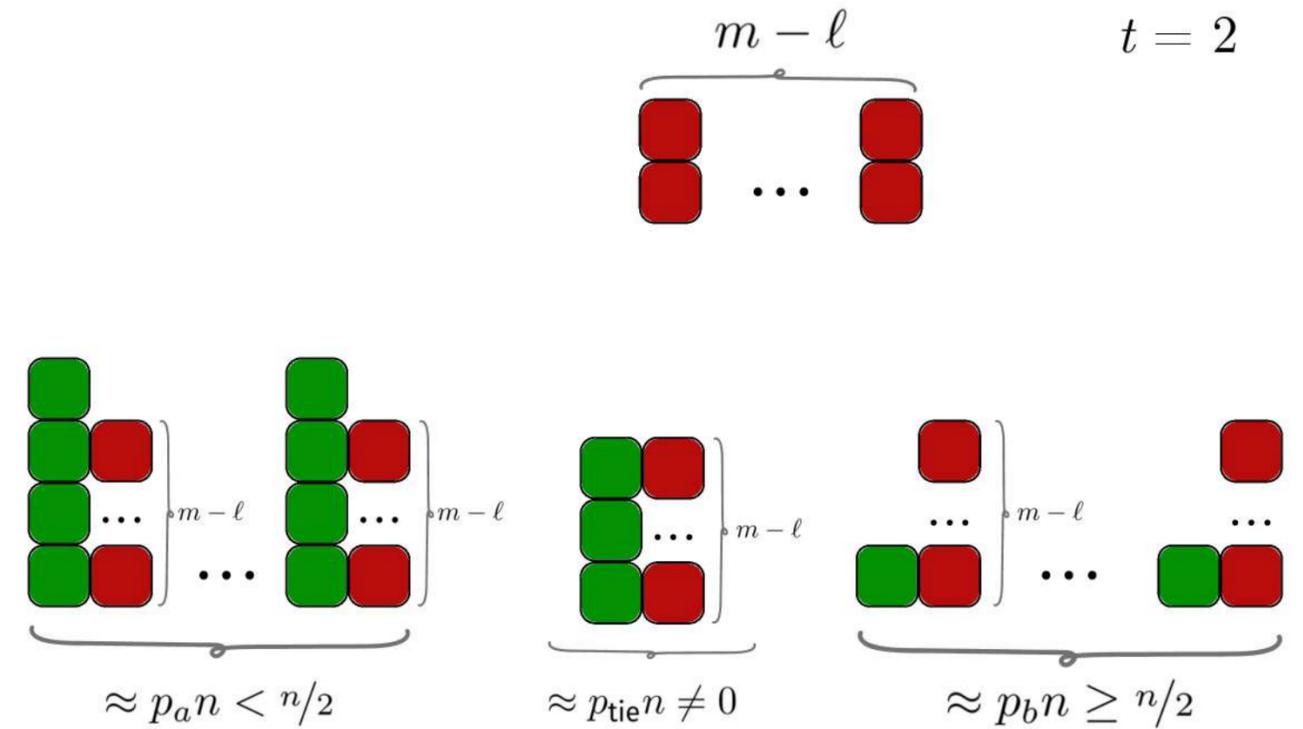
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⇓

#  $b$  voters  $\approx p_b n$ ,  
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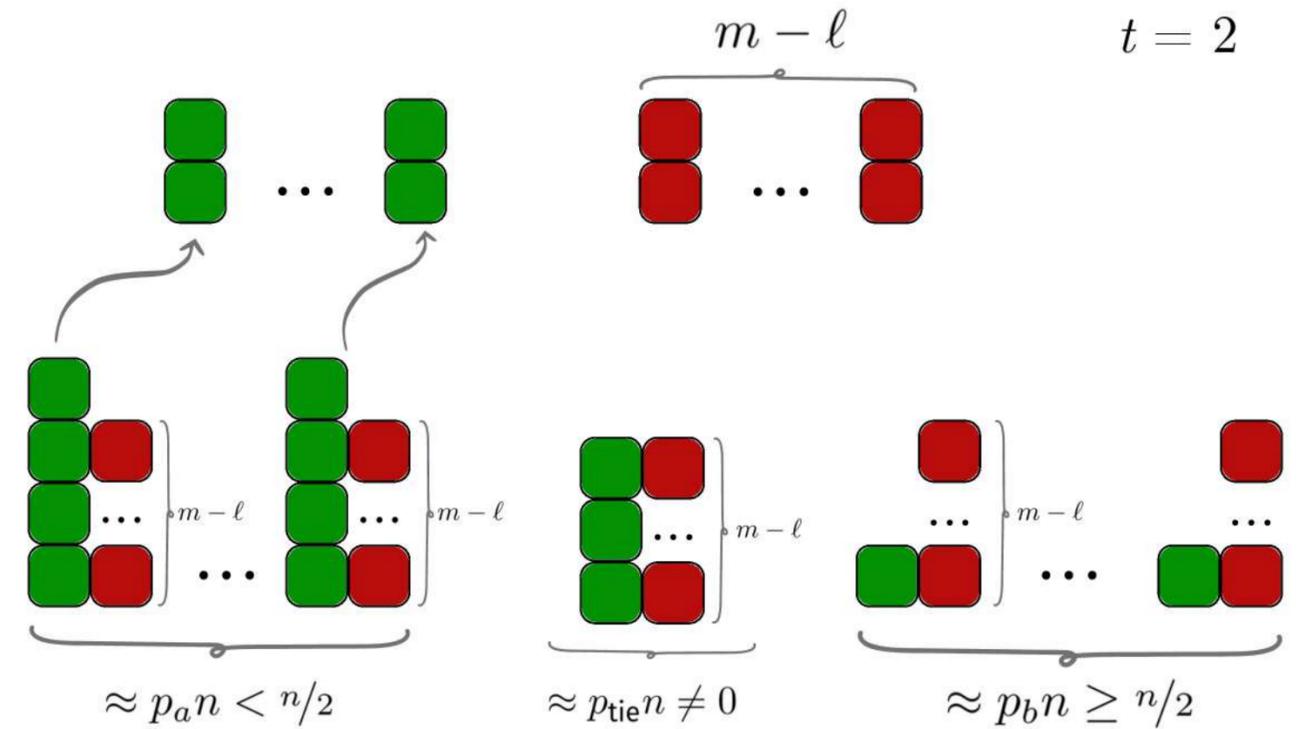
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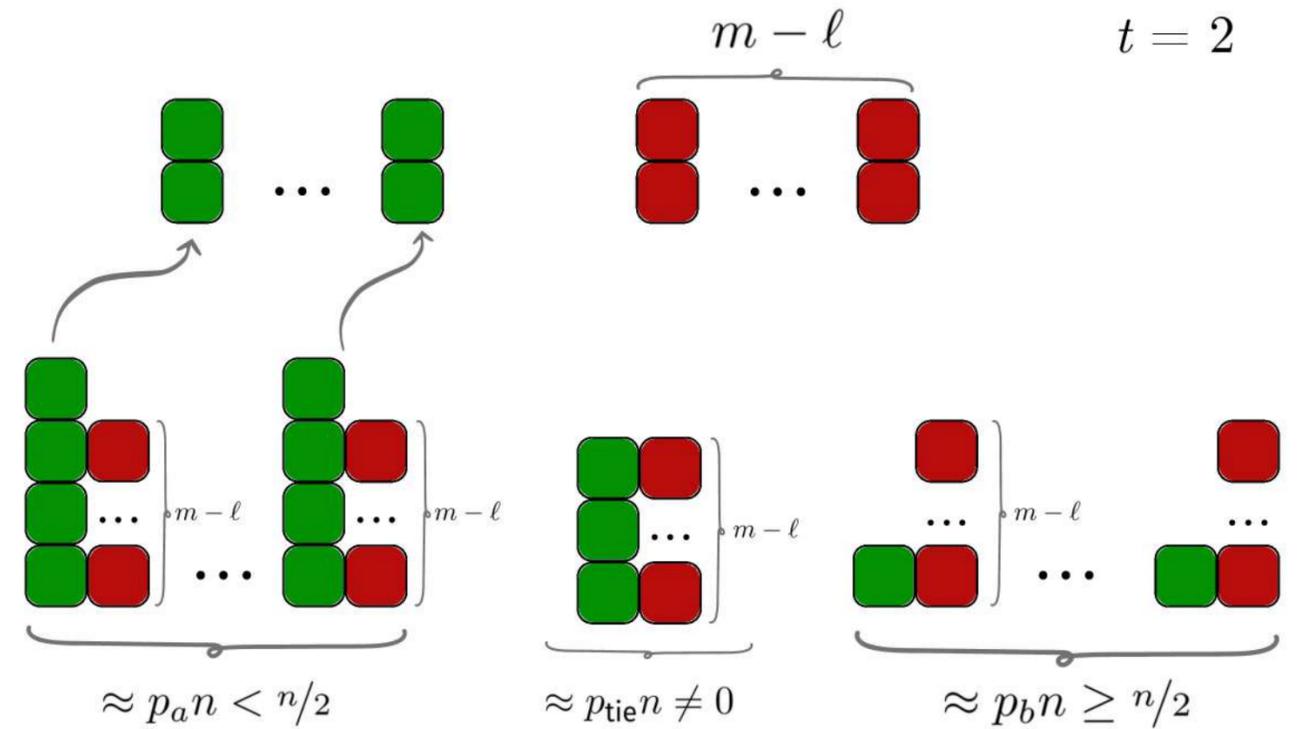
now the  $a$  voters are the dissenters



# ROUND 2: GENERAL RESULT

Take odd  $k$  and  $\ell/m > 1/2$ . As  $n \rightarrow \infty$ , it holds, for any agent  $i$ , that:

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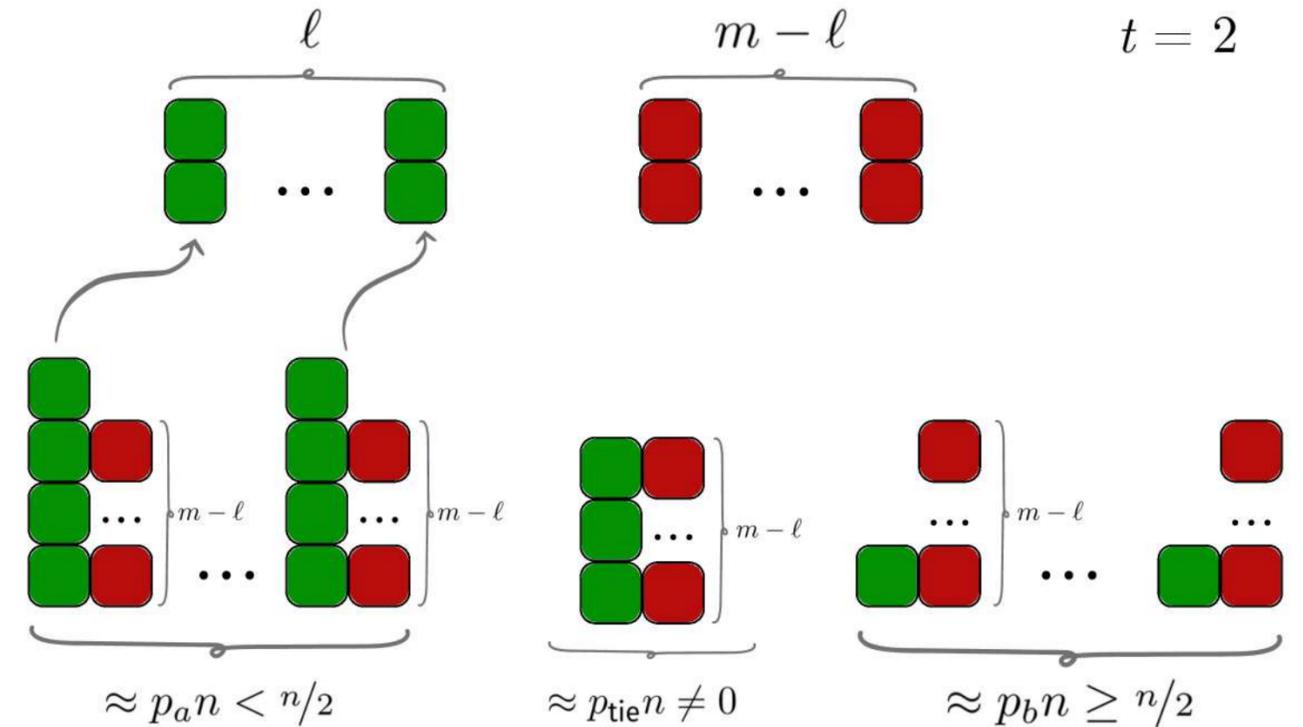


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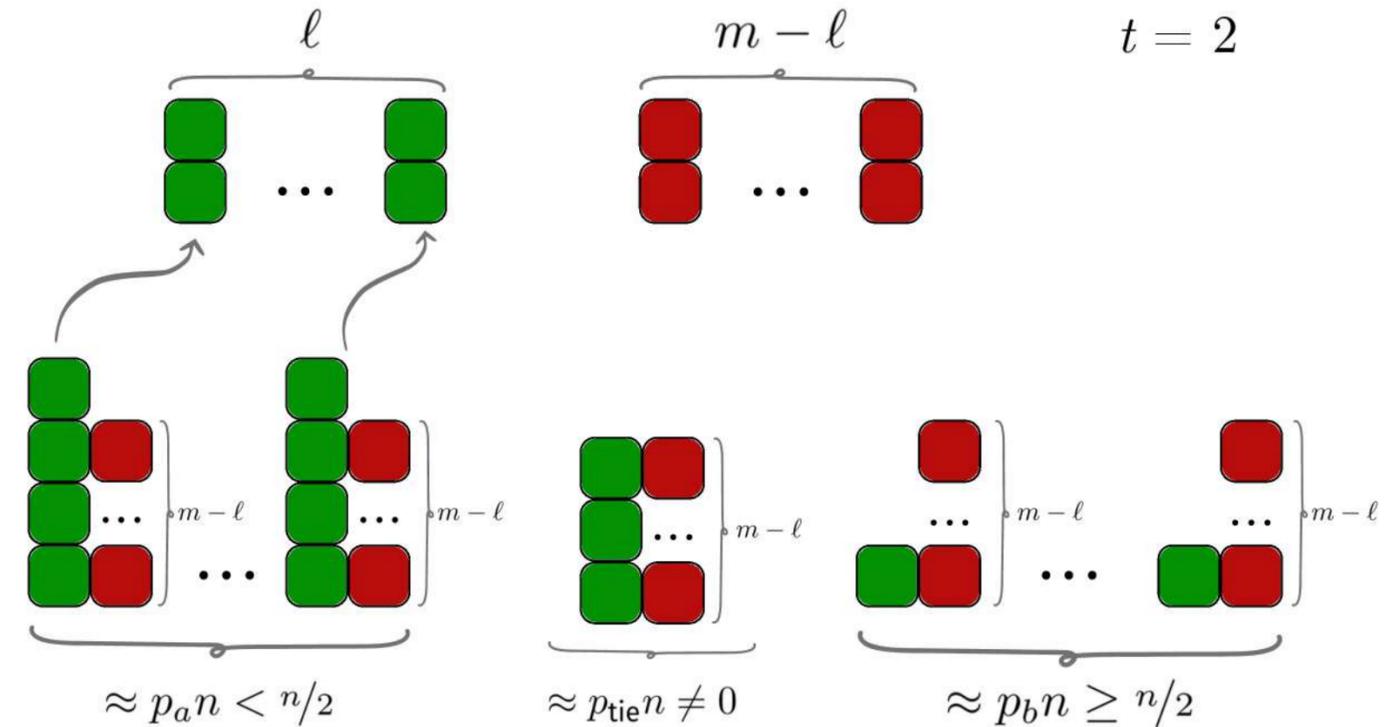


# ROUND 2: GENERAL RESULT

Take odd  $k$  and  $\ell/m > 1/2$ . As  $n \rightarrow \infty$ , it holds, for any agent  $i$ , that:

$$\text{if } m - \ell < k < \frac{m(m-\ell-1)}{\ell}, \text{ then } \Pr[v_i^2 = a] = 1.$$

Why? As  $n \rightarrow \infty$  it becomes very likely that the  $a$  minority holds all evidence for  $a$ . Once they disclose, everyone votes for  $a$ .



# ROUND 2: OUTCOME

For odd  $k$  and  $\ell/m > 1/2$ :

(ii) if  $m - \ell < k < \frac{m(m-\ell-1)}{\ell}$ , then:

$$\lim_{n \rightarrow \infty} \Pr[\text{Vote}^2(n) = a] = 1,$$

and deliberation terminates with  $a$  as unanimous winner.

// winner flips again

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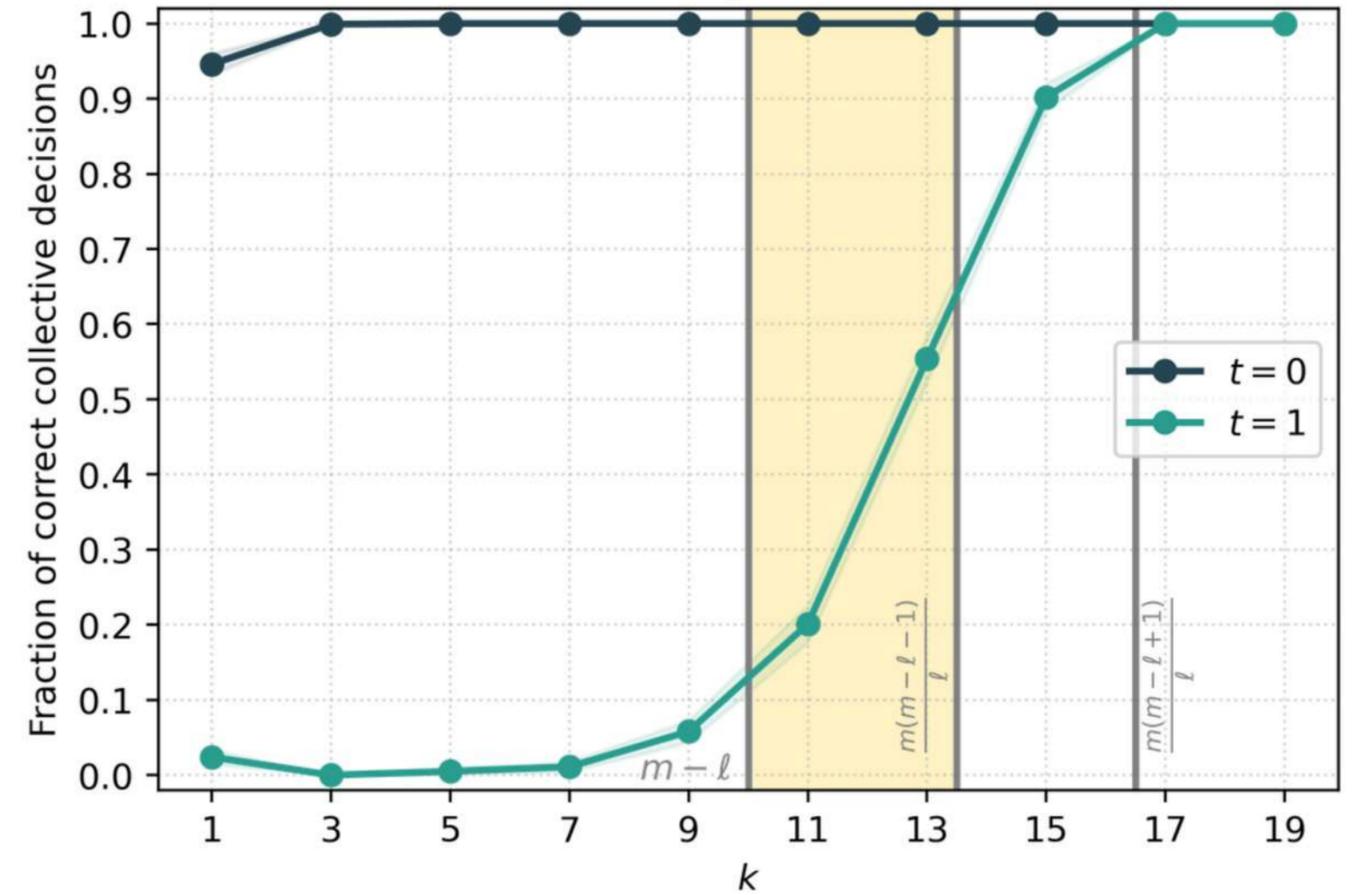
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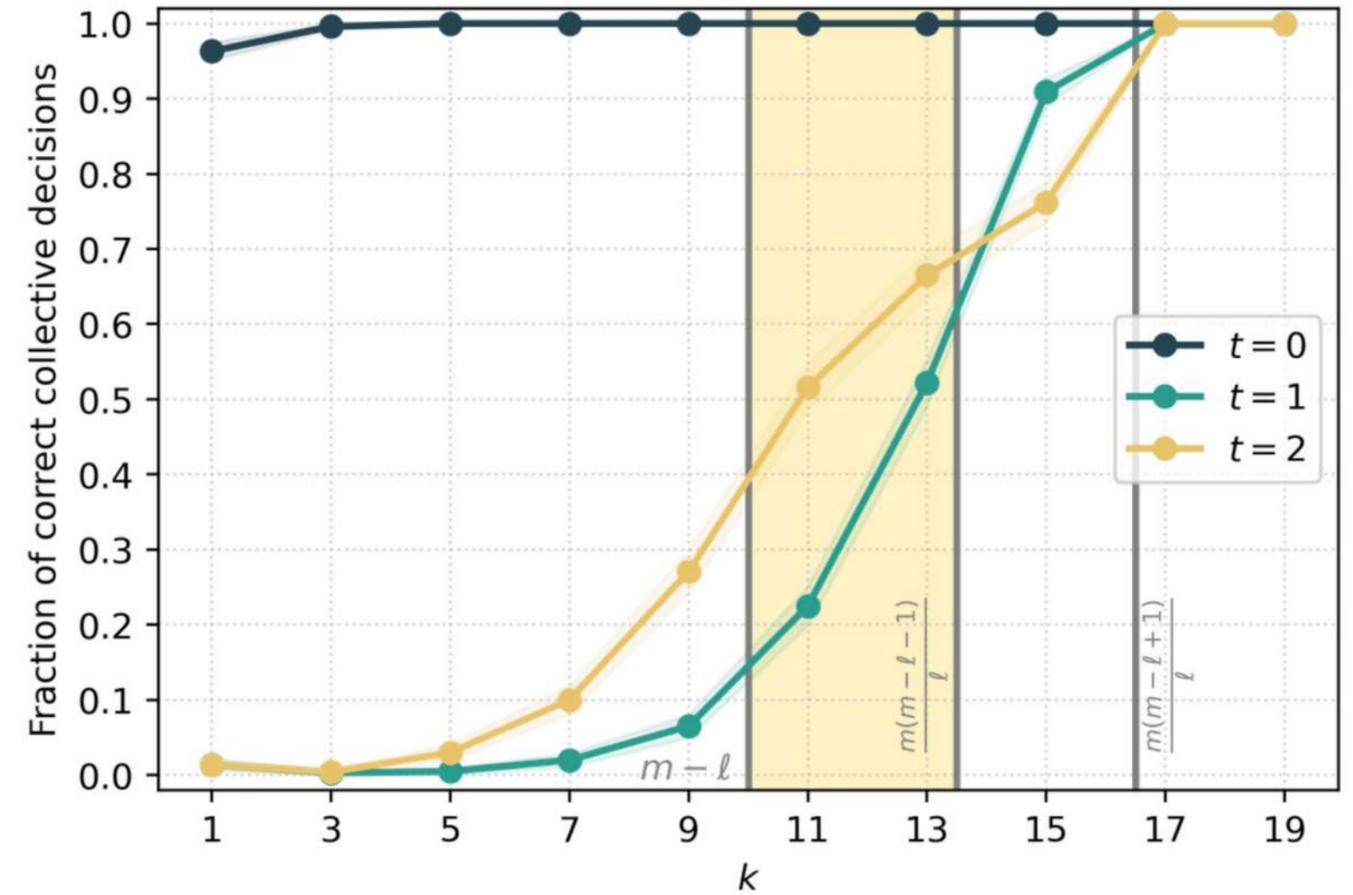
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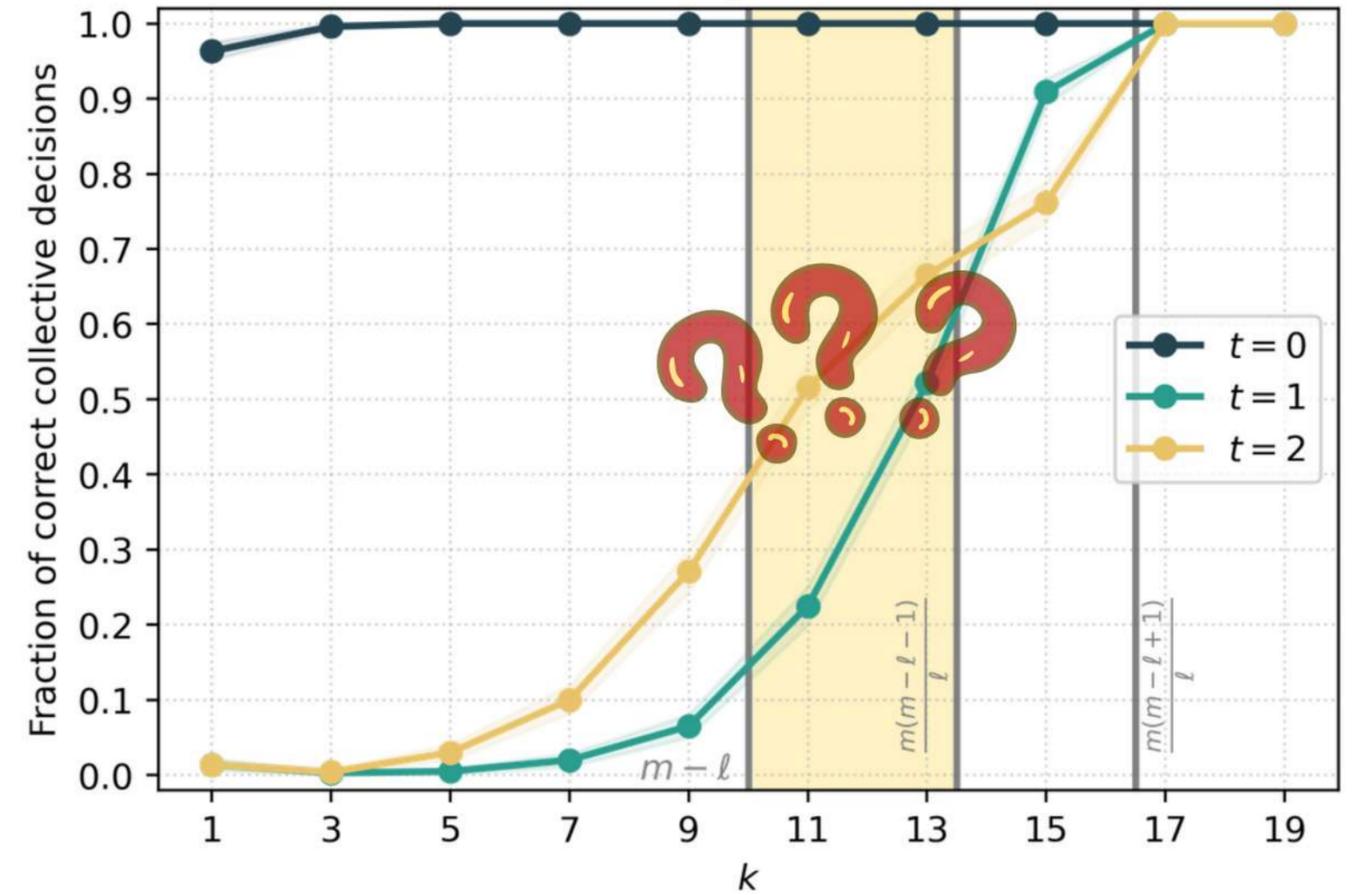
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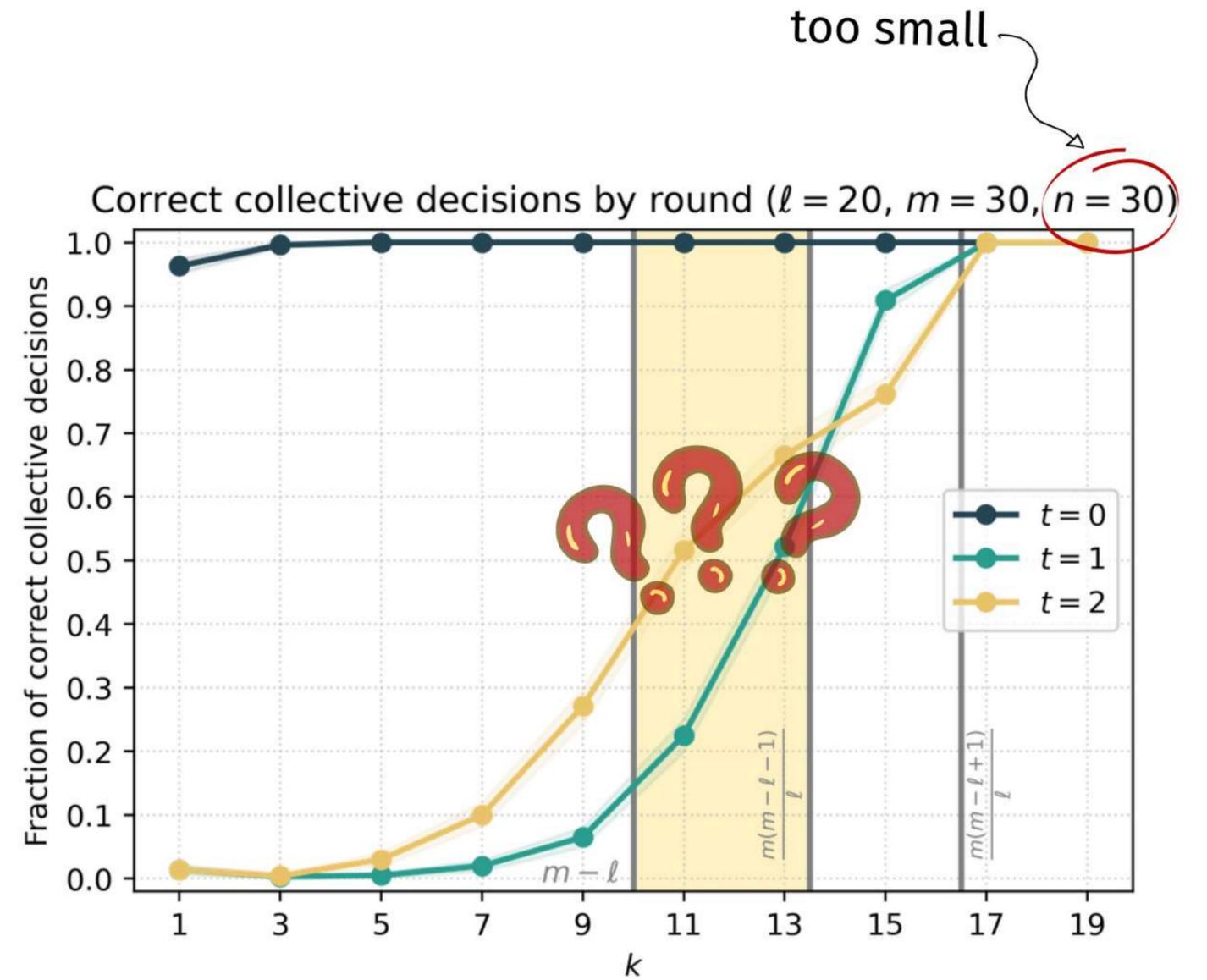
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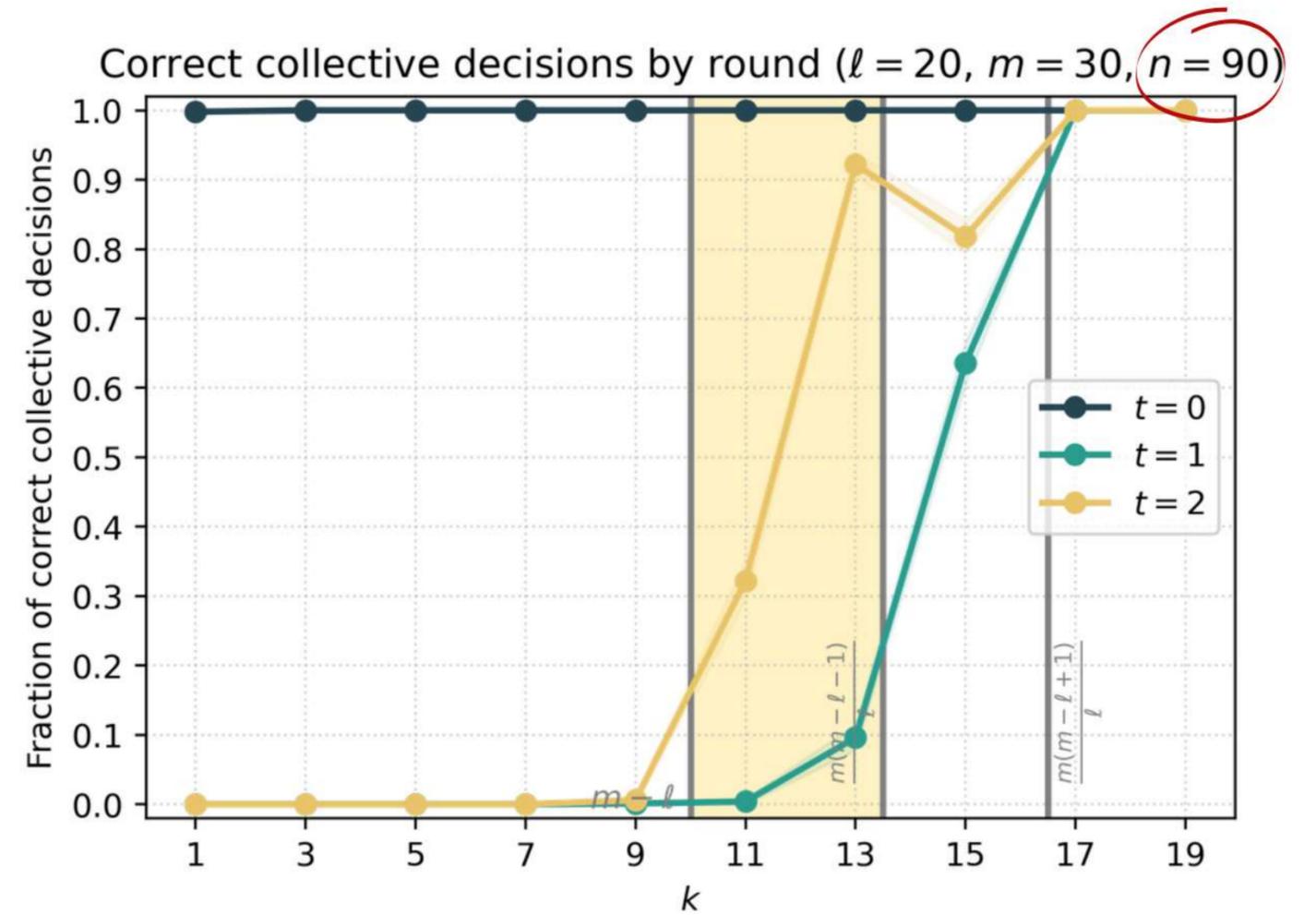
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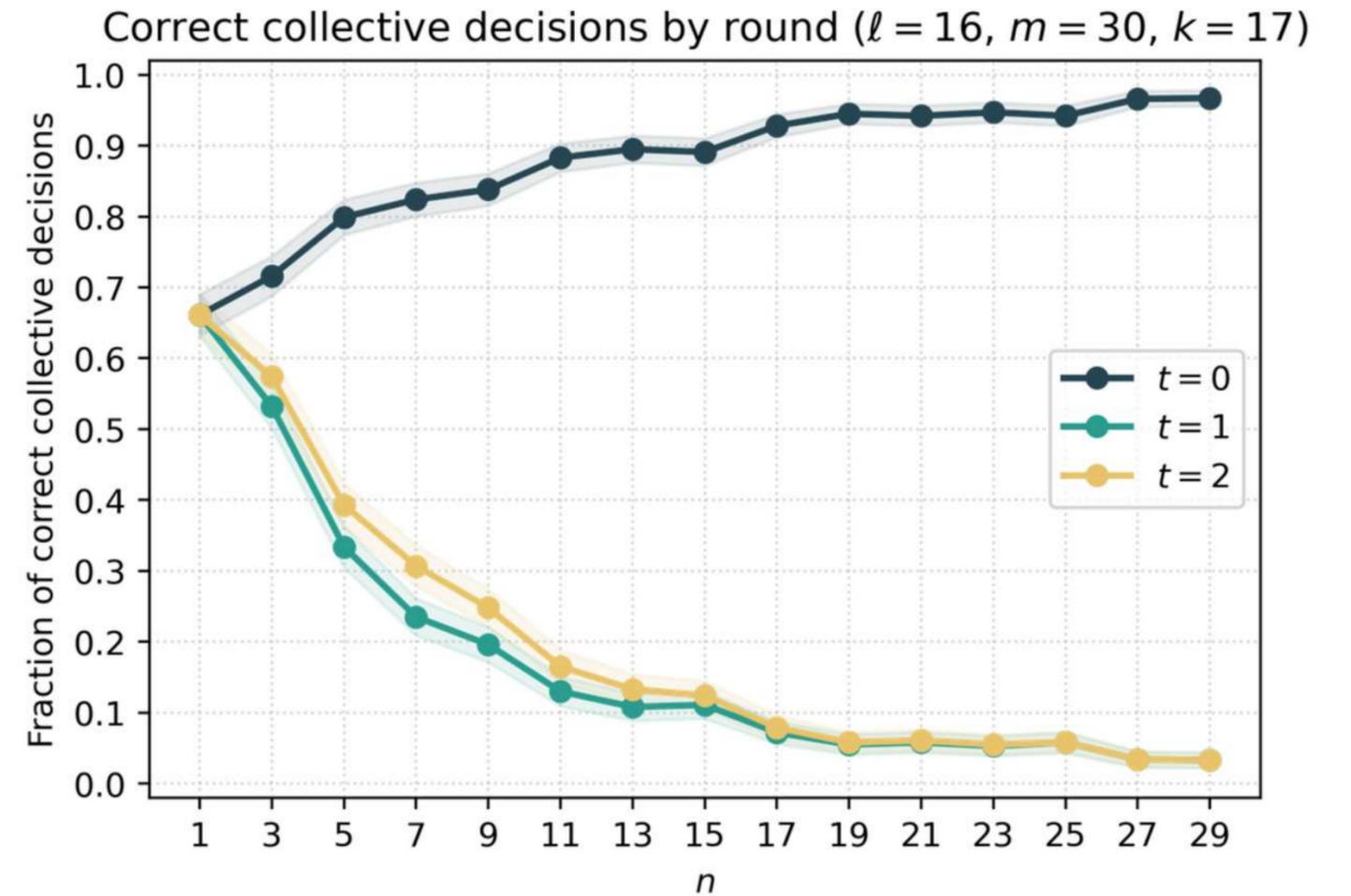
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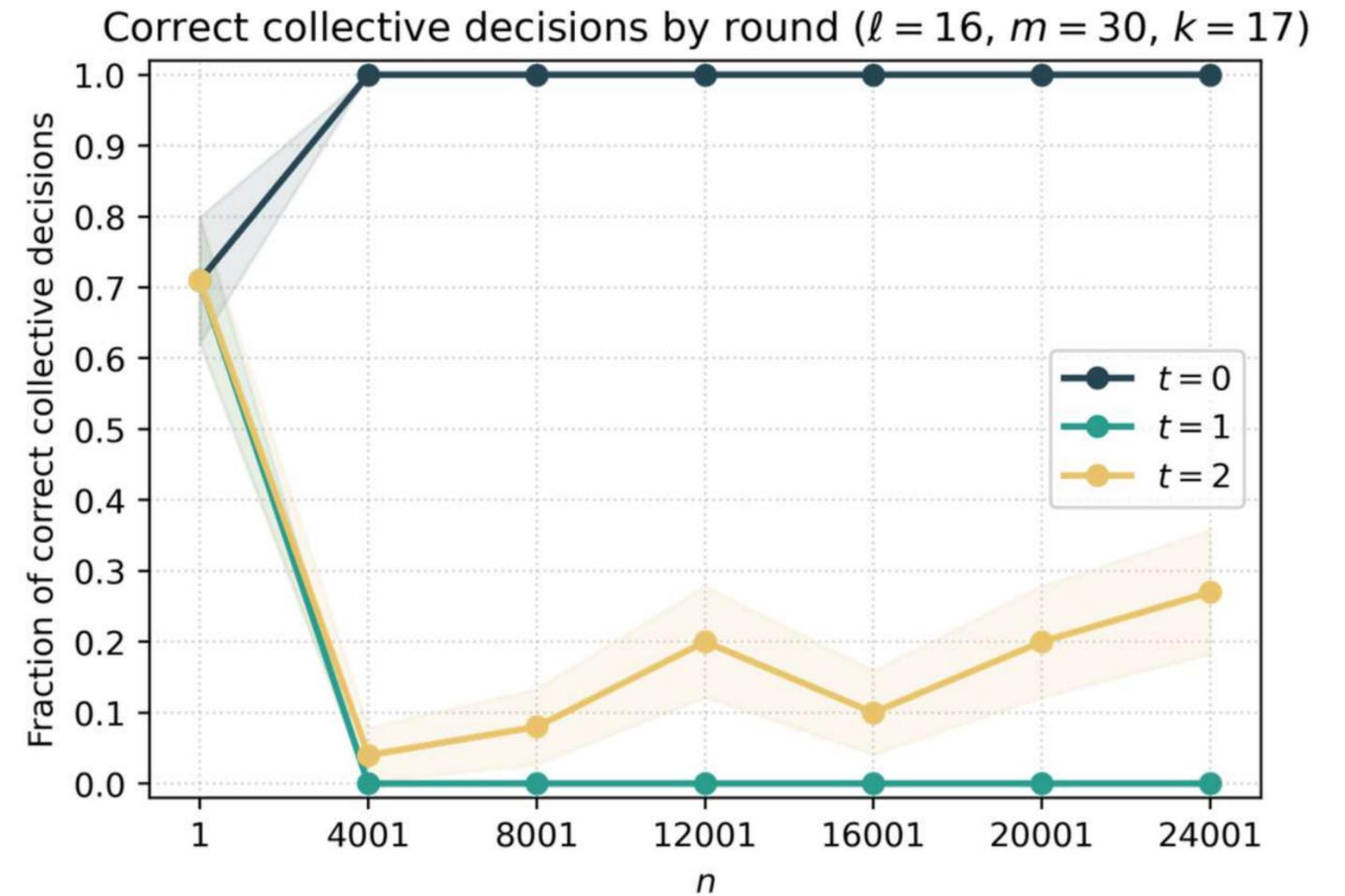
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# AND THE FIRST RESULTS?

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Again, the effect is visible only for large (very large!) groups.



# TODAY'S SECOND RESULT

## THEOREM

As  $n \rightarrow \infty$ , deliberation ends with high probability after at most two rounds, with sometimes  $a$  as winner, sometimes  $b$ , as shown before.

All in all, not super encouraging...



HÉLÈNE LANDEMORE

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ROSA

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ADRIAN

Something related to the distribution of evidence...



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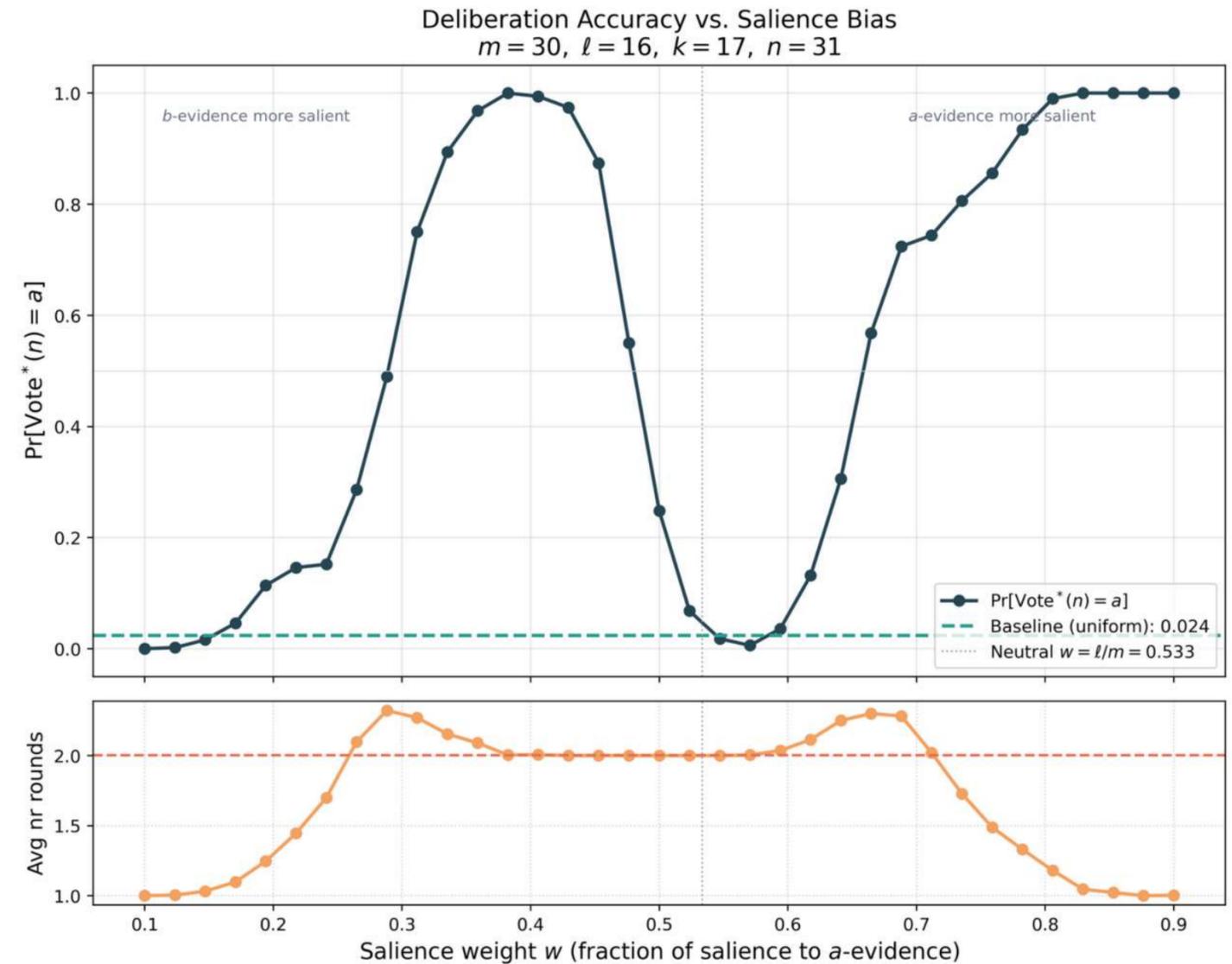
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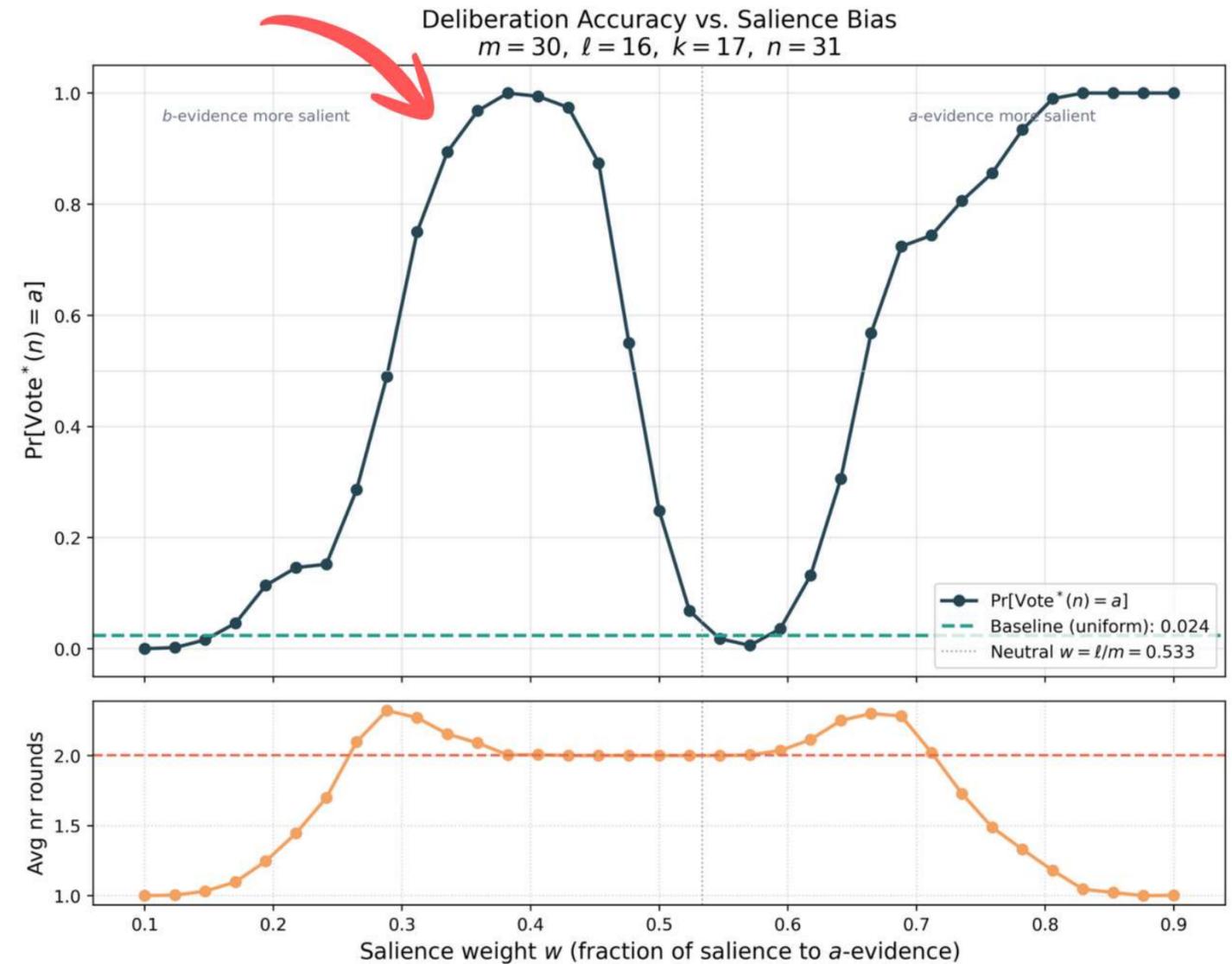
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In the second ( $0.7 < w < 0.9$ ):  $a$ -evidence more salient,  $a$  wins easily.

